Improving Accuracy of Long-term Prognostics of PEMFC stack to Estimate Remaining Useful Life

Kamran Javed, Rafael Gouriveau, Noureddine Zerhouni, Daniel Hissel
FEMTO-ST Institute (UMR CNRS 6174) Besançon France,
FCLAB (FR CNRS 3539) Belfort, France.
Email: firstname.lastname@femto-st.fr

***Abstract*—Proton Exchange Membrane Fuel cells (PEMFC) are energy systems that facilitate electrochemical reactions to create electrical energy from chemical energy of hydrogen. PEMFC are promising source of renewable energy that can operate on low temperature and have the advantages of high power density and low pollutant emissions. However, PEMFC technology is still in the developing phase, and its large-scale industrial deployment requires increasing the life span of fuel cells and decreasing their exploitation costs. In this context, Prognostics and Health Management of fuel cells is an emerging field, which aims at identifying degradation at early stages and estimating the Remaining Useful Life (RUL) for life cycle management. Indeed, due to prognostics capability, the accurate estimates of RUL enables safe operation of the equipment and timely decisions to prolong its life span. This paper contributes data-driven prognostics of PEMFC by an ensemble of constraint based Summation Wavelet- Extreme Learning Machine (SW-ELM) algorithm to improve accuracy and robustness of long-term prognostics. The SW-ELM is used for ensemble modeling due to its enhanced applicability for real applications as compared to conventional data-driven algorithms. The proposed prognostics model is validated on run-to-failure data of PEMFC stack, which had the life span of 1750 hours. The results confirm capability of the prognostics model to achieve accurate RUL estimates.**

I. Introduction

The Fuel cell (FC) technology is gaining popularity among various renewable energy sources due to their cleanliness, high efficiency and economical supply of power demand by the customers [1]. A FC is an electrochemical energy conversion system that can generate electricity as long as fuel is supplied. Basically, it converts the chemical energy released during an electrochemical reaction of hydrogen (fuel) and oxygen to electrical energy. The classification of FCs depends on the fuel and the choice of electrolyte. According to literature, among six major types of FCs, the Proton Exchange Membrane Fuel cell (PEMFC) are popular due to high power density, quick start up, low operating temperature and solid non-corrosive electrolyte [1]. Typically the PEMFC have short life duration around 2000 hours, whereas 6000 hours are necessary for some applications (including transportation) [2]. Therefore, durability and high maintenance costs are among main limiting factors for commercialization of PEMFC technology [3].

FC aging is an unavoidable process, the optimization of its service and minimization of life cycle costs / risks require continuous monitoring of aging process and accurate prediction of life time at which it will be unable to perform desired functionality. In this context, Prognostics and Health Management of fuel cells is an emerging field [4], which aims at extending their life span, while reducing exploitation and maintenance costs. More precisely, FC prognostics becomes a major area of focus nowadays. The core process of prognostics is to identify degradation at cell / stack level at early stages and to estimate its Remaining Useful Life (RUL) for life cycle management. This enables, managing operating conditions and performing timely maintenance or control to prolong the life span of the fuel cell. However, FCs are highly multiphysics and multiscale systems and it is not easy to access their internal parameters to fully understand the aging process. Therefore, building physics based prognostics models can be very difficult to achieve. Alternatively, data-driven prognostics model can learn behavior of degrading FC directly from the data, without any physical understanding about the aging phenomena.

According to authors knowledge, only two data-driven approaches have been applied so far for prognostics of PEMFC namely, Adaptive Neuro-Fuzzy Inference System (ANFIS) [5] and Echo state network (ESN) [6]. Basically, [5] studied the prediction of PEMFC stack voltage reduction caused by degradation under normal conditions (using ANFIS). Though, results appear to interesting, but still limited to a single prediction (for 500 hours horizon) which is not reliable in presence of uncertainty due to monitoring data and modeling phase. The prediction results for ANFIS model were based on *iterative approach*, which suffers from error accumulation problem. In [6] the application of ESN for RUL estimation was not clearly demonstrated. Because, the prediction results of PEMFC stack voltage were based on direct and parallel structures of ESN, that require *prior* knowledge of final horizon step preset by the practitioner. Indeed, for prognostics applications, the prediction horizon is unknown and cannot be set *a priori*. Moreover, both approaches (ANFIS & ESN) were dependent on several parameter set by the user, also they did not present any results on sensitivity of the prognostics model to failure thresholds (FT). Such issues limit the applicability of a prognostics approach, as for importance of FT see [7].

According to above limitations, this paper contributes with an ensemble of constraint based Summation Wavelet- Extreme Learning Machine (SW-ELM) algorithm. SW-ELM is inserted in the ensemble due to its enhanced applicability as compared to other data-driven approaches for prognostics of PEMFC. To account for issues of long-term prognostics, the ensemble is achieved by selecting those SW-ELM models that satisfy the constraints. These enable managing the uncertainty of prognostics and improving the accuracy of RUL estimates.

The paper is organized as follows. Section II elaborates background of PEMFC cell / stack, data-driven prognostics and highlight issues of long-term predictions with data-driven approach. Section III presents the proposed approach of long-term prognostics based on SW-ELM ensemble. Section IV validates our proposition on PEMFC stack data. Finally, section V concludes this work.

II. Backgrounds

A. Unit cell (PEMFC) and stack

The scheme of simplified unit cell (PEMFC) is shown in Fig. 1a. Hydrogen is supplied at the anode side of the membrane, where electrons and protons are split. The protons pass through the membrane (barrier) and electrons flow through the external circuit which produces electricity and combine with oxygen at cathode side. The byproducts of this electrochemical reaction are heat and water. Fig. 1b shows stacking of multiple cells in series (via bipolar plates) to increase the voltage of the stack ($Utot$) given in Eq. 1.

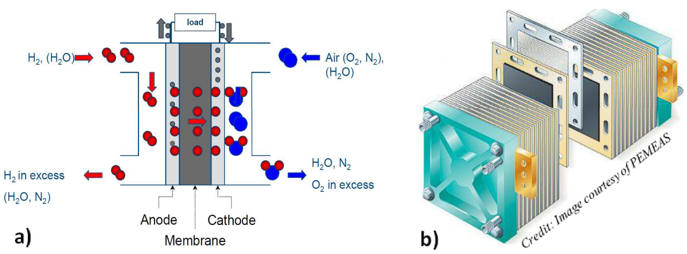

Fig. 1. a) Scheme of PEMFC & b) PEMFC stack

$$Utot = \sum_{k=1}^{c} Ucell^c \tag{1}$$

where c is the number of cells in the stack. Note with series connection current will be the same in each cell. Indeed, the stack is prone to degrade due to factors like material degradation, design and assembly, etc., and the performance decay induced is strongly associated to the operating conditions (e.g. operating temperature, current load, etc.,) [5], [8]. Moreover, the performance of the FCs stack is constrained by the worst performing cell [9]. Whatever the cause of stack degradation, it will result a voltage drop. Thus, stack voltage can be considered as a useful indicator for FC health assessment and prognostics.

B. From stack monitoring data to RUL

To transform the raw data into relevant behavior models, the frame of data-driven prognostics is based on following steps that are necessary to implement PHM of FCs.

- Data-acquisition: is a process of collecting and storing useful data from the stack, to identify the changes that can develop faults or can even lead to failures. The measurements that are economically possible from PEMFC stack are: current, voltage, air compressor speed, cooling water temperature, Air / H_2 temperatures and Electrical Impedance Spectroscopy.
- Data-processing: is a step for extracting and selecting health indicators that are sensitive to stack condition, preferably having monotonic trends. Selection of health indicators can be done by transforming them to another space or based on highest information content.
- Prognostics Modeling: aims at building an effective model that is capable of predicting the evolution of FC stack aging process and estimating unknown RUL. The data-driven approaches learn the degradation model from past observations and project current condition ($\mathbf{t}current$) of equipment up to defined FT, i.e., failure time ($\mathbf{t}fail$), Fig. 2a, where RUL is defined by Eq. 2. When new data arrives, prediction is updated and this process is repeated at frequent intervals.

$$RUL = \mathbf{t}fail - \mathbf{t}current \tag{2}$$

C. Long-term predictions & issues

Long-term predictions (or multi-steps ahead prediction "msp") with data-driven (connectionist) approaches can be achieved in different ways and by using different structure and algorithms like ANFIS, ESN, etc. [10] studied five structures, namely the Iterative, Direct, DirRec, Parallel, and MISMO for performing msp. According to this study, except *iterative approach* all other approaches require knowledge of prediction horizon H. With iterative approach, the msp is performed by a single model that is tuned to perform a one-step ahead prediction $\hat{x}_{t+1}$. This estimated value is used as the regressors of the model to estimate the following ones and the process is repeated until the estimation of $\hat{x}_{t+H}$ (see [10] for details). The prediction obtained in recursive manner result error accumulation with increasing H. Also, due to inherent uncertainties of deterioration phenomena, unclear future operating conditions, modeling errors, lack of data, and uncertain FTs, RUL estimation with a single prediction is not sufficient. Therefore, the unknown RUL (Fig. 2a) should be estimated from several predictions. However, in the presence of above issues it is not necessary that all predictions clearly reflect degradation and intersect the FT as well. Thus, poor predictions will result large uncertainty of RULs and the final value of estimated RUL is inaccurate. Fig. 2b illustrates this behavior with a large tail distribution, which is not useful to further plan of actions. Thus, it is necessary to ensure good predictions to achieve accurate RUL estimates, which is the aim of next section.

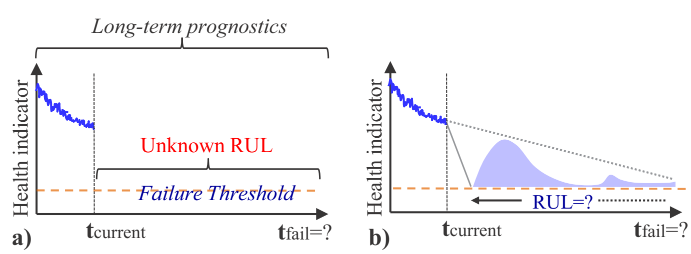

Fig. 2. Long-term prognostics & uncertainty of RUL estimate

III. Proposed approach for prognostics

Data-driven prognostics approaches have the advantage of better applicability, when there is absence of prior knowledge or human experts. They learn systems behavior directly from data and do not require any specific knowledge about the system. However, as mentioned in section I only two data-driven connectionist techniques have been applied so far for prognostics of PEMFC, i.e., ANFIS and ESN. This paper presents data-driven prognostics PEMFC by an ensemble of constraint based Summation Wavelet- Extreme Learning Machine (SW-ELM) algorithm. The proposed approach is based on following hypothesis.

- Stack voltage drop is a useful prognostics indicator.
- Stack aging process is irreversible degradation.

A. SW-ELM and constraints

Basically, SW-ELM in an improved variant of Extreme Learning Machine (ELM) algorithm to train single layer feed forward neural network (SLFN Fig. 3) [11], that avoids slow iterative tuning (e.g. ANFIS) and requires one-pass for learning. Unlike ESN and ANFIS, ELM has only two parameters to be set by the user. However, the solution with ELM algorithm vary for each run due to random parameters initialization of SLFN (i.e., weights & bias). Also the complexity of hidden layer and the choice of activation function influence performance of ELM, which is the case of ESN as well. To address these issues SW-ELM and constraints are given as follows.
SW-ELM is a combination of neural network and wavelet theory, and appears to be an effective prediction approach [12]. SW-ELM is a one-pass algorithm like ELM, also it benefits from an improved parameter initialization to minimize the impact of random weights and bias (of input-hidden layer) and an improved structure with dual activation functions. Also SW-ELM works on actual scales of the data (see Fig. 3).

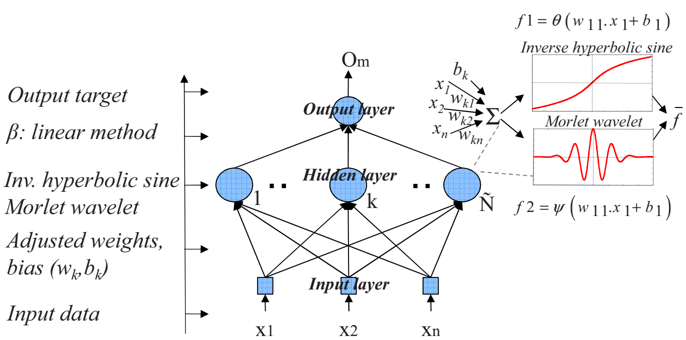

Fig. 3. Machine learning view of SW-ELM

Let note n and m the numbers of inputs and outputs, N the number of learning data samples (x_i, t_i), where $i \in [1 \dots N]$, $x_i = [x_{i1}, x_{i2}, ..., x_{in}]^T \in \Re^n$ and $t_i = [t_{i1}, t_{i2}, ..., t_{im}]^T \in \Re^m$, and $\tilde{N}$ the number of hidden nodes, each having activation functions (f_1 & f_2). To minimize the difference between output o_j and target t_j, there exist β_k, w_k and b_k such that:

$$\sum_{k=1}^{\tilde{N}} \beta_k \bar{f}\left[(\theta, \psi)(w_k.x_j + b_k)\right] = t_j \ , \ j = 1, 2, ..., N \tag{3}$$

where $\bar{f}$ is the average output from two different activation functions θ and ψ. $w_k = [w_{k1}, w_{k2}, ..., w_{kn}]^T \in \Re^n$ is an input weight vector connecting the k^{th} hidden to input layer neurons, $(w_k.x_j)$ is the inner product of weights and inputs, and $b_k \in \Re$ is the bias of k^{th} hidden neuron. Also, $\beta_k = [\beta_{k1}, \beta_{k2}, ..., \beta_{km}]^T \in \Re^m$ is the weight vector to connect k^{th} hidden neuron to output neuron. In matrix form Eq. 3 can be written as $H_{avg}\beta = T$, where T is target matrix and H_{avg} is hidden layer output matrix expressed as:

$$H_{avg}\left(w_1, \dots, w_{\tilde{N}}, x_1, \dots, x_{\tilde{N}}, b_1, \dots, b_{\tilde{N}}\right) =$$

$$\bar{f}(\theta, \psi) \begin{bmatrix} (w_1.x_1 + b_1) & \dots & (w_{\tilde{N}}.x_1 + b_{\tilde{N}}) \\ \vdots & \dots & \vdots \\ (w_1.x_N + b_1) & \dots & (w_{\tilde{N}}.x_N + b_{\tilde{N}}) \end{bmatrix}_{N \times \tilde{N}} \tag{4}$$

$$\beta = \left[\ \beta_1^T \cdots \beta_{\tilde{N}}^T \ \right]_{\tilde{N} \times m} and \ \ T = \left[\ t_1^T \cdots t_N^T \ \right]_{N \times m} \tag{5}$$

Finally, the least square solution of the linear system $H_{avg}\beta = T$, with minimum norm of output weights β is:

$$\hat{\beta} = H_{avg}^{\dagger} T = \left(H_{avg}^T H_{avg}\right)^{-1} H_{avg}^T T \tag{6}$$

where $H_{avg}^{\dagger}$ shows the Moore-Penrose generalized inverse for the hidden layer output matrix H_{avg} [13]. The SW-ELM algorithm can be synthesized as follows (see details in [12]).

Algorithm 1 Brief learning scheme of SW-ELM

Require
- N learning samples (x_i, t_i), n inputs, $\tilde{N}$ hidden nodes
- Arcsinh and Morlet activation functions (θ and ψ)

1: Initialize wavelet parameters (i.e., dilatation & translation).
2: Assign parameters of hidden nodes (w_k, b_k) randomly & adjust.
3: Obtain hidden layer output matrix H_{avg} using Eq. 4 .
4: Find the output weight matrix $\hat{\beta}$ in Eq. 6.

In order to achieve RUL estimates with iterative approach using SW-ELM and to account for issues of long-term predictions (section. II-C), constraints are included in the prognostics modeling phase to ensure that predictions decay properly over a long-term horizon (to reflect stack aging process) and intersect with FTs as well.

$$\frac{d}{dt}(\hat{x}_t) \neq 0 \tag{7}$$

$$x_{tcurrent} > \hat{x}_{t+i} \ , \quad i \in \mathbb{Z}_+ \tag{8}$$

$$\hat{x}_{t+h} \leq FT \tag{9}$$

The constraints given in Eq. 7 and Eq. 8 are based on assumption that stack aging process is irreversible (i.e., decreasing trend). Thus, the slope of recursive predictions at any step cannot be zero and predicted value $(\hat{x}_{t+i})$ at each step i should be less than current state of stack at time **t**current from which prediction is initiated. The constraint given in Eq. 9 ensures that predicted trends intersect the failure threshold (FT).

B. Ensemble modeling

Predicting behavior of aging FC is a complicated task, since there are various sources of uncertainty that impact RUL estimates (section II-C). Thus, its not feasible to estimate RUL with complete accuracy on the basis of a single prediction, which could lead to wrong decisions. Thus, prognostics with an ensemble model would be less likely to be in error than an individual model and appears to be meaningful. The proposed approach aims at building an ensemble using selective SW-ELM models that satisfy the constraints given in Eq. 7, 8 and 9. The main steps of ensemble modeling are as follows.

1) Set the required number of models for the ensemble.

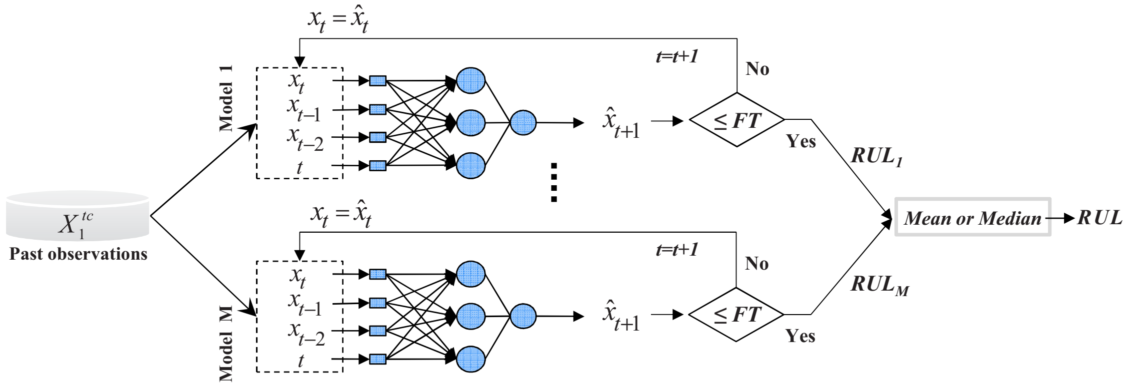

Fig. 4. Structure of SW-ELM ensemble modeling

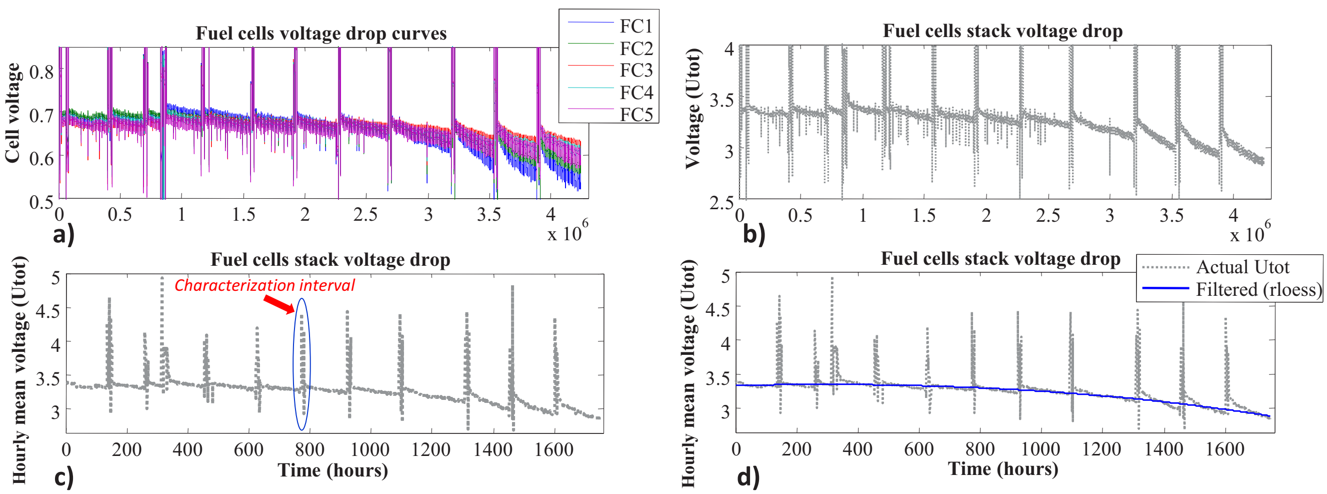

Fig. 5. Data-processing to extract voltage drop trend

2) Learn a group of SW-ELM models with same complexity & data, but different parameters initialization.
3) Choose the model with minimum learning error. However, the model may not be optimum, therefore it must satisfy the constraints given in Eq. 7, 8 & 9 in the test phase.
4) Test the model for performing msp (to estimate RUL) under constraints, to be selected for ensemble.
5) Repeat steps 2-4 until the given number of SW-ELM models for an ensemble is met.
6) Compute mean / median of RULs from ensemble .

With proposed ensemble, the final RUL value is the outcome of several predictions from selective SW-ELM models which will improve the accuracy and robustness of long-term prognostics, see Fig. 4.

IV. Experiment and results discussions

The data of PEMFC stack used for the validation of proposed approach were provided by FCLAB Research Federation (FR CNRS 3539, France). The data were obtained from a PEMFC test bench that enables aging of FCs stacks under actual operating conditions. For the experiment, 5-cells stack was assembled at FCLAB and was operated under constant current of 60A approximately. The experiment last for the duration of 1750 hours. The monitoring data collected from run-to-failure experiment are composed of voltage measurements, load measurements, temperatures, air stoichiometry rates, hydrogen, etc., (see [2] for further understanding and details from similar experiments).

A. *Data-processing & simulation settings*

As mentioned in section II stack voltage is an important prognostics indicator and economically possible. During experiment the stack voltage is obtained by adding cell voltages as given in Eq. 1. Fig. 5a shows voltage curves acquired from 5-cells in the PEMFC stack, that are added to get stack voltage (Utot) shown in Fig. 5b. One can note that voltage drops as the time grows. The peaks in the hourly voltage curves are mainly due to characterization phases that also facilitate to monitor FC aging process (Fig. 5c). According to these plots, 11 characterizations were performed to observe the response of PEMFC stack under varying loads, controlled gas and environmental conditions. However, the peaks due to characterization can impact data-driven model performances. Prior prognostics modeling the stack voltage signal is smoothed to extract a monotonic trend by applying **rloess** filter with span value 0.9 Fig. 5d. Note that, rloess is a robust local regression filter

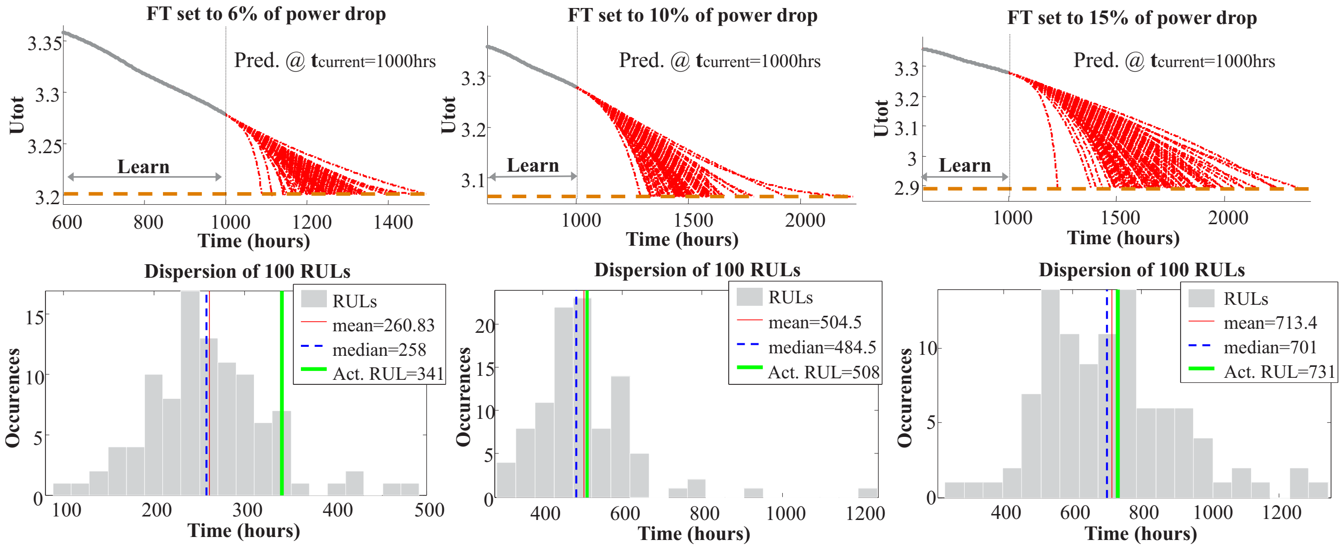

Fig. 7. Analysing sensitivity of prognostics model by changing FTs

that allocate lower weight to outliers, see [14]. According to procedure of ensemble modeling in section III-B, the selective models for ensemble are set to 100 (Fig. 4). A group of 100 SW-ELM models are learned, and the best model with minimum learning error is selected for testing. The structure of each model is set to 4 input neurons, 15 hidden neurons and 1 output neuron. The inputs of each SW-ELM model are (3 regressors from) $Utot$ and aging time. The parameter (w_k, b_k) adjustment constant C=0.01 (see [12]). Prognostics performances are assessed by: 1) Sensitivity analysis to FT defined at 6%, 10% and 15% of hourly power drop signal (i.e., P=VxI) Fig. 6, and 2) RUL estimation at frequent intervals.

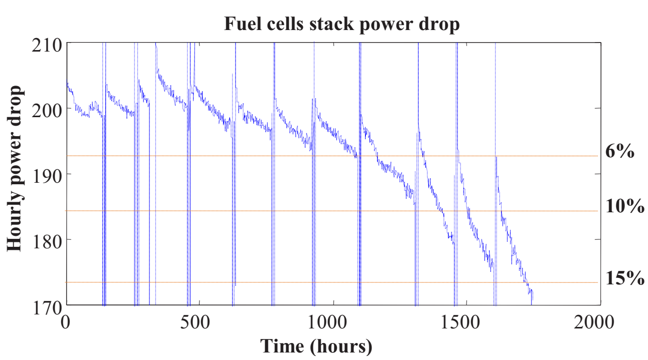

Fig. 6. Fuel cell stack power drop curve & FTs

B. Prognostics results

1) Sensitivity to failure thresholds: basically, thresholds aim at defining the failure time to stop the prediction process. Surely, the uncertainty of FTs can impact prognostics performances. Fig. 7 show prognostics results with proposed ensemble (of 100 models) for three different FT 6%, 10% & 15% of power drop (for which Utot are 3.207, 3.067 & 2.888 respectively). For each FT, plots show predictions from ensemble and their dispersion. Note that, for each case, 1000 hours data were learned and this index is considered as **t**current from which prognostics is initiated. Most importantly, whatever the FT is, the constraints based strategy ensures that predictions from ensemble decay properly and intersect the FT as well. But, obviously not all the models in an ensemble have the same accuracy, which can be seen by the dispersion of RUL estimates for each FT (Fig. 7). Mainly, the uncertainty is due to lack of data, parameter initialization (weights & bias) and FT defined on raw power drop signal. Nevertheless, the final RUL estimates (mean or median) from the RULs dispersion for each case of FT enable managing the uncertainty of RULs, which shows the significance of ensemble strategy over single prediction that are sensitive to FT. For all cases the final RUL value obtained by computing the *mean RUL* from an ensemble are close to the actual RUL, as compared to *median RUL* estimates. However, it is necessary to thoroughly investigate SW-ELM ensemble performance over long-term horizon at frequent intervals. This will also enable us to compare RUL errors with mean or median RULs.

2) Rul estimation & accuracy performances: to see overall performance of the proposed approach like a real situation, prognostics is initiated at 850 hours that is almost the half life of the PEMFC stack used for the testing. Therefore, initial learning frame consists of only 850 samples from stack voltage and corresponding aging time. During the tests, RUL is estimated after every 50 hours interval. Fig. 8 shows this situation, where estimated RUL value is updated when new data arrives at given time interval. The accuracy of RUL estimates (mean or median) increase with time, as more data are available. In PHM context, it is generally desirable to have early RUL estimates rather than late RULs to avoid failures. The qualitative analysis in Fig. 8 shows that *median RUL* give early estimates and have better accuracy when more data are available. This can be seen by the closeness of median RULs to actual RULs as compared to mean RUL estimates.
To validate the findings from the prognostics results (i.e., mean vs. median) Fig. 8, the RUL estimation procedure is repeated for 10 trials and the average errors from median RULs and mean RULs are compared. Fig. 9 shows these results by comparing the *pdfs* of RUL errors. The mean RUL error

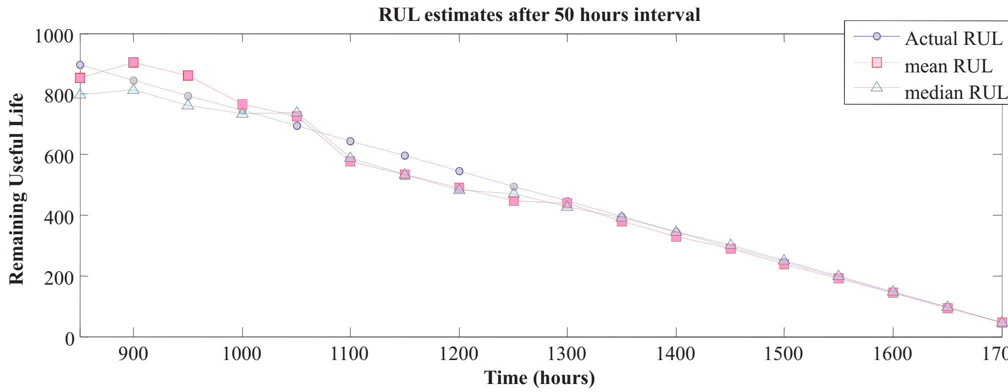

Fig. 8. Comparison of RUL estimates at frequent intervals

distribution has a wide spread and shows negative RUL errors which mean late estimates. The median RUL errors distribution has preference for early RUL estimates and has narrowness of error interval with large area close to 0 error, which means more on-time RULs as depicted in Fig. 8.

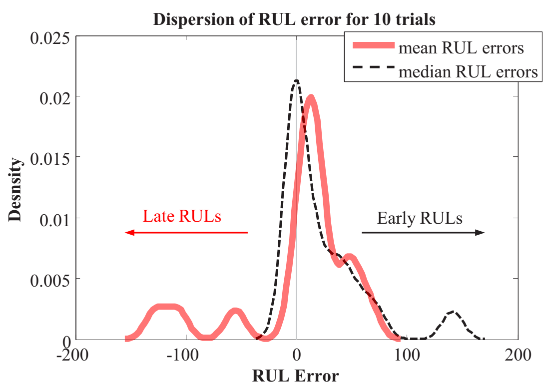

Fig. 9. SW-ELM ensemble RUL error dispersion (850 hrs to 1750 hrs)

V. Conclusion

This paper presents prognostics of PEMFC stack that enable timely decisions to prolong its life span. The development focuses on improving the accuracy of long-term prognostics of PEMFC with relevantly a new data-driven approach. More precisely, an ensemble of constraints based SW-ELM algorithm is proposed, that benefits from ease of implementation for prognostics of PEMFC. The proposed approach is applied to real data of fuel cell stack with a life span of 1750 hours. Prognostics performances are thoroughly investigated to show improvements. From results analysis we find that SW-ELM ensemble manages uncertainty and avoids late RUL estimates, which is useful in terms of safety. Also it is computationally less expensive (due to one-pass SW-ELM), which makes it suitable for on-line decisions to reduce FC operational costs.

Acknowledgment

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