DINAME 2015 - Proceedings of the XVII International Symposium on Dynamic Problems of Mechanics
V. Steffen, Jr; D.A. Rade; W.M. Bessa (Editors), ABCM, Natal, RN, Brazil, February 22-27, 2015

Smart periodic structures for vibroacoustic control

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Abstract: Periodic structures have been at the center of many research works in various fields of physics. Structural dynamics and vibroacoustic developments have found their first applications in the late 1970s. One of the main features associated to the periodic distribution of mechanical devices is the ability of the system to block the propagation of waves in specific frequency ranges. This is physically due to waves interferences (non-resonant systems) which can be combined to TMD-like effects (resonant systems). Early developments were based on analytical techniques, therefore many results are available for simple systems (MDOF, beams, plates). More recently, the combination of periodicity theories with the finite element method has allowed designers to handle complex geometries and materials. This has induced a renewal in the interest for periodic structures, which is still growing in the community. In parallel, research activities on smart materials and structures have also contributed to the development of passive and active strategies to enhance the vibroacoustic behavior of structures. In this talk, after some reminders concerning historical aspects, the main methodologies for the design of periodic structures will be given, before showing several applications combining periodicity and smart structures (auxetic materials, semi-active and active strategies) for vibroacoustic applications. All practical aspects will be discussed, from the numerical tools for design and optimization, to the practical implementation for experimental validation, together with robustness issues.

Keywords: Smart structures, periodic structures, phononics, piezo-shunt, auxetic structures

NOMENCLATURE

$\boldsymbol{w}(\boldsymbol{x}) \in \mathbb{R}^3$ = displacement vector
ρ = density of the material
$\boldsymbol{C}(\boldsymbol{x})$ = Hooke elasticity tensor
ω = frequency
$\boldsymbol{k}$ = wave vector
Ω_R = primitive cell of the periodic problem
$\boldsymbol{\varepsilon}(\boldsymbol{x}) = \nabla_{sym}(\boldsymbol{w}(\boldsymbol{x})) = \frac{1}{2}(\nabla \boldsymbol{w}^T(\boldsymbol{x}) + \boldsymbol{w}(\boldsymbol{x})\nabla^T)$ = strain tensor
$\boldsymbol{R}$ = matrix containing the three lattice vectors (elementary cell)
$\boldsymbol{n}$ = outpointing unit normal vector
ϕ = wave vector angle in the 2D lattice

INTRODUCTION

Periodic structures exhibit wave propagation properties that find application in many fields of physics. Structural vibrations and acoustics of periodic structures found their first formulations during the 17th century, while practical applications in the mechanical engineering field can be found from the late 1970s. One of the main features associated to the periodic distribution of mechanical devices is the ability of the system to block the propagation of waves in specific frequency ranges. This is physically due to waves interferences in non-resonant systems (corresponding to the so-called phononic structures), which can be combined to TMD-like effects with resonant systems (corresponding to the so-called metamaterials). Early developments were based on analytical techniques, therefore many results are available for simple systems (MDOF, beams, plates). More recently, the combination of periodicity theories with the finite element method has allowed designers to handle complex geometries and materials. This has induced a renewal in the interest for periodic structures, which is still growing in the community. A very detailed review of historical origins, recent progress and future outlook of this topic has been published recently by Hussein et al. (2014). The reader is invited to refer to this article and the following discussion by Mace (2014) that cover the most important aspects of this topic.

In parallel, research activities on smart materials and structures have also contributed to the development of passive and active strategies to enhance the vibroacoustic behavior of structures. In particular, piezoelectric materials have been deeply employed to improve the vibroacoustic quality of structural components, especially in the low frequency range (see references Preumont (1997), Nelson and Elliott (1992) or Banks and R.C. Smith (1996) among many others). Recently, much effort has been spent on developing new multi-functional structures integrating electro-mechanical systems in order to optimize their vibroacoustic behavior over a larger frequency band of interest, among which Thorp et al. (2001) or Collet et al. (2009).

The problem addressed here is a contribution to the challenges of designing and implementing a new class of integrated smart metacomposites capable of improved engineering performances in terms of mechanical and vibroacoustic behavior as compared to strictly passive structures.

The definition of a metacomposite combines two different aspects of vibration control. The first aspect is connected to periodic structure theories, which are usually associated with metamaterial developments. In the field of light propagation, research has explored how to design and construct photonic crystals exhibiting photonic band gaps that prevent light from propagating in certain directions with specified frequencies. Other efforts have explored creation of photonic crystals able to propagate light in anomalous and useful ways (i.e. negative refraction and artificial magnetism). In the acoustic domain, similar studies were carried out with the aim of preventing the propagation of elastic waves within a medium. For both light and acoustic waves, the band gap is obtained by periodically modulating some electromagnetic or mechanical properties as shown by Yang et al. (2002). This technique presents two main problems: the spatial modulation must be of the same order as the wavelength in the gap, and the position of the band gap cannot be easily changed since it strongly depends on the materials employed (Bragg's band gap). A possible solution for these problems is found using composites with locally resonant units. The periodicity of the crystal creates a stop band that can be shifted by modifying the properties of the resonators. Liu et al. (2000) had demonstrated that a resonant sonic crystal with building blocks of rubber-coated lead balls exhibits a low-frequency sonic band gap, and the resonance can provide a maximum impedance mismatch to shield against airborne sound. The same effect can be obtained using Helmoltz resonators as showed by Fang et al. (2006); Ambati et al. (2007) or Hu et al. (2005). The same idea was extended in the vibroacoustic domain for the control of elastic waves propagating into a waveguide. The resonant units in this case were obtained using RL circuits shunted to piezoelectric ceramics embedded on the structure's surface. Numerous works have been published by Park and Baz (2005) that present analyses of the capability and efficiency of a shunted piezoelectric patch for structural damping and wave cancellation. An elegant formulation of passive shunting was first proposed Hagood and von Flotow (1991) and is still commonly used. The study showed how a piezoelectric material shunted through a series RL circuit, i.e., a resonant shunt, which would exhibit a behavior analogous to the well-known mechanical tuned mass damper. Periodically induced impedance-mismatch zones generate broader stop bands, i.e., frequency bands where waves are attenuated. The tunable characteristics of shunted piezo-patches allow the equivalent mechanical impedance of the structure to be tuned so that stop bands are generated over desired frequency ranges. The presence of a resistance in the shunt circuit generates a damped resonance of the electrical network. The resistance also allows the energy dissipation mechanism of shunted piezos to be exploited, which dampens the amplitude of vibration also outside the stop bands. The original periodic shunting concept was numerically demonstrated by Thorp et al. (2005) on rods and fluid-loaded axisymmetric shells. More recently, this strategy was extended to plates by Casadei et al. (2010); Spadoni et al. (2009); Chen et al. (2013), where the Bloch theorem was used to predict the dispersion properties of the resulting periodic assembly. However the limitation of this approach is the narrow-band effectiveness of resonant circuits. For that reason a different circuit layout was proposed. A very effective method is based on the use of negative capacitance shunts, as originally proposed by Forward (1979). In this configuration, a piezoelectric patch is shunted through a passive circuit to a negative impedance converter. In this way, the internal capacitance of the piezoelectric ceramic is artificially canceled, and the impedance of the shunt circuit reduces to that of the passive circuit. Optimization of the electrical impedance for modal damping is well described by Livet et al. (2011). Although the negative capacitance shunting strategy has been experimentally validated, it must be used with caution since it requires active elements that can destabilize the structure if improperly tuned. Efficiency band and stabilty can be improved by using specific parameters and circuit architecture as illustrated by Beck et al. (2014). This technique requires in fact to tune the circuit very close to the stability limit as indicated by Fukada et al. (2004); Kim and Jung (2006). The second concept includes the definition of composite conceived in a broader sense, in which shunted piezoelectric materials, electronic components, controllers and the structure are intimately connected to each other. In this respect, the notion of programmable matter coined by Toffoli and Margolus (1991) to refer to an ensemble of computing elements arranged in space is now extended to smart materials based on distributed piezoelectric actuators able to modify the inherent vibroacoustic properties based on an input signal. Applications of distributed shunted patchs concept on controlling vibroacoustic energy diffusion is really novative and can induce significant capacity to absorb or reflect vibration field, as shown by Tateo et al. (2014a,b); Collet et al. (2014b).

This paper aims at showing examples of such metacomposites for controlling acoustic and mechanical power flows. In the first part, after introducing some numerical tools for damping management in periodic structures models, an auxetic kirigami application is given. The second part deals with smart mechanical interfaces for controlling absorption and transmission of elastodynamical energy.

FORMULATION OF THE FLOQUET-BLOCH THEOREM FOR ELASTODYNAMICS

This section recalls the shifted-cell version of the Floquet-Bloch theorem for to elastodynamics. The formulations of Floquet (1883) and Bloch (1928) respectively for one dimensional (1D) and two dimensional (2D) systems governed by differential equations with periodic coefficients are here presented in light of their application to the analysis of damped periodic mechanical systems. The full presentation of this formulation can be found in the work of Collet et al. (2011).

Shifted-cell elastodynamics formulation

Let us consider an infinite periodic conservative elastic problem. The harmonic homogeneous dynamical equilibrium of system is driven by

$$\rho(\boldsymbol{x})\omega^2\boldsymbol{w}(\boldsymbol{x}) + \nabla\boldsymbol{C}(\boldsymbol{x})\nabla_{sym}(\boldsymbol{w}(\boldsymbol{x})) = 0\ \forall\boldsymbol{x} \in \mathbb{R}^3 \tag{1}$$

(signification of symbols is given in nomenclature). By considering a primitive cell of the periodic problem Ω_R, the Bloch eigenmodes and the dispersion functions $\boldsymbol{k}(\omega)$ can be found by searching for the eigen solutions of the homogeneous problem (1) as

$$\boldsymbol{w}(\boldsymbol{x}) = \boldsymbol{w}_{n,k}(\boldsymbol{x}, \boldsymbol{k})e^{i\boldsymbol{k}.\boldsymbol{x}}, \tag{2}$$

where $\boldsymbol{w}_{n,k}(\boldsymbol{x}, \boldsymbol{k})$ are Ω_R-periodic functions.

While the classical approach consists in solving (1) with adequate shifted boundary conditions, the "shifted-cell" version of the Bloch theorem proposed by Bensoussan et al. (1978); Wilcox (1978) used here considers continuity on boundary conditions with a shifted operator:

$$\frac{\partial}{\partial \boldsymbol{x}} \rightarrow \frac{\partial}{\partial \boldsymbol{x}} + i\boldsymbol{k}. \tag{3}$$

In that case $\boldsymbol{w}_{n,k}(\boldsymbol{x}, \boldsymbol{k})$ and $\omega_n(\boldsymbol{k})$ are the solutions of the following generalized eigenvalues problem:

$$\begin{aligned}
\rho(\boldsymbol{x})\omega_n(\boldsymbol{k})^2\boldsymbol{w}_{n,k}(\boldsymbol{x}) + \nabla\boldsymbol{C}(\boldsymbol{x})\nabla_{sym}(\boldsymbol{w}_{n,k}(\boldsymbol{x}))& \\
-i\boldsymbol{C}(\boldsymbol{x})\nabla_{sym}(\boldsymbol{w}_{n,k}(\boldsymbol{x})).\boldsymbol{k} - i\nabla\boldsymbol{C}(\boldsymbol{x})\frac{1}{2}(\boldsymbol{w}_{n,k}(\boldsymbol{x}).\boldsymbol{k}^T + \boldsymbol{k}.\boldsymbol{w}_{n,k}^T(\boldsymbol{x}))& \\
+\boldsymbol{C}(\boldsymbol{x})\frac{1}{2}(\boldsymbol{w}_{n,k}(\boldsymbol{x}).\boldsymbol{k}^T + \boldsymbol{k}.\boldsymbol{w}_{n,k}^T(\boldsymbol{x})).\boldsymbol{k} &= 0 \quad \forall \boldsymbol{x} \in \Omega_R, \qquad (4)\\
\boldsymbol{w}_{n,k}(\boldsymbol{x} - \boldsymbol{R}.\boldsymbol{n}) - \boldsymbol{w}_{n,k}(\boldsymbol{x}) &= 0 \quad \forall \boldsymbol{x} \in \Gamma_R. \qquad (5)
\end{aligned}$$

Eq. (4) is simply obtained by introducing the spectral shift into (1), while equation (1), while eq. (5) corresponds to the continuity boundary conditions expressed on boundary faces of the lattice polyhedron for a rectangular parallelepiped cell. This Quadratic Eigenvalue Problem (QEP) can be solved by fixing two of the constants ω, $|\boldsymbol{k}|$ (the complex amplitude) or cosine directions of $\boldsymbol{k}$ and compute the last one.

The proposed formulation is based on the computation of the Floquet vectors from equation (4), instead of computing the Floquet propagators commonly used for elastodynamic applications. The methodology allows the computation of the full complex map of the dispersion curves incorporating computation of evanescent waves and allowing the introduction of any frequency-dependent characteristic in the model.

Finite element formulation and computation of waves dispersion curves in periodical lattice

The numerical implementation is obtained by using a standard finite elements method to discretize the weak formulation as shown by Collet et al. (2011). The assembled matrix equation is

$$(\boldsymbol{K} + \lambda\boldsymbol{L}(\boldsymbol{\Phi}) - \lambda^2\boldsymbol{H}(\boldsymbol{\Phi}) - \omega_n^2(\lambda, \boldsymbol{\Phi})\boldsymbol{M})\boldsymbol{w}_{n,k}(\boldsymbol{\Phi}) = 0, \tag{6}$$

where $\lambda = ik$, $\boldsymbol{k} = k\begin{bmatrix}\sin(\theta)\cos(\phi)\\ \sin(\theta)\sin(\phi)\\ \cos(\theta)\end{bmatrix}$, θ and ϕ represent the direction angles in the reciprocal lattice domain, $\boldsymbol{\Phi} = \begin{bmatrix}\sin(\theta)\cos(\phi)\\ \sin(\theta)\sin(\phi)\\ cos(\theta)\end{bmatrix}$, $\boldsymbol{M}$ and $\boldsymbol{K}$ are respectively the standard symmetric definite mass and symmetric semi-definite stiffness matrices, $\boldsymbol{L}$ is a skew-symmetric matrix and $\boldsymbol{H}$ is a symmetric semi-definite positive matrix:

$$\begin{aligned}
\boldsymbol{M} &\rightarrow \int_{\Omega_R} \rho(\boldsymbol{x})\omega_n^2(\boldsymbol{k})\tilde{\boldsymbol{w}}_{n,k}(\boldsymbol{x})\boldsymbol{w}_{n,k}(\boldsymbol{x})d\Omega,\\
\boldsymbol{K} &\rightarrow \int_{\Omega_R} \tilde{\boldsymbol{\varepsilon}}_{n,k}(\boldsymbol{x})\boldsymbol{C}(\boldsymbol{x})\boldsymbol{\varepsilon}_{n,k}(\boldsymbol{x})d\Omega,\\
\boldsymbol{L} &\rightarrow \int_{\Omega_R} -\tilde{\boldsymbol{\kappa}}_{n,k}(\boldsymbol{x})\boldsymbol{C}(\boldsymbol{x})\boldsymbol{\varepsilon}_{n,k}(\boldsymbol{x}) + \tilde{\boldsymbol{\varepsilon}}_{n,k}(\boldsymbol{x})\boldsymbol{C}(\boldsymbol{x})\boldsymbol{\kappa}_{n,k}(\boldsymbol{x})d\Omega,\\
\boldsymbol{H} &\rightarrow \int_{\Omega_R} \tilde{\boldsymbol{\kappa}}_{n,k}(\boldsymbol{x})\boldsymbol{C}(\boldsymbol{x})\boldsymbol{\kappa}_{n,k}(\boldsymbol{x})d\Omega.
\end{aligned} \tag{7}$$

When k and $\boldsymbol{\Phi}$ are fixed, eq. (6) is a linear eigen value problem allowing computation of the dispersion functions $\omega_n^2(\boldsymbol{k}, \boldsymbol{\Phi})$ and associated Bloch eigenvector $\boldsymbol{w}_{n,k}(\boldsymbol{\Phi})$.

This approach has been widely used for developing homogenization techniques and spectral asymptotic analyses like in the work of Allaire and Congas (1998). It can also be applied for computing wave's dispersion even if Floquet propagators are preferred for 1D or quasi 1D computation, as indicated by Ichchou et al. (2007), Houillon et al. (2005) or Mencik and Ichchou (2005). Nevertheless these approaches have been only developed for undamped mechanical systems that is to say represented by a set of real matrices. In this case, most of the previously published works present techniques based on the mesh of a real k-space (i.e k or λ and $\mathbf{\Phi}$) inside the first Brillouin zone for obtaining the corresponding frequency dispersion diagrams and the associated Floquet vectors. For undamped systems, only propagative or evanescent waves exist, corresponding to families of eigen solutions purely real or imaginary. Discrimination between each class of waves is easy. If a damped system is considered, that is to say if matrices $\boldsymbol{K}, \boldsymbol{L}, \boldsymbol{H}$ are complex, evanescent part of propagating waves appear as the imaginary part of $\omega_n^2(\lambda, \mathbf{\Phi})$ and vice versa. It then becomes very difficult to distinguish the two families of waves but also to compute the corresponding physical wave's movements by applying spatial deconvolution.

Another possibility much more suitable for computing damped system, dedicated for time/space deconvolution and for computation of diffusion properties as defined by Collet et al. (2009) or Mencik and Ichchou (2005), is to consider the following generalized eigen value problem:

$$(\boldsymbol{K} - \omega^2 \boldsymbol{M}) + \lambda_n(\omega, \mathbf{\Phi})\boldsymbol{L}(\mathbf{\Phi}) - \lambda_n^2(\omega, \mathbf{\Phi})\boldsymbol{H}(\mathbf{\Phi}))\boldsymbol{w}_{n,k}(\mathbf{\Phi}) = 0. \tag{8}$$

In this problem, the pulsation ω and the propagative angle $\mathbf{\Phi}$ are fixed real parameters. Wave's numbers $\lambda_n = ik_n$ and associated Floquet vectors $\boldsymbol{w}_{n,k}$ are then computed by solving the quadratic eigen problem (8). This approach allows introduction of frequency dependent matrices corresponding to generalized damping terms (viscoelasticity), multiphysic coupling (especially electromechanical with electronic ordinary differential equation), foams (Biot-Allard model) or open domain boundary conditions (Sommerfeld condition).
Based on this approach, an inverse Fourier transformation in the k-space domain can lead us to evaluate the physical wave's displacements and energy diffusion operator when the periodic distribution is connected to another system, like in the work by Collet et al. (2009). Another temporal inverse Fourier transformation can furnish a way to access spatio-temporal response for non-homogeneous initial conditions. As $\boldsymbol{L}$ is skew-symmetric, the obtained eigen values are quadruple $(\lambda, \bar{\lambda}, -\lambda, -\bar{\lambda})$ collapsing into real or imaginary pairs (or a single zero) when all matrices are real (i.e. for an undamped system). In this case a real pair of eigen values correspond to evanescent modes oriented in two opposite directions on the k-space and imaginary values to two traveling waves propagating in opposite direction. The obtained eigen solutions are similar in 1D to those given by SAFE method and additional important properties can be extrapolated from Garvic (1995).

Characterization of damped power flow

When designing engineering periodic structures, suitable criterion for power flow characterization are needed. In particular, we need to define suitable indicator for distinguishing propagative and evanescent behavior especially when damped system is concerned. The capability of a given Bloch wave to transport energy is given by its group velocity. Indeed, they indicate how energy is transported into the considered system and allow to distinguish the 'propagative' and 'evanescent' waves. If a Bloch eigen solution (i.e $\boldsymbol{u}_n(\omega, \phi),\ \boldsymbol{k}_n(\omega)$) is considered, the associated group velocity vector is given by Maysenhölder (1994):

$$\boldsymbol{C}_{g_n}(\omega, \phi) = \nabla_{\boldsymbol{k}}\omega = \frac{\langle\langle \boldsymbol{S} \rangle\rangle}{\langle\langle e_{tot} \rangle\rangle} = \frac{\langle \boldsymbol{I} \rangle}{\langle E_{tot} \rangle} \tag{9}$$

where $\langle\langle : \rangle\rangle$ is the spatial and time average respectively on one cell and one period of time, $\boldsymbol{S}$ is the density of energy flow, $\boldsymbol{I}$ the mean intensity and e_{tot}, E_{tot} the total energy and its time average on a period (see Maysenhölder (1994) for details).

The intensity vector $\boldsymbol{I}$ is expressed as:

$$\langle \boldsymbol{I}_n \rangle = -\frac{\omega}{2} Re \left(\int_{\Omega_x} C(\varepsilon_n(\boldsymbol{x}) + ik\Xi_n(\boldsymbol{x})).(\boldsymbol{w}_n^*(\boldsymbol{x})) \frac{d\Omega}{V_{ol}} \right) \tag{10}$$

where $.^*$ is the complex conjugate, Re stands for real part and V_{ol} the domain volume.

As the spatio-temporal average of the system Lagragian is null (see Maysenhölder (1994)), the total energy average is approximated by only computing the kinetic energy average:

$$\langle E_{tot} \rangle = \frac{1}{2V_{ol}} \left(\int_{\Omega_x} \rho \omega^2 \boldsymbol{w}_n(\boldsymbol{x}, \omega, \phi).\boldsymbol{w}_n^*(\boldsymbol{x}, \omega, \phi) d\Omega \right). \tag{11}$$

The group velocity vectors $\boldsymbol{C}_{g_n}(\omega, \phi)$ can then be computed for all wave numbers at each frequency, providing a useful tool for structural design, as it will be shown later in the paper.

APPLICATION: DESIGN OF A DAMPED KIRIGAMI AUXETIC PYRAMIDAL CORE

In this part, we propose an application of the above methodology for designing a smart passive periodic structure, namely damped kirigami auxetic pyramidal core. A special attention is paid to the directivity properties of the core, and particularly to the impact of the damping in the wave propagation properties.

Kirigami is the ancient Japanese art of folding and cutting paper, and it has been applied to produce complex 3D cellular structures using modular moulding techniques and mathematical representation of the honeycomb lattice proposed by Scarpa et al. (2013), with a special topology resulting in the auxetic character of the structure. Auxetic solids have been extensively studied during the past two decades as indicated by Scarpa et al. (2000). The term "auxetics" indicates a wide range of materials and structures exhibiting a negative Poisson's ratio. In cellular configurations, a negative Poisson's ratio can be achieved in re-entrant centre-symmetric (butterfly) honeycombs as proposed by Gibson et al. (1982); Scarpa et al. (2000), rotating rectangles and triangles (see Grima et al. (2011)), as well as arrow-head proposed Larsen et al. (1996) and star-shaped configurations that can be found in the work of Grima et al. (2005). The centre-symmetric auxetic configuration has also been considered as a basis for gradient cellular structures by Lira et al. (2011); Prall and Lakes (1997). All these strategies have been investigated in terms of manufacturing possibilities and mechanical performances, mainly in the static domain. However, new research activities in the areas of vibroacoustics of auxetic structures have been performed in recent years. The negative Poisson's ratio, which provides an unusual large volume deformation during loading induces tunable wave propagation directivities not commonly observed in classical systems. Recently, some Kirigami auxetic cellular structures have also been deeply investigated in terms of wave propagation by Scarpa et al. (2013) as presented here, and the concept has been pushed toward its limits with a null Poisson's ratio that induces negative stiffness regime under nonlinear deformation and high energy dissipation under cyclic loading as shown by Virk et al. (2013).

The figure 1 shows the production process and the first Floquet vector of the unit cell. All details can be found in the paper by Scarpa et al. (2013). The Floquet vector which is shown here corresponds formally to the A0 (flexural) propagating mode observed on homogeneous 2D media.

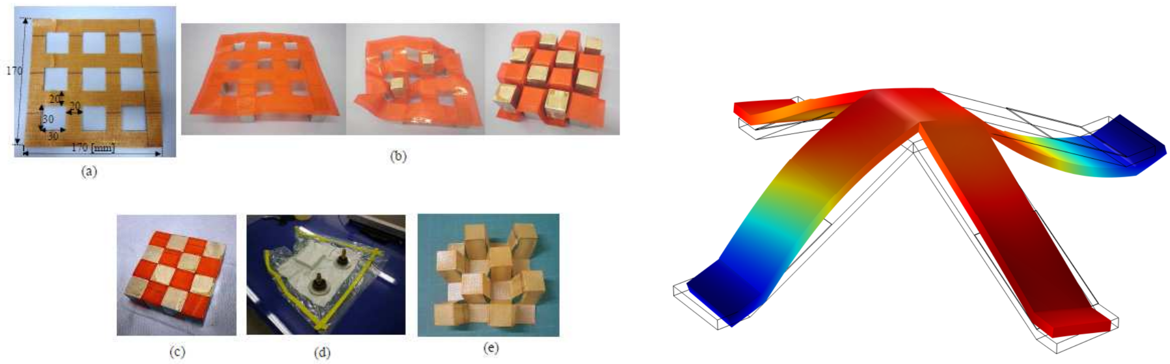

Figure 1 – Production process and first Floquet vector of the unit cell (after Scarpa et al. (2013))

The wave propagation in this structure is very well impacted by the geometric nature of the lattice. The geometry being not simple, the band structure of the system is quite complex (see the paper by Scarpa et al. (2013) for details), but simple indicators can help the designer to handle the main features of the structure. One of the well-known features in phononic structures is the band gap effect, namely frequency bands in which no wave can propagate. This effect is illustrated on the right picture of figure 2, which shows the "evanescence ratio" value as a fonction of ϕ (wave vector angle in the plane) and frequency. This indicator measures the ability of the waves to propagate by

$$\mathrm{Ind}(\omega, \phi) = \min_n \left| \frac{\mathrm{Real}(\lambda_n)}{|\lambda_n|} \right|. \tag{12}$$

For the undamped case, its value is either 0 (which means that some waves can propagate in the structure), or 1 (which means that no wave can propagate at angle and frequency of interest. Intermediate values shown in figure 2 are only due to interpolation between sampling points and do not correspond to any real physical feature of the system. In this case, it is clear that the band gaps are almost omnidirectionnal: for example, no wave can propagate in the structure around 4.5 kHz whatever the incident angle is. Some very selective directional in both frequency and angle can be observed but seem very difficult to exploit in practical applications. Only the 45 degrees focusing at 6.5 kHz is large enough to be considered as useful. At this point, we must mention that with the proposed approach, all the waves are included in the analysis, there is no a priori selection of the nature of the waves: the eigenvalue problem provides the full dynamics of the system.

Including the damping in the procedure is very natural, and it allows designers to check the viability of the system for real-life applications. The right picture of figure 2 shows the directivity diagram for the damped case. The omnidirectionnal band gaps are almost unchanged, while we can observe clearly the selective frequencies corresponding to spatial filtering of the propagation. Ther are several frequency bands where only propagation at $\phi = 45$ degrees is possible. All other directions will lead to evanescent waves that will not propagate in the structure. An interesting thing is the emergence of a 0 degree focusing juste below 5 kHz, even if its practical viability would require more investigations. It should be mentioned that this structure has been recently used as a basis of metacomposite in the work presented by Collet et al. (2014a).

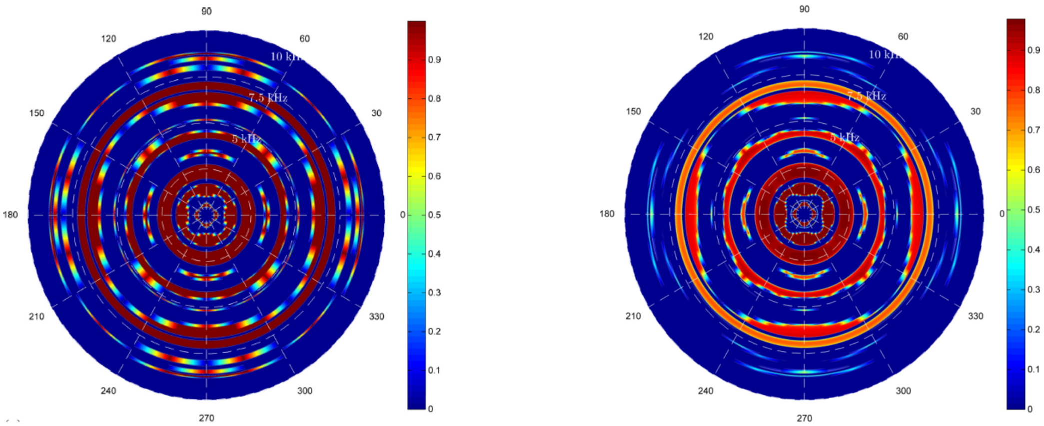

Figure 2 – Directivity of the kirigami auxetic pyramidal core. Left: undamped case; right: damped case (after Scarpa et al. (2013))

APPLICATION: DESIGN OF A RECONFIGURABLE METACOMPOSITE FOR SEMI-ACTIVE CONTROL OF STRUCTURAL VIBRATIONS

In this section, the concept of design of a metacomposite for semi-active control of structural vibrations is presented. The full details are given in the papers by Collet et al. (2012); Tateo et al. (2014a,b); Collet et al. (2014b). The system of interest is constituted by a host plate, with a periodic distribution of piezo-shunt devices. The unit cell is presented in figure 3. The objective is to design an interface which can be used between (unknown) structures, which allows control of the structural waves. This control is semi-active, ie there is no computation loop, only analogical electric circuits are used in the shunt, and reconfigurable with the ability to pass from one function to another. The first function of interest is a vibration barrier, and the second one is a dissipative interface.

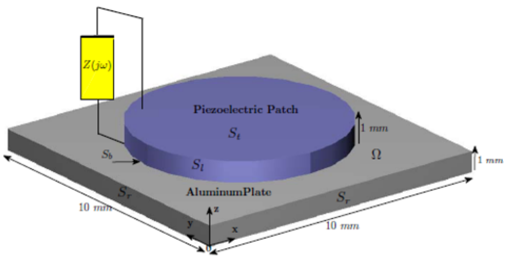

Figure 3 – Unit cell of the system of interest (after Collet et al. (2012))

These two functions are realized through the use of the electric shunt, which is optimized such that the multiphysics system exhibits the awaited properties in terms of wave propagation. The vibration barrier is obtained by minimizing the group velocity of the flexural wave in the system, while the dissipative interface is obtained by maximizing the energy dissipated in the electric circuit. The optimization procedures gives the evolution of the optimal shunt circuit, which is interpreted here as equivalent frequency-dependent resistance R_{opt} and capacitance C_{opt}. For the barrier, the optimal values are shown in the left part of figure 4 for all angles ϕ. The optimal impedance values almost correspond to a constant negative capacitance in all directions. Equivalent resistances corresponding to the active part of the shunt impedance are negative which indicates that the optimization leads to provide energy to the system for controlling mechanical barrier effects. The optimized configuration also tends to converge toward a fully conservative system. This last point is of paramount importance in both physical and technological points of view. Indeed, this result tends to show, on this example, that a pure band gap effect is only reachable if the system is completely reactive, which means that no dissipation occurs in the system. Another important issue is that the optimal shunt does not depend on the incident wave angle, which means that the system will be efficient for all waves that will interfere with it. Results given in the right part of figure 4 for the dissipative interface indicate that a positive resistance is required to dissipate energy, while the optimal capacitance is almost the same as for the barrier. This value is close to the stability limit of the system, and require precise management for the practical implementation of the system. Nevertheless, the reconfigurable character of the system is somehow independent from this limit since it requires only a change in the (synthetized) resistive part of the electric circuit.

The effect of the shunt on wave propagation properties together with numerical validations on finite structures and effect of design variables can be found in the paper by Collet et al. (2012); Tateo et al. (2014a).

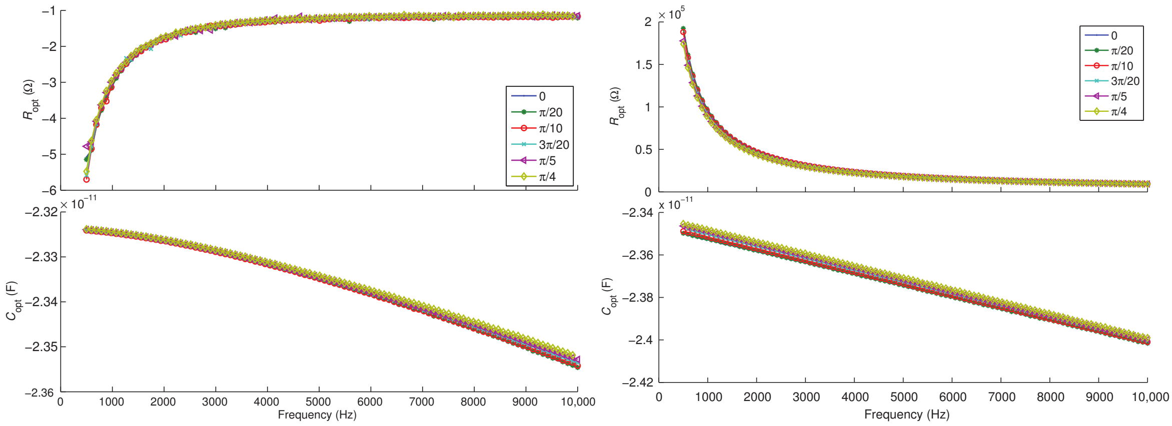

Figure 4 – Optimal electric impedances for the barrier (left) and dissipative (right) interfaces (after Collet et al. (2012))

Experimental results

The system has been tested experimentally. The interface is located between two passive plates, as shown in figure 5. Details about the practical implementation are given in the paper by Tateo et al. (2014b). The structure is suspended to a rigid frame through metallic wires. The plate is equipped with 75 piezoelectric patches from $PZ26$ series (Ferroperm industries) arranged in a regular 15×5 array. Each actuator is connected to an independent analog circuit reproducing the negative capacitance effect together with the appropriate value of (constant) resistance. The circuits must be powered since an operational amplifier is required for the synthesis of the impedance. All components in the circuits are analogical. The circuits require in-situ tuning for the synthetic capacitance, which is facilitated by a dedicated saturation-driven LED.

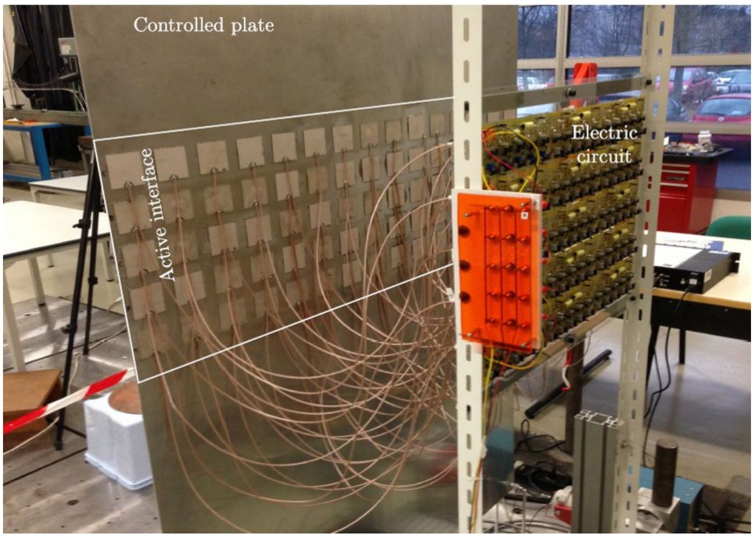

Figure 5 – Picture of the metacomposite

The system is excited by a point load located on the bottom of the plate, and full-field measurement are performed using a scanning laser vibrometer. The results show that the strategy controls the plate in the $50 - 5000\ Hz$ range. The lower limit is due to the fact that evanescent waves exist due to boundaries, with a spatial length which is much larger than the interface length, while the higher limit is related to the length of the unit cell compared to the wavelength. The size of the frequency band is very large compared to classical control strategies, this is probably one of the main features of the concept. The figure 6 shows some examples of full-field measurements on the uncontrolled structure, and with the two functionnalities activated. Results are given for two frequencies of interest, namely 25 Hz (below the efficiency limit) and 3000 Hz (in the frequency range). From these pictures it can clearly be observed that the awaited functionnalities are reached, which validates the concept of metacomposite.

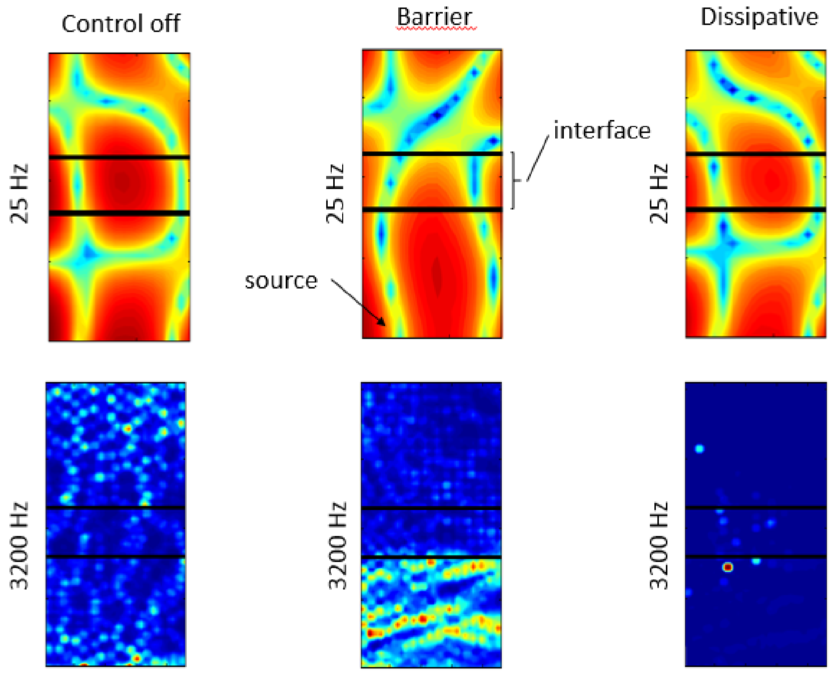

Figure 6 – Full-field measurements: displacement field on the whole structure

CONCLUSIONS

This article presents a numerical procedure able to compute the damped wave dispersion functions in the whole first Brillouin domain of multidimensional damped elastodynamical systems. The method has been first applied to design a damped kirigami auxetic pyramidal core, which exhibits some very specific wave propagation properties, including pass-bands, stop-bands and directivity signature. In a second step, a smart metacomposite has been designed and included in a finite structure to confer it new functionalities. On the basis of wave propagation properties in frequency-dependent periodic media, optimal values of shunting devices for piezoelectric patches have been found and correspond to awaited functionnalities.

ACKNOWLEDGMENTS

This work was co-financed by The French National Research Agency under grants number NT09-617542-CALIOP, ANR-12-JS09-008-COVIA. It has been performed in cooperation with the Labex ACTION program (ANR-11-LABX-0001-01).

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