Efficiency comparison of CMS vibroacoustic formulations for uncertain damped problems

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Abstract When dealing with internal vibroacoustics, many formulations are available. The classical (u,p) approach uses physical variables (displacement of the structure, pressure in acoustic cavity), and alternatives exist based on a displacement or velocity potential variable instead of the acoustic pressure in the fluid. Each formulation has its own advantages and drawbacks which are addressed in this paper in the context of Component Mode Synthesis for model reduction, but globally all formulations exhibit the same behavior. Thus, the decoupled mode bases which are classically considered for CMS are identical in all formulations providing that the static pressure is not included in the formulation. So, the question addressed in this work deals with the strategies to take the static response of the fluid domain into account in the projection and on the ability of each formulation to adapt to these strategies. Many options are presented for each of the formulations and are applied for two study cases: a shoe box, that is a parallepipedic cavity and a curved box with a more complex geometry.

1 Introduction

This paper is mainly focused on analyses of acoustic cavities closed by elastic structures, in presence of absorbing materials. In particular, for optimization or uncertainties propagation purposes, strategies of model reduction are addressed. The classical $(\mathbf{u}, p)$ approach uses physical variables (displacement of the structure, pressure in acoustic cavity), and alternatives exist based on a displacement or velocity potential

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variable instead of the acoustic pressure in the fluid [7]. In the context of model reduction, it can be shown that in terms of convergence properties, the main point is related to the availability of the formulation to include static effects of the acoustic cavity. This is due to the fact that whatever the formulation is, the acoustic modes are identical, thus implying equivalent convergence properties using a Ritz based method. The main topic addressed here is then to focus on the ability of the method to take into account the static acoustic mode, since, as indicated in the literature, this mode is theoretically not required [8], while it helps to converge [2]. We propose here an illustration to this fact, together with a new formulation that can naturally include static effect. In a second step, it will be shown that using residue corrections [1, 12] is very efficient to improve convergence with only a few additional computational cost.

2 Formulation of structural-acoustic problem

2.1 Coupled formulation

This part recalls the basic equations of the coupled problem, which are classically available in literature [5, 7, 10].

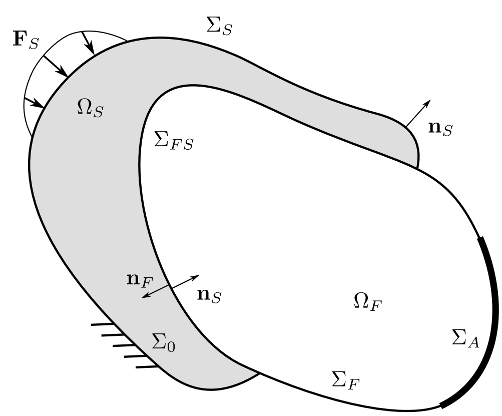

Fig. 1 Description of vibroacoustic problem

The internal vibroacoustic problem which is considered in this paper is presented in figure 1. Let Ω_F be the fluid domain, Ω_S the structural domain. The partition of boundaries is done according to the mechanical conditions: Σ_{FS} is the structural-acoustic coupling surface, Σ_F is the part of the acoustic border on which a Neumann condition is applied, corresponding to a rigid wall (a homogeneous Dirichlet condition could also be considered without loss of generality), Σ_A is the part of the acoustic border on which a Robin condition is considered, corresponding typically to an absorbing material, Σ_S is the structural boundary on which a Neumann condi-

tion is applied, corresponding to a prescribed force, Σ_0 is the structural boundary on which a homogeneous Dirichlet condition is applied, corresponding to a clamped area. $\mathbf{n}_S$ and $\mathbf{n}_F$ are respectively the outgoing unit normals of structural and fluid domain.
The physical variables which are used to describe the behavior of the system are the displacement $\mathbf{u}$ for the structure and the acoustic pressure p for the fluid.

For the structural part, the linearized strain tensor is denoted as $\varepsilon(\mathbf{u})$ and the associated stress tensor is denoted as $\sigma(\mathbf{u})$. The choice of acoustic pressure p to describe the fluid behavior instead of a fluid displacement vector field reduces the number of degrees of freedom and avoids the discretization of the fluid irrotationality constraint. This is valid while $\omega \neq 0$. The particular case $\omega = 0$ will be discussed in the next section.
The variational formulation of this problem consists in finding $\mathbf{u}$ and p in $\mathscr{C}_\mathbf{u} = \left\{\mathbf{u} \in [H^1(\Omega_S)]^3 \,/\, \mathbf{u} = \mathbf{0} \,\text{on}\, \Sigma_0\right\} \times \mathscr{C}_p = \left\{p \in H^1(\Omega_F)\right\}$ such that,
for all $(\delta\mathbf{u}, \delta p) \in \mathscr{C}_\mathbf{u} \times \mathscr{C}_p$:

$$\begin{cases} \displaystyle\int_{\Omega_S} \sigma(\mathbf{u}) : \varepsilon(\delta\mathbf{u})\, d\Omega - \omega^2 \int_{\Omega_S} \rho_S \mathbf{u}\cdot\delta\mathbf{u}\, d\Omega \\ \qquad\qquad \displaystyle - \int_{\Sigma_{FS}} p\, \mathbf{n}_F \cdot \delta\mathbf{u}\, d\Sigma = \int_{\Sigma_S} \mathbf{F}_S \cdot \delta\mathbf{u}\, d\Sigma, & (a) \\ \displaystyle\frac{1}{\rho_F} \int_{\Omega_F} \nabla p \cdot \nabla \delta p\, d\Omega - \frac{\omega^2}{\rho_F c^2} \int_{\Omega_F} p\delta p\, d\Omega \\ \qquad\qquad \displaystyle -\omega^2 \int_{\Sigma_{FS}} \mathbf{u}\cdot\mathbf{n}_F\, \delta p\, d\Sigma + \frac{i\omega}{Z_a(\omega)} \int_{\Sigma_A} p\delta p\, d\Sigma = 0, & (b) \end{cases} \tag{1}$$

where ρ_S is the structure mass density, $\mathbf{F}_S$ is the complex amplitude of the force exciting the structure at frequency ω, c is the sound speed in the fluid and ρ_F the mass density of the fluid at rest. $Z_a(\omega)$ is the complex impedance of absorbing material. The FE discretization of this variational formulation can be written as:

$$\begin{cases} [K_S]\{U\} - \omega^2 [M_S]\{U\} - [L]\{P\} = \{F_S\}, & (a) \\ [K_F]\{P\} - \omega^2 [M_F]\{P\} - \omega^2 [L]^T\{U\} + \frac{i\omega}{Z_a(\omega)}[A_F]\{P\} = \{0\}, & (b) \end{cases} \tag{2}$$

or

$$\left(\begin{bmatrix} K_S & -L \\ 0 & K_F \end{bmatrix} - \omega^2 \begin{bmatrix} M_S & 0 \\ L^T & M_F \end{bmatrix} + \frac{i\omega}{Z_a(\omega)} \begin{bmatrix} 0 & 0 \\ 0 & A_F \end{bmatrix} \right) \begin{Bmatrix} U \\ P \end{Bmatrix} = \begin{Bmatrix} F_S \\ 0 \end{Bmatrix}, \tag{3}$$

where:

$$\begin{aligned} [K_S] &\to \int_{\Omega_S} \sigma(\mathbf{u}) : \varepsilon(\delta\mathbf{u})\, d\Omega, & [M_S] &\to \int_{\Omega_S} \rho_S \mathbf{u}\cdot\delta\mathbf{u}\, d\Omega, \\ [K_F] &\to \frac{1}{\rho_F} \int_{\Omega_F} \nabla p \cdot \nabla\delta p\, d\Omega, & [M_F] &\to \frac{1}{\rho_F c^2} \int_{\Omega_F} p\delta p\, d\Omega, \\ [L] &\to \int_{\Sigma_{FS}} p\, \mathbf{n}_F \cdot \delta\mathbf{u}\, d\Sigma, & [A_F] &\to \int_{\Sigma_A} p\delta p\, d\Sigma. \end{aligned} \tag{4}$$

N.B. For a sake of clarity, no structural damping is considered in the weak formulations and associated FE discretizations presented here, but it can naturally be integrated in the formulations.

2.2 Considerations about the static case

As indicated in reference [8], the considered problem is not valid for $\omega = 0$ since when the frequency tends to 0, the "movement" of the fluid tends to static irrotational motion. In that case, the pressure can be decomposed in two terms:

$$p = p^S + \tilde{p}, \tag{5}$$

where $\tilde{p}$ is a dynamic pressure and p^S is a so-called static pressure, which is constant in space and differs from the static solution which could be obtained by extending the considered problem (2) to $\omega = 0$, since it would result in a static pressure which is constant in space but undetermined in amplitude. The uniqueness of $\tilde{p}$ is guaranteed by the condition

$$\int_{\Omega_F} \tilde{p} d\Omega = 0. \tag{6}$$

If Σ_F is rigid, the static pressure can be directly derived from the normal displacement of the structure:

$$p^S = -\rho_F \frac{c^2}{V_F} \int_{\Sigma_{FS}} \mathbf{u} \cdot \mathbf{n}_F d\Sigma - \rho_F \frac{c^2}{V_F} \int_{\Sigma_A} \mathbf{u}^F \cdot \mathbf{n}_F d\Sigma, \tag{7}$$

where V_F is the measure of the volume occupied by Ω_F. This means that several ways can be considered to solve the considered problem using a displacement/pressure formulation. The variable which describes the movement of the structure is the displacement $\mathbf{u}$, while for the fluid one can use the following strategies:

- use of pressure p for fluid description. This formulation has been presented in section 2.1, it is not valid for $\omega = 0$.
- use of dynamic pressure $\tilde{p}$ for fluid description. This formulation is valid for $\omega = 0$. The constraint (6) has to be considered to solve the problem. In particular, in a finite elements context, this constraint has to be discretized and included in the final system. The total pressure p is then obtained by summing $\tilde{p}$ and p^S which comes from equation (7). This requires in particular the discretization of p^S from equation (7) in which $\mathbf{u}^F \cdot \mathbf{n}_F$ must be expressed on Σ_A in function of the dynamic pressure $\tilde{p}$ using the impedance condition. The corresponding system has large expressions and exhibits no special interest comparing to other strategies.
- use of both dynamic pressure $\tilde{p}$ and static pressure p^S for fluid description. This formulation is valid for $\omega = 0$. The authors did not found any mention of this possibility in the literature, even if a $(\mathbf{u}, \tilde{p}, \varphi, p^S)$ formulation can be found in

[3]. To obtain this formulation, the constraint (6) and the equation (7) have to be considered and discretized. The total pressure p is then obtained by summing $\tilde{p}$ and p^S.

Following this approach, one can define the subspace $\mathscr{C}_{\tilde{p}} = \left\{ \tilde{p} \in \mathscr{C}_p \,/\, \int_{\Omega_F} \tilde{p}\, d\Omega = 0 \right\}$. In a weak form, one has then to find $\mathbf{u}$, $\tilde{p}$ and p^S in $\mathscr{C}_{\mathbf{u}}$, $\mathscr{C}_{\tilde{p}}$ and $\mathscr{R}$ such as, for all $(\delta\mathbf{u}, \delta\tilde{p}, \delta p^S) \in \mathscr{C}_{\mathbf{u}} \times \mathscr{C}_{\tilde{p}} \times \mathscr{R}$:

$$
\begin{cases}
\displaystyle\int_{\Omega_S} \sigma(\mathbf{u}) : \epsilon(\delta\mathbf{u})\, d\Omega - \omega^2 \int_{\Omega_S} \rho_S \mathbf{u} \cdot \delta\mathbf{u}\, d\Omega \\
\qquad \displaystyle - \int_{\Sigma_{FS}} \tilde{p}\, \mathbf{n}_F \cdot \delta\mathbf{u}\, d\Sigma - \int_{\Sigma_{FS}} p^S\, \mathbf{n}_F \cdot \delta\mathbf{u}\, d\Sigma = \int_{\Sigma_S} \mathbf{F}_S \cdot \delta\mathbf{u}\, d\Sigma, \; (a) \\
\displaystyle\frac{1}{\rho_F} \int_{\Omega_F} \nabla\tilde{p} \cdot \nabla\delta\tilde{p}\, d\Omega - \frac{\omega^2}{\rho_F c^2} \int_{\Omega_F} \tilde{p}\delta\tilde{p}\, d\Omega - \omega^2 \int_{\Sigma_{FS}} \mathbf{u} \cdot \mathbf{n}_F\, \delta\tilde{p}\, d\Sigma \\
\qquad \displaystyle + \frac{i\omega}{Z_a(\omega)} \int_{\Sigma_A} \tilde{p}\delta\tilde{p}\, d\Sigma + \frac{i\omega}{Z_a(\omega)} p^S \int_{\Sigma_A} \delta\tilde{p}\, d\Sigma = 0, \qquad (b) \\
\displaystyle\frac{1}{\rho_F c^2} p^S V_F \delta p^S + \int_{\Sigma_{FS}} \mathbf{u} \cdot \mathbf{n}_F\, d\Sigma\, \delta p^S \\
\qquad \displaystyle + \frac{1}{i\omega Z_a(\omega)} \int_{\Sigma_A} \tilde{p}\, d\Sigma\, \delta p^S + \frac{1}{i\omega Z_a(\omega)} p^S S_A \delta p^S = 0, \qquad (c)
\end{cases}
\tag{8}
$$

where V_F is the volume of the fluid domain and S_A the surface of the absorbing area. The corresponding finite element formulation can be written as:

$$
\left(\begin{bmatrix} K_S & -L & -\ell \\ 0 & K_F & 0 \\ \ell^T & 0 & \frac{V_F}{\rho_F c^2} \end{bmatrix} - \omega^2 \begin{bmatrix} M_S & 0 & 0 \\ L^T & M_F & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{i\omega}{Z_a(\omega)} \begin{bmatrix} 0 & 0 & 0 \\ 0 & A_F & a_F \\ 0 & \frac{-1}{\omega^2} a_F^T & \frac{-1}{\omega^2} S_A \end{bmatrix} \right) \begin{Bmatrix} U \\ \tilde{P} \\ p^S \end{Bmatrix} = \begin{Bmatrix} F_S \\ 0 \\ 0 \end{Bmatrix}, \tag{9}
$$

where the acoustic displacement potential dofs $\tilde{P}$ are associated to $\tilde{p}$ and:

$$
[a_F] \rightarrow p^S \int_{\Sigma_A} \delta\tilde{p}\, d\Sigma, \;\; [\ell] \rightarrow p^S \int_{\Sigma_{FS}} \delta\mathbf{u} \cdot \mathbf{n}_F\, d\Sigma. \tag{10}
$$

One should emphasize that this system must be solved under the constraint $\int_{\Omega_F} \tilde{p} d\Omega = 0$, which corresponds to

$$
[C]^T \{\tilde{P}\} = 0 \tag{11}
$$

where

$$
[C] \rightarrow \int_{\Omega_F} \tilde{p} d\Omega. \tag{12}
$$

One should emphasize that this system can be easily symmetrized by dividing the equations related to dynamic fluid dofs by ω^2 and changing sign on the last line.

Equivalent $(\mathbf{u}, \varphi, p^S)$, $(\mathbf{u}, \psi, p^S)$ can also be derived in the same way using velocity or displacement potentials φ and ψ.

3 Model reduction of structural-acoustic problem

3.1 Classical reduction using decoupled basis

In a generic way, all previous problems can be written as:

$$[K - \omega^2 M + \frac{i\omega}{Z_a(\omega)} A]\{Y\} = \{F\}, \tag{13}$$

where $\{Y\}$ includes a partition of structural and acoustic dofs. In this generic notation, the matrices can possibly depend on frequency. A very classical way to reduce the size of the harmonic problem is to search the response on a given vectorial space, typically built from the associated undamped problem. In our case, on can define :

- the *in vacuo* structural modes, which are the normal modes of the structure without wet surface (i.e. for which Σ_{FS} is replaced with a free surface), these modes have shapes that can be stored in the structural modal matrix T_S;
- the *blocked* acoustic modes, which are the normal modes of the cavity in which both Σ_{FS} and Σ_A are replaced by rigid wall conditions. The associated shapes are stored in the acoustic modal matrix T_F.

One should emphasize that the decoupled mode shapes are identical in all formulations providing that the static pressure is not included in the formulation. The global projection matrix is then built as:

$$[T] = \begin{bmatrix} T_S & 0 \\ 0 & T_F \end{bmatrix} \tag{14}$$

One can reduce now the initial problem using the projection $\{Y\} = [T]\{q\}$ with $\{q\} = \begin{Bmatrix} q_S \\ q_F \end{Bmatrix}$:

$$[\bar{K} - \omega^2 \bar{M} + \frac{i\omega}{Z_a(\omega)} \bar{A}]\{q\} = \{\bar{F}\}, \tag{15}$$

where:

$$[\bar{K}] = [T^T K T], \quad [\bar{M}] = [T^T M T], \quad [\bar{A}] = [T^T A T], \quad \{\bar{F}\} = [T^T]\{F\}. \tag{16}$$

3.2 Considerations about the static case

Concerning the remarks presented in the previous section about the static case, one should underline that in literature, the constraint (6) is generally omitted in the FE formulation. This is valid since the acoustic modes are calculated with rigid boundary conditions, which implies that they automatically verify the constraint. On the other side, for a full model computation, the constraint must be taken into account for proper estimation of the low frequency content of the response.
Several strategies are available to take into account static response of the fluid domain in the projection:

- The mode p_0 can simply be added in the Ritz basis, even if this is not correct in a mathematical point of view, as indicated in [8].
- The proposed $(\mathbf{u}, p, p^S)$ formulation can be used without condensation of p^S.
- The impact of p^S on the structure can be evaluated using elastic modes, as indicated in [9]. In this case, its contribution is interpreted in terms of added mass and stiffness. $M_C = \sum_{\alpha=1}^{n} \frac{\rho_F}{\omega_\alpha^2} L P_\alpha P_\alpha^T L^T$ is the added mass matrix and K_C is the added stiffness matrix obtained by the discretization of $p_0^2 \int_{\Sigma_{FS}} \mathbf{u} \cdot \mathbf{n} d\Sigma \int_{\Sigma_{FS}} \mathbf{n} \delta \mathbf{u} dS$. In this case the reduction of the structural part is performed using the modified structural eigenvalue problem including added effects on the structure [6, 9]. It should be emphasized that this approach leads to convenient formulation without considering the acoustic absorbing area Σ_a. When Σ_a, characterized by Z_a, is present, the methodology used to derive the previous system leads to complex relationships which have no special interest compared to the alternative ways to take into account the static behavior of the fluid cavity.

3.3 Description of the test structures

In order to illustrate the convergence properties of the reduced models, one will exhibit the results coming from two models. The first one, called *shoe box*, is a parallelepiped cavity, with 5 rigid faces, closed by an elastic simply supported plate. This model has a very simple geometry in order that anyone can easily reproduce the results presented here. The second one, called *curved box*, has a more complex geometry, which induces couplings between space directions for the modal shapes of the cavity.

3.3.1 The shoe box

The shoe box is an acoustic cavity ($c = 342.2\, m.s^{-1}$, $\rho = 1.213\, kg.m^{-3}$) with size $0.654 \times 0.527 \times 0.6\, m^3$, closed by a simply supported plate, made of aluminum ($E = 7.2 \times 10^{10}\, Pa$, $\nu = 0.3$, $\rho = 2700\, kg.m^{-3}$ with a thickness of 3 mm. It has 3 non parallel sides covered by an absorbing layer. The excitation force is a normal point force located at coordinates (0.19075,0.197625) on the plate. The geometry and one

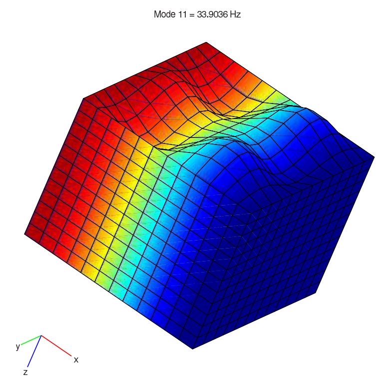

Fig. 2 The shoe box, example of coupled mode (color: acoustic pressure, shape: displacement)

of the coupled modes is presented in figure 2.

3.3.2 The curved box

The curved box has a more complex geometry, which is illustrated in figure 3. The upper face of the box is an elastic plate, and an absorbing layer covers the lower face. The structural excitation is distributed on a surface which is about 1/20th of the structural area.

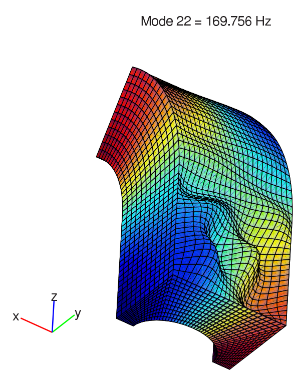

Fig. 3 The curved box, example of coupled mode (color: acoustic pressure, shape: displacement)

3.4 Results concerning the convergence of the $(\mathbf{u}, p)$ formulation

In this part we are interested in the convergence properties of the $(\mathbf{u}, p)$ formulation when the number of modes in the fluid basis increases. The first model of interest is the shoe box. The figure 4 shows frequency evolution of indicators when the absorbing area is not considered, while the figure 5 corresponds to similar results with absorbing area. On both figures, three results are shown: the reference curve (full model), and two curves corresponding to reduced models, one of them corresponding to reduction without the static acoustic mode, while the last one includes the static mode, either by added mass and stiffness for the “undamped” case, or by simply adding a constant vector in the fluid Ritz basis for the “damped” case. In all cases, the system includes dissipation since the structural part is considered to have a complex Young’s modulus $E(1+i\eta)$ where $\eta = 4\%$. It can be observed that, in

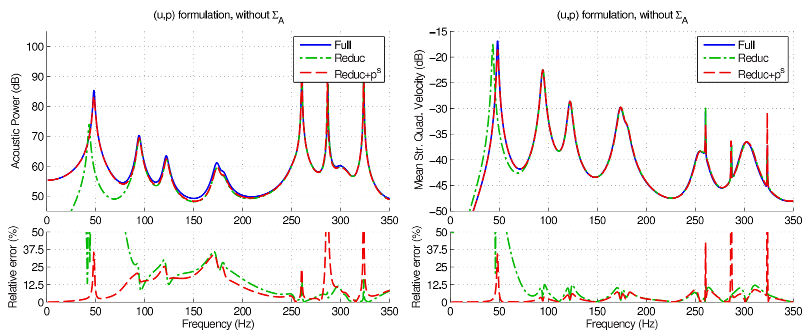

Fig. 4 Indicators for $(\mathbf{u}, p)$ formulation without Σ_A, including or not the static mode

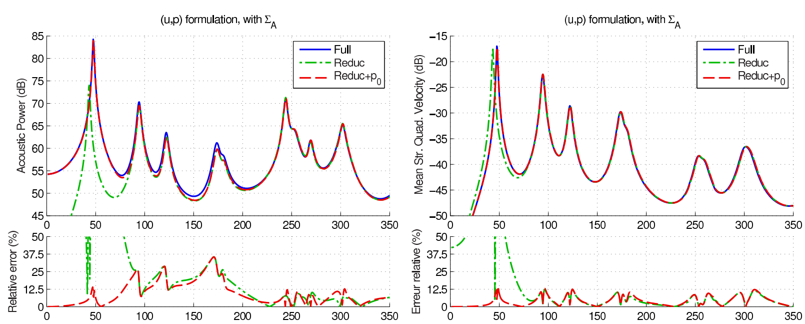

Fig. 5 Indicators for $(\mathbf{u}, p)$ formulation with Σ_A, including or not the static mode

both cases, the reduced models are not able to properly estimate the low frequency behavior of the coupled system if the acoustic static mode is not considered in the

projection. In the middle part of the frequency range of interest, the error (in linear scale) can still be large even if the static mode is considered, particularly in the acoustic domain. This is due to missing information in the fluid basis, which is not rich enough to properly represent the acoustic behavior of the system. In these calculations, 50 structural modes have been considered for T_S (up to 1493 Hz). In this case, using 200 structural modes and a static residual associated to the point force has no effect on the results in the frequency range of interest.
The missing informations are clearly associated to the fluid basis, since the modes have been estimated using rigid boundary conditions, which makes very difficult for the pressure to converge towards continuity on the coupling area and on the absorbing area. As illustrated in figure 6, the convergence is very low. This figure shows the evolution of acoutic power estimated by the reduced model when the number of modes in the fluid basis increases.

A more systematic study can be performed to evaluate the performance of the re-

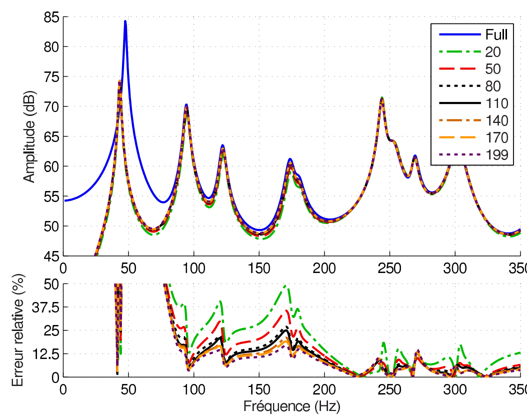

Fig. 6 Example of convergence : acoustic power estimated by reduced model vs. number of modes in the fluid basis

duced basis in terms of convergence related to the number of acoustic modes in the Ritz basis. The figure 7 illustrates the results of this analysis, by showing convergence curves of the mean relative error on indicators on the frequency range of interest, when using the following strategies in the reduction:

- direct reduction by using only “elastic” acoustic modes (i.e. without p_0 or p^s), referred as “Reduc” in the figure;
- reduction with static acoustic mode directly included in the projecting basis, referred as “Reduc + p_0” in the figure;
- reduction with static acoustic mode considered through added mass and stiffness, referred as “Reduc + p_S” in the figure (this case is only considered on the configuration without absorbing area);
- reduction of $(\mathbf{u}, p, p^S)$ formulation, without considering the static acoustic mode in the projecting basis, this mode being considered through p^S which is kept as a degree of freedom in the model. This case is referred as “$(\mathbf{u}, p, p^S)$ Reduc” in the figure.

The results are presented on both configurations (with or without the absorbing layer in the cavity). It can clearly be observed that:

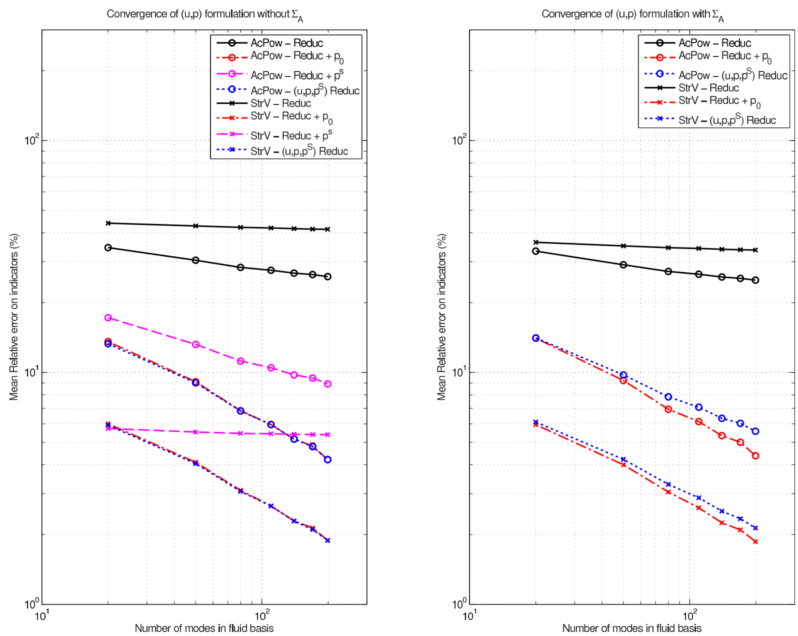

Fig. 7 Convergence of $(\mathbf{u}, p)$ for the shoe box

- in all cases, the convergence rate is very low;
- the direct reduction by using only "elastic" acoustic modes leads to large errors, it is then clear that including static contribution for the fluid part is necessary for correct estimation of low-frequency content of the responses;
- considering static contribution through added mass and stiffness is efficient for a low number of modes, but the convergence is very low, due to the fact that the added mass is evaluated from modal reduction and that no information is provided in the fluid domain to improve convergence;
- the best convergence rate corresponds to direct inclusion of constant vector p_0 in the fluid basis and also to projection using the $(\mathbf{u}, p, p^S)$ formulation. In both cases, the mean error on quadratic velocity (resp. acoustic power) reduces from 6% (resp. 10.5%) for 20 modes in the fluid basis to 1% (resp. 3%) for 200 modes.

There is no fundamental difference between the analysis performed with or without the absorbing layer in the fluid domain. The small difference (1% in mean value) which can be observed is due to the fact that the exact value of static pressure given in equation (7) is better estimated by the contribution of projection of p_0 in the response than by the reduction of $(\mathbf{u}, p, p^S)$ formulation, which implies a projection of (7) on the modes of interest. The results on the curved box clearly exhibit similar

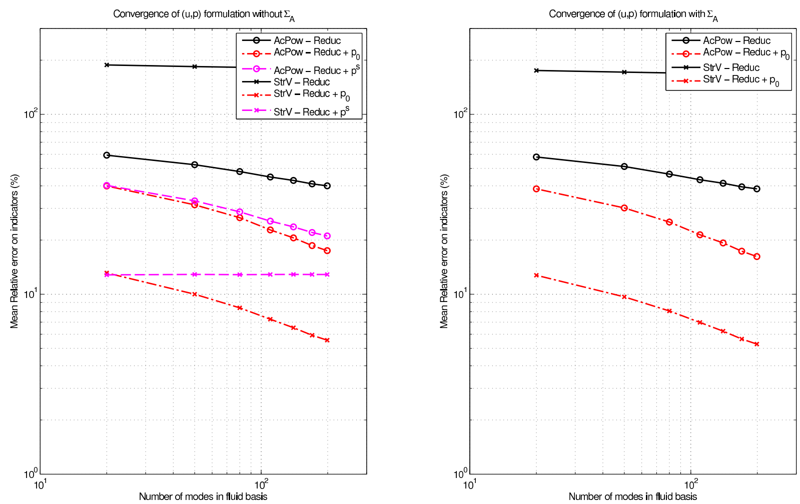

Fig. 8 Convergence of $(\mathbf{u}, p)$ for the curved box

trends, as illustrated in figure 8, which indicates that above results are not specific to the simple geometry of the shoe box case or to the fact that point source is used in this case: more complex geometry together with distributed excitation lead to similar results.

As a conclusion, it has been illustrated that including the static acoustic cavity mode in the Ritz basis, even if this is not required in theory, helps to improve convergence of reduced model. The convergence is equivalent to the one obtained with a new $(\mathbf{u}, p, p^S)$ formulation which includes rigorously the static effects at the price of a little more computation cost (assembly of terms corresponding to an additional dof in the full model). This new formulation provides a nonsingular stiffness matrix, which can be interesting for estimation of static residuals. Nevertheless, if the application does not require a definite stiffness matrix, the analyst can use the cheapest strategy for model reduction, which is the simple addition of the rigid cavity mode in the projecting basis.

4 Strategies for bases enrichment

In this section, two approaches are considered to enrich the decoupled bases in order to illustrate the efficiency of bases enrichment using residual information instead of completing Ritz bases with uncoupled modes.

4.1 Residue iterations

Residue iterations consist in evaluating the error in the estimation of the response, and building associate vectors to enrich the basis [4]. This iterative approach can be summarized as follows. Starting from a decoupled projecting basis, at step j of the iterative procedure, the basis T_j is used to project the unknown dofs vector: $\{q_j\} = [T_j]\{Y_j\}$, where

$$\{q_j\} = [\bar{K} - \omega^2 \bar{M} + \frac{i\omega}{Z_a(\omega)} \bar{A}]^{-1} \{\bar{F}\}. \tag{17}$$

The residue at iteration j is then evaluated:

$$\{R_j\} = [K_0]^{-1} [K - \omega^2 M + \frac{i\omega}{Z_a(\omega)} A]\{Y_j\} - \{F\}, \tag{18}$$

where K_0 is a matrix representative of the stiffness of the problem. It can typically be either K or a filtered version of it. $\{R_j\}$ being complex and frequency-dependent, T_j is combined with real and imaginary parts of $\{R_j\}$ taken at several frequency steps (in particular those corresponding to the largest errors in the response estimation). The new projecting basis T_{j+1} is finally obtained from the principal directions of the set of vectors using a singular value decomposition. This strategy is very general and can take into account any frequency dependency of system's characteristics.

4.2 Robust bases

The vibroacoustic robust bases [12] correspond to a non-iterative procedure and also start from uncoupled bases which are enriched to take into account the effect of the moving structure on the fluid and of the presence of an absorbing material. For weak vibroacoustic coupling (i.e. with light fluid like air), the approach leads to enrichment of the fluid part only and T_F is enriched by ΔT_{Fs} and ΔT_{Fa}:

$$\begin{aligned} \Delta T_{Fs} &= \left(K_F - \omega_c^2 M_F\right)^{-1} L^T T_S, \\ \Delta T_{Fa} &= \left(K_F - \omega_c^2 M_F\right)^{-1} A_F T_F, \end{aligned} \tag{19}$$

where ω_c is a reference frequency (or a set of references) in the band of interest. An orthogonalization is obviously required to ensure good conditioning of the procedure. The approach can also deal with strong coupling and parametric changes, in particular for uncertainties propagation. These points are not discussed here. Compared with the residue iteration, one can qualitatively expect a solution which is equivalent to the one obtained after one full iteration, but with a lower computational time since only one resolution of the problem is required. For more difficult cases, in particular if only a few modes are kept in the uncoupled bases, the residue iteration technique will surely lead to more precise results after few iterations.

4.3 Numerical results

Figures 9 and 10 show improvement of convergence of reduced model when above residue terms are added to the projecting bases. In the simulations, 50 modes have been considered in both structural and acoustic bases. The results are shown in terms of relative errors, and indicate that both procedures lead to almost similar error reduction. The global error is very small in both cases. The calculation cost is of course higher for the iterative procedure that requires a little more calculation effort, but allows new iterations to improve results if the convergence is not achieved yet, which is not the case for the robust basis.

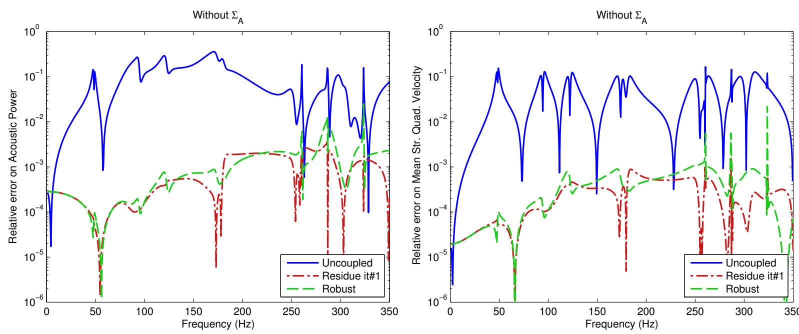

Fig. 9 Comparison of results errors (shoe box) compared with full response for uncoupled basis, residue iteration (after first iteration), robust basis. Case without absorbing area.

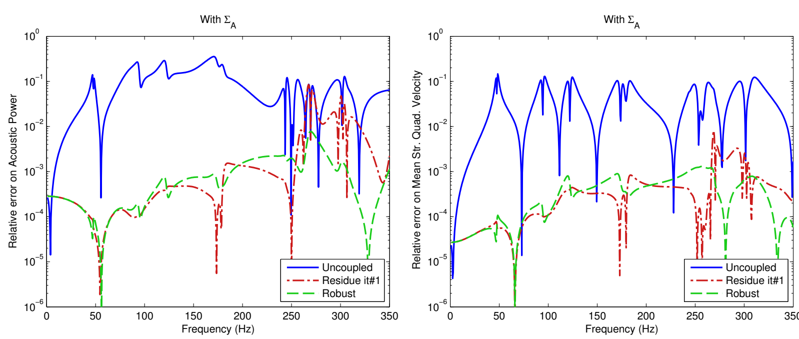

Fig. 10 Comparison of results errors (shoe box) compared with full response for uncoupled basis, residue iteration (after first iteration), robust basis. Case with absorbing area.

4.4 Conclusions

In this paper, it has been shown that the consideration of static effects of cavity was mandatory to obtain precise results concerning the vibroacoustic response of damped systems. For model reduction purpose, including the static mode by itself helps improving convergence in a fast and easy way. A new $(\mathbf{u}, p, p^S)$ formulation has been proposed to automatically include static effects in a proper way, leading to definite reduced stiffness matrix. Finally, efficiency of residual terms added to the uncoupled bases is demonstrated using two different methodologies that allows great improvement of the convergence, even in presence of absorbing layers in the fluid cavity.

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