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Effect of frictional sliding on the unilateral damaged behaviour for laminate composite

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Abstract

This paper presents a unilateral damage model coupled with an orthotropic elastic behaviour, to analyse the closure/opening effect on the behaviour of a representative elementary volume. Indeed, the oriented nature of the cracks along the fibres and the unilateral contact of the cracks lips (according to the opening or closure of cracks) lead to complex anisotropic behaviour. The anisotropic nature of the damage is reported through a scalar variable D acting on different ways according to the active stress components. The closure/opening effect of damage is recorded in two steps. The sign of the normal stress component to the fibre direction, defines the crack state (opened or closed). It acts firstly, on the damage kinetic and secondly on the softening of the stress components due to damage. If the crack is open, the damage effect is applied on all the stress components. If the crack is closed, the virgin elastic behaviour is recovered in the direction normal to the crack and a friction coefficient is then used to detect if the closed crack lips are stuck or able to slide, according to the relative absolute value of the normal and the shear stresses. If the two lips are stuck the behaviour is considered as virgin for shearing as well, otherwise, the shear behaviour is partially damaged. The damage can increase if the crack is either opened or closed and sliding or stuck. This analyse could easily be extended to a non linear anisotropic behaviour (including plasticity and visco-plasticity) through the concept of effective stress [1], [2]. Some numerical results are presented to illustrate the different situations.

Keywords : Closure/opening effect, anisotropic damage, composites,

1. Introduction

For several decades, many models have been proposed to describe composite behaviour, and especially the asymmetry in traction and compression loads [5,6,7,8]. A previous model of unilateral damage has been developed by the authors [3] which was able to report matrix micro-cracking taking account for the cracks states. When cracks were opened, a damaged behaviour was used, when cracks were closed and stuck the virgin behaviour was recovered and when cracks were closed and sliding, virgin behaviour was recovered in compression and damaged shearing behaviour was present. Actually, when cracks were closed with a slight negative transverse stress component and loaded with a large shearing component, no damage increased contrary to expected. The new version of the model is able to let damage increase even with closed cracks.

2. Unilateral damaged elastic behaviour

2.1. Damage variable definition

The damage model presented here is proposed for laminates made of layers of polymer matrix reinforced with long glass fibres. Using a meso-macro approach, the material behaviour is only needed at the level of one ply. It represents the degradation of the elastic properties due to the initiation and the growth of cracks in the matrix along the fibres direction (See fig.1).

Although the damage is anisotropic, it can be modelled by a single damage variable D differently acting, according to the cracks opening modes through a damage tensor $\underline{\underline{H}}(D)$ (See fig.2). This tensor increases some components of the virgin compliance tensor $\underline{\underline{S}}$ when damage is acting.

The compliance of the damaged elastic material is called $\underline{\underline{\hat{S}}}$ and the elastic strain is obtained as follows:

$$\underline{\varepsilon}^e = \underline{\underline{\hat{S}}} : \underline{\sigma} = \left(\underline{\underline{S}} + \underline{\underline{H}}\right) : \underline{\sigma} \tag{1}$$

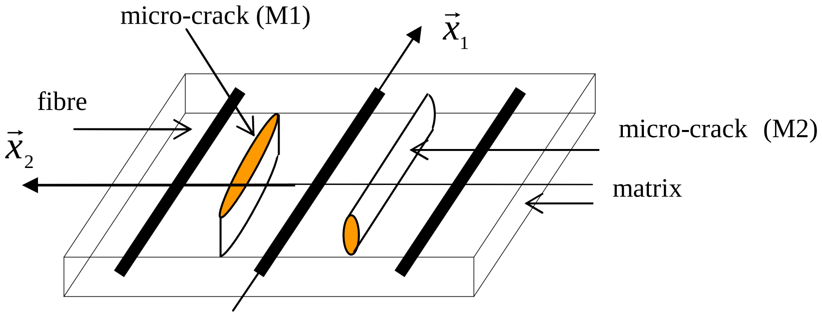
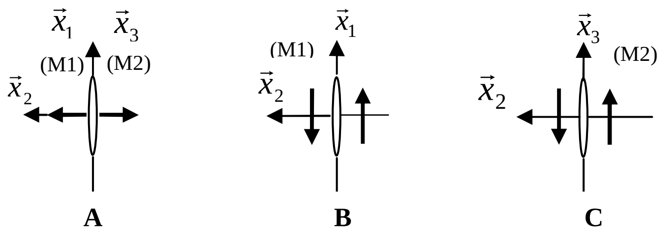


Figure 1 : Micro-cracks orientation in the matrix



$$\underline{\underline{H}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & H_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & H_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & H_{66} \end{bmatrix}$$

$$H_{22} = \frac{D}{1-D} S_{22} \qquad H_{44} = \frac{D}{\sqrt{1-D}} \sqrt{S_{11} S_{22}} \qquad H_{66} = \frac{D}{\sqrt{1-D}} S_{22}$$

Figure 2 : Micro-cracks opening modes and associated damage tensor

2.2. Damage kinetic with opened cracks

The thermodynamics of irreversible processes associated to the local state method is used to describe the damage evolution. Choosing the specific free energy as follow:

$$\psi\left(\underline{\varepsilon}^e, D\right) = \frac{1}{2\rho}\left(\underline{\varepsilon}^e : \underline{\underline{\hat{S}}}^{-1}(D) : \underline{\varepsilon}^e\right) \tag{2}$$

the Clausius-Duhem inequality gives the state law and the driving force of damage:

$$\underline{\sigma} = \rho \frac{\partial \psi}{\partial \underline{\varepsilon}^e} = \underline{\underline{\hat{S}}}^{-1} : \underline{\varepsilon}^e \qquad \text{and} \qquad Y = \frac{1}{2} {}^{T}\underline{\sigma}\, \underline{\underline{H}}' \underline{\sigma} \quad \text{with} \quad \underline{\underline{H}}' = \frac{\partial}{\partial D} \underline{\underline{H}}(D) \tag{3}$$

According to these definitions, only $\sigma_{22}, \sigma_{12}, \sigma_{23}$ are able to activate damage increase through $\underline{\underline{H}}'$.

This formulation corresponds to the one developed by Boubakar et al. in [1]. The evolution law of D is classically obtained from the principle of maximal dissipation. A yield damage function is chosen in the following form:

$$f_D = Y - (Y_c + qD^p) \tag{4}$$

Y_c, q, p being material constants.

2.3 Cracks closure

As soon as the cracks close ($\sigma_{22} = 0$), the behaviour is modified by the presence of the cracks which are closed but whose lips are still able to slide. A new damaged compliance $\underline{\underline{\tilde{S}}}^*$ is defined from the virgin compliance modifying only the terms possibly affected by damage: $\underline{\varepsilon}^e = \underline{\underline{\tilde{S}}}^* \underline{\sigma}$ where

$$\underset{\approx}{\tilde{S}}^{*}=\begin{bmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ S_{21} & S_{22}^{*} & S_{23} & 0 & 0 & 0 \\ S_{21} & S_{23} & S_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44}^{*} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44} & 0 \\ 0 & 0 & 0 & 0 & & S_{66}^{*} \end{bmatrix} \quad (5)$$

To insure the strain continuity at the closure, the condition $\left(\underset{\approx}{\tilde{S}}-\underset{\approx}{\tilde{S}}^{*}\right)\underline{\sigma}=\underline{0}$ should be imposed. Then, $S_{22}^{*}=S_{22}$, $S_{44}^{*}=S_{44}+H_{44(Dc)}$ and $S_{66}^{*}=S_{66}+H_{66(Dc)}$ where D_c is the damage value at the closure. The new compliance can be written in the following way $S_{44}^{*}=\left(S_{44}+\zeta\left\langle f_{12}^{s}\right\rangle H_{44(Dc)}\right)+\left(1-\zeta\left\langle f_{12}^{s}\right\rangle\right)H_{44(Dc)}$ (and idem for S_{66}^{*} with the sliding function f_{23}^{s}). D_c is the damage value at the cracks closure. The second part of S_{44}^{*} will give a blocked strain due to the cracks closure. The first part will let the compliance to be either the virgin one or a partly damaged one according to the sign of the sliding function f_{12}^{s}. The new coefficient and the sliding function will be discussed further.

2.4. Damage kinetic with closed cracks

When cracks are closed $(\sigma_{22}<0)$, the virgin elastic behaviour is recovered in compression thanks to S_{22} which is kept equal to the value of the virgin material all along the closure of the cracks. At the same time, the shear behaviour is supposed to depend on the contact between the cracks lips. If the compression stress is much more important than the shearing one, cracks are assumed to be stuck and the virgin shearing behaviour should then be recovered through the nullity of the sliding function f_{ij}^{s}. If compression is much weaker than shearing, cracks lips are assumed to be sliding one with respect to the other, leading to a more important strain than the previous case. The virgin compliance is then increased through ζ and f_{ij}^{s}. In order to record as closely as possible the sliding of the cracks lips, the sliding function is defined as follows:

$$f_{ij}^{s}=\left|\sigma_{ij}\right|-f\left|\sigma_{22}\right| \quad (6)$$

where *f* is a friction coefficient.
The strain increment is now split in two parts:

$$\underline{\varepsilon}=\underline{\varepsilon}^{e}+\underline{\gamma}^{0} \quad (7)$$

The first term is able to report the virgin/partly damaged behaviour in case of stuck/sliding cracks; the second is constant and obtained at the closure of the cracks.
When σ_{22} is negative, the free specific energy is chosen as follows:

$$\psi\left(\underline{\varepsilon}^{e},D\right)=\frac{1}{2\rho}\left(\underline{\varepsilon}^{e}:\left(\underset{\approx}{S}+\zeta\underset{\approx}{H}^{*}(D)\right)^{-1}:\underline{\varepsilon}^{e}\right) \quad (8)$$

where ζ is a parameter which deals with the effect of damage on closed sliding cracks. f_{ij}^{s} is a criterion function to detect whether cracks will be able to slide when they are closed. The damage tensor will act or not according of to the sign of the criterion.

$$\underset{\approx}{H}^{*}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \left\langle f_{12}^{s}\right\rangle H_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \left\langle f_{23}^{s}\right\rangle H_{66} \end{bmatrix} \quad \text{where} \quad \left\langle f_{ij}^{s}\right\rangle=\begin{cases}1 & if \quad f_{ij}^{s}\geq 0\\ 0 & if \quad f_{ij}^{s}<0\end{cases} \quad (9)$$

The state law, the damage force and the intrinsic dissipation becomes now:

$$\underline{\sigma} = \rho \frac{\partial \psi}{\partial \underline{\varepsilon}^e} = \left(\underline{\underline{S}} + \zeta \underline{\underline{H}}^*\right)^{-1} : \underline{\varepsilon}^e \qquad \text{with} \qquad Y = \frac{1}{2} \zeta \, \underline{\sigma} : \underline{\underline{H}}^{*\prime} : \underline{\sigma} \tag{10}$$

The damage dissipation is considered in the same way than opened cracks. The ζ coefficient must be positive but smaller than one to record an increase of compliance if cracks slide but not more than when cracks are opened. The damage driving force will be smaller than with opened cracks and as a result, damage increase will be harder with closed sliding cracks. The virgin shear behaviour is recovered as soon as the sliding function is negative, what means when cracks are stuck. When they can slide, a partly damaged behaviour is obtained through the ζ coefficient. The important point here is that damage can increase even with closed cracks if the damage force becomes high enough.

In order to insure the strain continuity when cracks slide, $\left(S_{44} + \zeta H^*_{44}\right)\sigma_{12} = S_{44}\sigma_{12}$ must be verify when cracks start to slide or to stick. This explains the presence of the sliding function in the H^* tensor.

2.5 Cracks opening

As soon as the cracks open ($\sigma_{22} \geq 0$), the damaged behaviour is recovered

$$\begin{aligned} \varepsilon_{12}^{opening} &= \left(S_{44} + H_{44(Dop)}\right)\sigma_{12}^{opening} \\ \varepsilon_{23}^{opening} &= \left(S_{66} + H_{66(Dop)}\right)\sigma_{23}^{opening} \end{aligned} \tag{11}$$

Dop is the damage value at cracks opening. This could result in a jump of strain due to the blocked part of the strain which is subtly released by the cracks opening.

3. Numerical results

Different simulations will be presented here in order to show the ability of the model to easily describe different behaviours of a representative volume element of plate according to the cracks states. The results obtained with only tensile-compression stress component in [3] are not affected by this new formulation and will not be related here. The only shearing stress is σ_{12} . The transverse normal stress σ_{22} is chosen negative to close the cracks and weak enough to let them slide. Three criteria have to be taking account for to develop the model. They are written in terms of stress, so as soon as the stress state is known the behaviour is identified. The loading is applied step by step and the strain increment is deduced from the behaviour laws corresponding to the current damage and cracks state.

The numerical results presented here have been obtained with the following fictive material characteristics:

E_1=45680 MPa; E_2=16470 MPa; G_{12}=76920 MPa, ν_{12} =0.3, ν_{23} =0.3;

Y_C =0.0043MPa; q=1.37; p=0.96

The material coefficients related to the sliding strains are studied below.

3.1 Effect of the sliding coefficient ζ

To point out the possibilities of the current model, shearing tests are presented here with alternatively closed and opened cracks. The compressive stress value is small enough to let the cracks slide when they are closed. Two kinds of test are carried out: a) shear loading is applied with opened cracks and unloading is performed with closed cracks; b) shear loading is applied with close cracks and unloading with opened cracks.

Two different values ($\zeta = 0.15$ and 0.75) have been chosen for the sliding coefficient. If ζ was equal to 0, the material would recover virgin shear behaviour as soon as cracks would be closed because no damage would be taken into account in the stress-strain law, and no stress would appear in the damage driving force. If ζ was equal to 1, the material would damage in the same way in shearing than with opened cracks as soon as cracks slide.

The effect of this coefficient is very sensitive to the value of damage.

During the first unloading of figure 3.a), no difference is visible between the two curves, but during the two last unloading when damage is important, the slope of the plain curve remains high because material is not supposed to be altered a lot by damage with a small value of ζ when cracks are closed, which is different with high value of ζ where the decreasing slope is closed to the one with opened cracks.

In figure 3.b) shear loading is applied with closed cracks which are then opened at different value of the shearing stress. The strain jump records the quick release of the blocked strain and the damage increase as soon as compressive stress disappears. At the cracks opening, both curves reach the curve corresponding to opened cracks. When ζ is small, the damage driving force can not be high enough to activate damage increase during the loading and as a result, the plain slope is straight up to the opening. On the other hand, when ζ is high enough, it could even make the damage increase as we can see around 120MPa. The unloading curve is common to the two curves

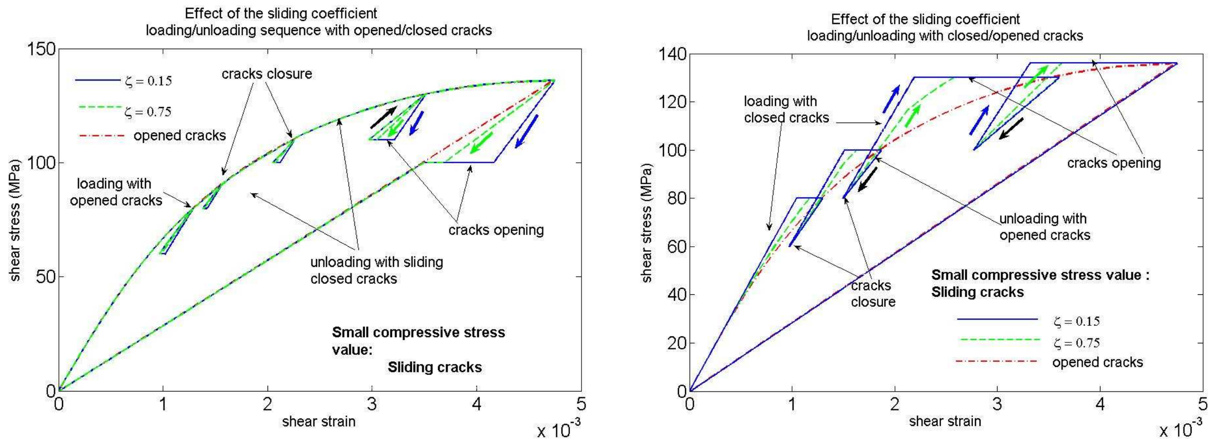


Figure 3 : Shearing stress-strain curve. a) loading with opened cracks, b) loading with closed cracks

The figure 4 shows how damage is affected by the sliding coefficient. The damage increase with closed cracks can be seen especially for the high value of ζ . The jump of damage can also be pointed out on this figure.

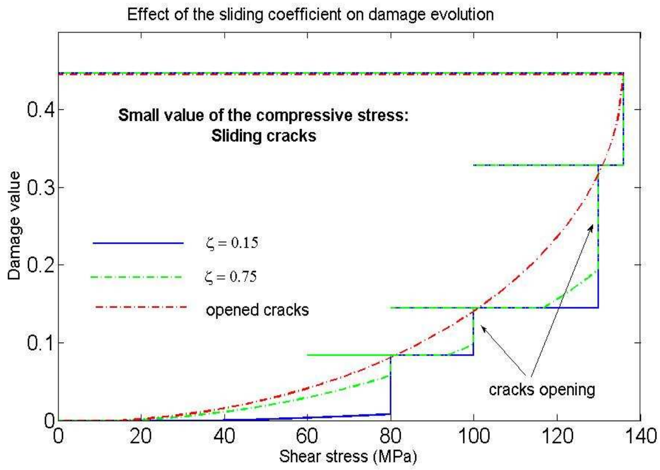


Figure 4 : Damage evolution according to the cracks state

3.2 Effect of the friction coefficient *f*

This coefficient let the sliding criterion be more (or not) easily reached to let the cracks slide. If it is high, it can delay the cracks sliding and prevent damage from increasing ($f = 0.7$ for instance in figure5). As soon as cracks start sliding, a slight (*f=0.1*) or high (*f=0.4*) jump is observed on the stress/strain curve corresponding to a jump in damage. When the friction coefficient is high (over 0.4), damage brusquely increases when the sliding function becomes positive because the damage force is much larger than the damage criterion and results in a jump of damage. This jump induces a loss of stiffness and results in a jump of shearing strain. Compared to the value of damage reached with opened cracks, the curve of damage with the sliding cracks is always below. The lower value of ζ , the lower the curve is. As far as unloading is concerned, cracks are closed before unload. For *f=0.1*, cracks are sliding down to *60 MPa* following a partly damaged slope. For medium value of *f*, *(f=0.4)* cracks are first sliding (down to *120 MPa*) and become stuck when the shearing stress goes on decreasing. When cracks are stuck, the virgin behaviour is recovered. At σ_{12} reaches *60MPa*, cracks are opened and the damaged behaviour is observed.

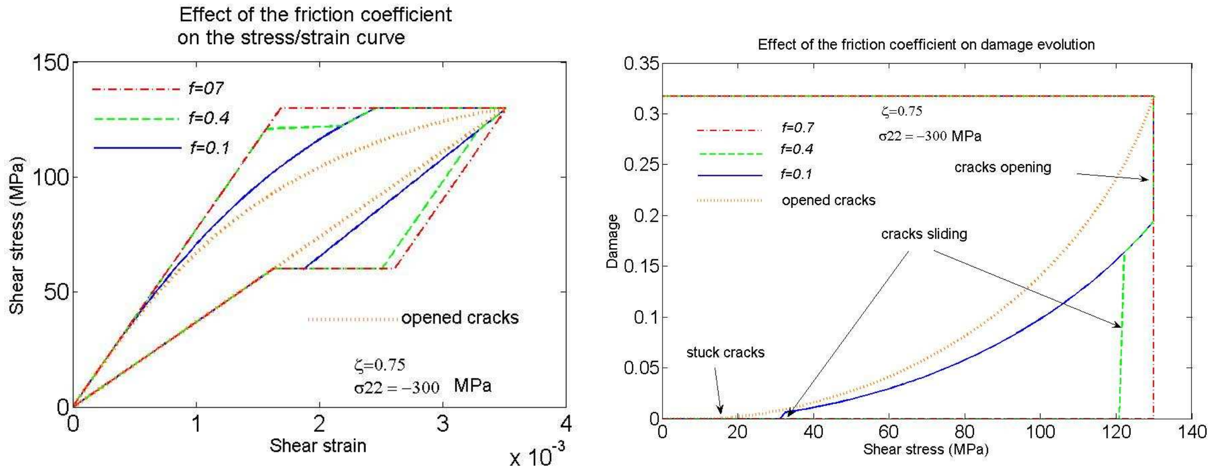


Figure 5 : Stress/strain curve (a) and damage evolution (b) according to the friction coefficient

3.3 Strain driven numerical integration

As soon as the behaviour has to be implemented in a finite element code, the numerical integration is strain driven[4]. In this model, three criteria written in terms of stress have to be used. The point is: knowing the stress state $\underline{\sigma}_n$, damage value D_n at the end of the previous step and the new strain increment $\Delta\underline{\varepsilon}$, find the stress increment $\Delta\underline{\sigma}$ which let the three criteria be verified. This results in a serial of predictor-corrector schemes to evaluate the stress increment related to the given strain increment.

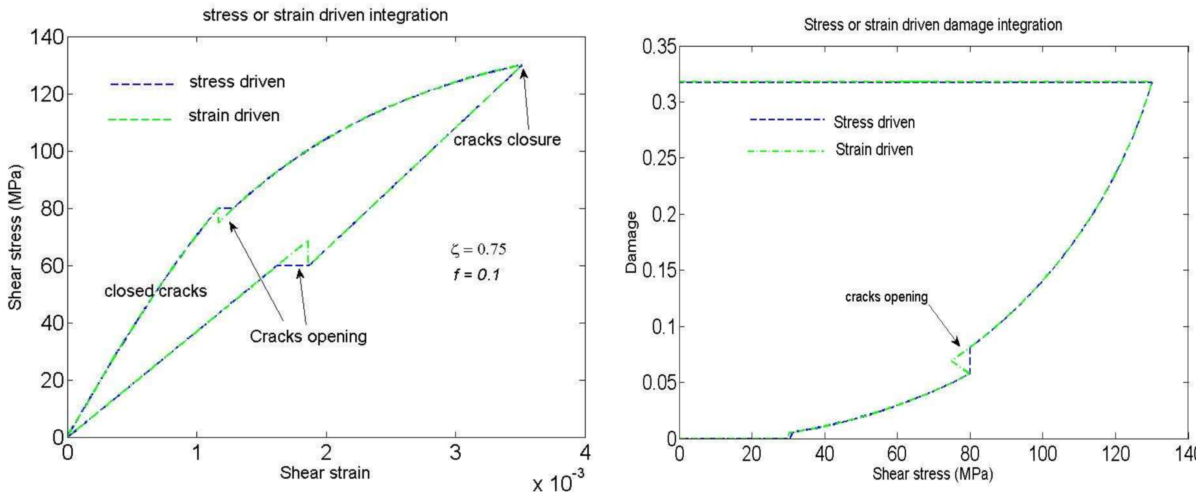


Figure 6 : Stress/strain curve(a) and damage evolution (b) according to integration scheme

➔ If cracks were opened at the end of the previous increment, we predict they will remain open

⇨ A new prediction is made with no damage increase

If damage criteria is verified $\left(f_d \leq 0\right)$, the stress increment is kept, ***otherwise,*** damage increment is computed (with the damage driving force $Y = \frac{1}{2}\underline{\sigma} : \underline{\underline{H}}^{*}{}' : \underline{\sigma}$) to check the damage criterion and a new stress increment is calculated

If the new stress state verifies the cracks state criterion $\left(\sigma_{22} \geq 0\right)$, the stress increment is kept, ***otherwise***, cracks have been closed during the step. As a result, the step is split in two parts to treat the first part as above (opened cracks behaviour) and the second part as below (closed cracks behaviour)

➔ If cracks were closed, we predict they will remain closed and stuck

⇨ ***If*** the sliding criterion is verified $\left(f^{s} < 0\right)$, the stress increment is kept, ***otherwise***, the sliding behaviour is used. In this case, the damage yield function is tested (with the damage force $Y = \frac{1}{2}\zeta \, \underline{\sigma} : \underline{\underline{H}}^{*\prime} : \underline{\sigma}$) and, according to the criterion, a damage increment is computed if necessary and a new stress increment is obtained.

If the new computed stress state verifies the cracks state criterion $(\sigma_{22} < 0)$, the stress increment is kept, ***otherwise***, cracks have been opened during the step. As a result, the step is split in two parts to treat the first part as above (closed cracks) and the second part as opened cracks. Just before computing the second part, the new shear stress is evaluated through the damaged behaviour with the value of damage at the cracks opening.

The figure 6 shows results when the integration is strain or stress driven. The applied strain state, when the computation is strain driven, is chosen in a way to give the same stress state than the stress driven computation. The two stress-strain curves are slightly different. When it is stress driven, the shear stress is kept constant during the opening of the cracks, the opening results then in a jump of strain because material becomes more compliant. When it is strain driven, shear strain is kept constant when cracks open and as a result, shear stress useful to keep the shear strain constant decreases because of the increasing compliance.

3. Conclusion

A new damage model accounting for the opening/closure effect is presented. This work follows a previously developed model whose weaknesses have been removed by letting damage increase even with closed cracks. It was already able to describe unsymmetrical behaviour in tensile or compression loading. A first criterion based on the sign of the transverse normal stress let decide if cracks are opened or closed. Using a new formulation and adding two new material coefficients and one new criterion, it is now able to report different behaviour according to the status of the closed cracks. The new criterion introduced here compares the absolute value of the compressive stress and the shearing stress and let decide if cracks are sliding or stuck. When cracks are supposed to be stuck, virgin shear behaviour is recovered. When cracks are supposed to be sliding, damage increased is possible as soon as the damage driving force is high enough to violate the damage criterion.

Acknowledgements

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