Analytical investigation of the pull-in voltage in capacitive mechanical sensors

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ABSTRACT

A MEMS capacitive sensor is basically an electrostatic transducer and an analytical approach to evaluate the pull-in voltage associated with a clamped circular plate or a circular membrane due to a bias voltage is presented. The approach is based on a linearized uniform approximation of the nonlinear electrostatic force due to a bias voltage and the use of a 2D load-deflection model for the clamped plate or membrane. In particular, the large deflection of the clamped thin, circular and isotropic plate is investigated when a transverse loading and an initial in-plane tension load are applied. The transition from plate behavior to membrane behavior is analyzed. The edge zone region is explored and properties of this region are given. The resulting electrostatic pressure on the diaphragm of the MEMS capacitive sensor and the pull-in voltage are studied.

Keywords: MEMS, von Karman plate theory, circular plate, circular membrane, pull-in voltage, edge zone

1. INTRODUCTION

Microsystems contain many different kinds of structural elements. The devices and the packages that hold them are solid objects governed by the laws of continuum solid mechanics, and the moving parts within the microsystems are usually critical to overall operation. An important structural element in a MEMS capacitive sensor is the diaphragm which can be modeled by a clamped thin circular plate or a clamped membrane. Furthermore, an initial tension, due to stretching and packaging during micro-fabrication processes, is often encountered. The effect of initial tension leads the appearance of severe variation of physical behavior around the clamped edge and the onset of nonlinearity. A degradation of the diaphragm is then possible. Little is known concerning the relationship between initial tension and the characteristic stress and strain fields, which determine the performance of the transducer. Therefore, it is desirable to examine the effects of internal stress on the structural behavior of these devices. The communication deals specifically with the deflection of a clamped circular plate with an initial in-plane tension load, subjected to a uniform pressure load over its surface. In particular, the effects of varying the initial tensile load from zero: pure plate case to very large values: pure membrane case are explored. The large deflection is studied by the von Karman theory[1-5] and analytically results in terms of stress, transverse deflection, deflection slope and curvature are derived. In the following section the governing equations are developed and the appropriate non dimensional scaling parameters identified.

In the second part of the communication an accurate determination of the pull-in, or collapse voltage, is presented. The approach is based on a linearized uniform approximation of the nonlinear electrostatic force due to a bias voltage and the use of a 2D load-deflection model for the clamped circular diaphragm. The central component in micro-electro-mechanical systems is the mechanical resonator which constitutes a capacitive transducer and is formed with two plates: a fixed plate and a movable plate (or a diaphragm). Due to the electrical force, when the gap between the two plates becomes two thirds of the initial gap, the movable plate is not stable, we have a ''push-down'' phenomenon and the MEMS fails. From the dynamics point of view, the system loses its stability. The gap being equal to two-thirds of the initial gap is termed the minimum gap in MEMS[6]. In fact, stoppers are used in the design of MEMS for avoiding such a failure of the capacitor. This capacitive-type sensor is then basically an electrostatic transducer that depends on electrical energy in terms of constant voltage (voltage drive) to facilitate monitoring of capacitance change due to an external mechanical excitation such as a force or an acoustical pressure. The electrostatic force associated with the voltage is non linear due to its inverse square relationship with the airgap thickness between the capacitor electrodes. This gives rise to a phenomenon known as ''pull-in'' that causes the movable structure to collapse if the bias voltage exceeds the pull-in limit and limits the effective range of deformation of the structure. Accurate determination of the pull-in voltage, or the collapse voltage, is critical in the design process to determine the sensitivity, harmonic distortion and the dynamic range of a MEMS-based capacitive transducer. Pull-in instability is fundamental to the understating of many MEMS devices and is one of the basic parameters of the design of many electrostatic MEMS devices. Lack of an accurate model for predicting pull-in voltage for basic beam structures such as fixed-fixed beams, fixed-free beams or for basic membranes necessitates the clear need for a closed form expression for the pull-in voltage. Attempts have been made by several authors in the past[6,7] to derive a closed-form expression for the pull-in voltage. In this communication a new analytical solution is presented to calculate the pull-in voltage and plate (or membrane) deflection for a clamped circular plate (or membrane) under electrostatic actuation.

2. DEFLECTION OF A CLAMPED CIRCULAR PLATE

2.1 Problem formulation

Consider a circular plate of radius a and constant thickness h under a uniform transverse load $p_z = p_0$ and an initial tension load $N_r = N_0$, as shown in Figure 1.

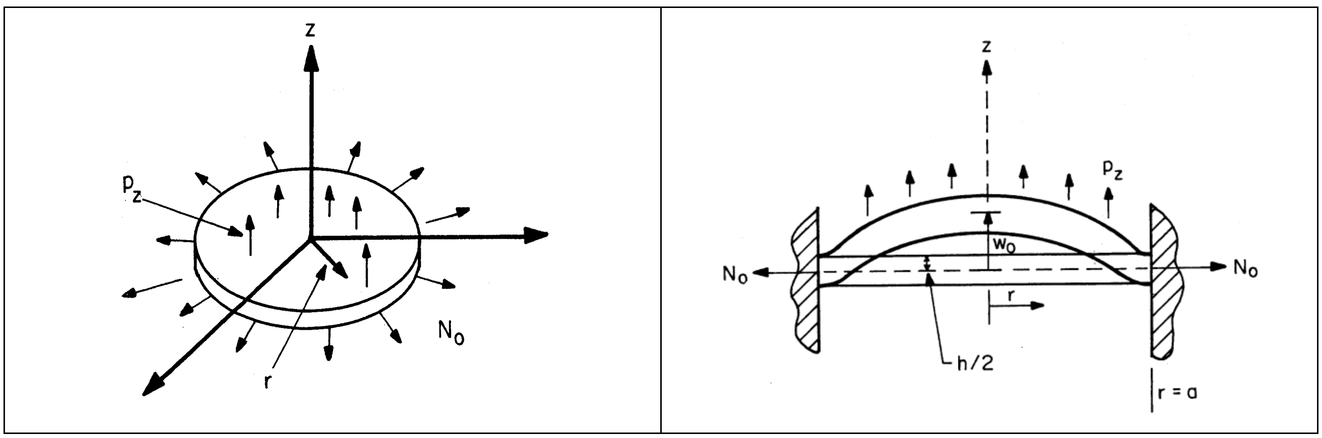

Figure 1. Schematic of a clamped circular plate under an initial in-plane stress N_0

Based on von Karman plate theory for large circular plate deflections, the radial and tangential midplane strains are[1]

$$\varepsilon_r = u_{,r} + \frac{1}{2}(w_{,r})^2 \qquad ; \qquad \varepsilon_\theta = \frac{u}{r} \tag{1}$$

Where $(\ )_{,r}$ indicates the differentiation with respect to the radial coordinate r, u the radial displacement and w the normal displacement or the deflection of the plate in the z-direction . The radial and tangential curvatures are defined as

$$K_r = -\, w_{,rr} \qquad ; \qquad K_\theta = -\frac{1}{r} w_{,r} \tag{2}$$

Note that a distance z from the middle surface there are additional strain due to bending, namely

$$\varepsilon'_r = z\, K_r = -z\, w_{,rr} \qquad ; \qquad \varepsilon'_\theta = z\, K_\theta = -\, z\, \frac{1}{r} w_{,r} \tag{3}$$

Applying Hooke’s law, the radial N_r and tangential N_θ loads per unit length for large plate deflections are[1]

$$\begin{bmatrix} N_r \\ N_\theta \end{bmatrix} = \left(\frac{E\,h}{1 - v^2} \right) \begin{bmatrix} 1 & v \\ v & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_r \\ \varepsilon_\theta \end{bmatrix} \tag{4}$$

where E is the modulus of elasticity and v the Poisson's ratio. The inversion of (4) yields

$$\begin{bmatrix} \varepsilon_r \\ \varepsilon_\theta \end{bmatrix} = \frac{1}{E\,h} \begin{bmatrix} 1 & -v \\ -v & 1 \end{bmatrix} \begin{bmatrix} N_r \\ N_\theta \end{bmatrix} \tag{5}$$

The radial M_r and tangential M_θ bending moments per unit length for large plate deflections are[1]

$$\begin{bmatrix} M_r \\ M_\theta \end{bmatrix} = D \begin{bmatrix} 1 & v \\ v & 1 \end{bmatrix} \begin{bmatrix} K_r \\ K_\theta \end{bmatrix} \tag{6}$$

where $D = Eh^3/12(1- v^2)$ is the flexural rigidity of the plate. The moment M_r acts along circumferential sections of the plate and M_θ acts along the diametral section rz of the plate.

The governing equations for the symmetrical bending of this circular plate are based on force and moment equilibrium. They can be formulated in terms of lateral slope $w_{,r}$, in-plane lateral loads N_r and N_θ as well as bending moments M_r and M_θ

$$(r\, N_{,r})_{,r} - N_\theta = 0 \tag{7}$$

$$(r\, Q_r)_{,r} + (r\, N_r\, w_{,r})_{,r} + r\, p_0 = 0 \tag{8}$$

$$r\, Q_r = (r\, M_r)_{,r} - M_\theta \tag{9}$$

where Q_r is the shear force which can be obtained by integrating (8)

$$Q_r + N_r\, w_{,r} + \frac{1}{2}\, p_0\, r = 0 \tag{10}$$

Placing relations (2) and (6) into (9) produces

$$Q_r = - D \left(w_{,rrr} + \frac{1}{r} w_{,rr} - \frac{1}{r^2} w_{,r} \right) \tag{11}$$

and combining (10) and (11) we obtain

$$w_{,rrr} + \frac{1}{r} w_{,rr} - \frac{1}{r^2} w_{,r} - \frac{N_r}{D} w_{,r} = \frac{p_0\, r}{2\,D} \tag{12}$$

Moreover, by considering the compatibility form of equation (1)

$$\varepsilon_r = r\, \varepsilon_{\theta\,,r} + \varepsilon_\theta + \frac{1}{2} (w_{,r})^2 \tag{13}$$

and substituting equation (5) in this compatibility equation, we obtain from (7)

$$N_{\theta\,,r} + \frac{N_\theta - N_r}{r} + \frac{E\,h}{2\,r} (w_{,r})^2 \tag{14}$$

Equations (7), (12) and (14) represent the three famous von Karman equations for the determination of the three unknowns w, N_r and N_θ . The boundary conditions for the clamped plate are $w = 0$, $w_{,r} =0$ and $Ehu/r = N_\theta - \nu N_r = 0$ at $r = a$ and $w_{,r} =0$ at $r=0$.

For the problem considered here, the circular plate is first stretched by an in-plane tension load $N_r = N_\theta = N_0$ around its circumference and then subjected to a uniform load p_0 . The solution of the initial in-plane tension problem is obtained by setting w=0 and p_0 = 0. This yields to $N_r = N_\theta = N_0$ and $u = N_0(1- \nu)r/Eh$. After being initially stretched by the load N_0 the plate is then subjected to a vertical load p_0. The lateral forces can be written in an incremental form

$$N_r = N_0 + \tilde{N}_r \quad \text{and} \quad N_\theta = N_0 + \tilde{N}_\theta \tag{15}$$

where $\tilde{N}_r$ and $\tilde{N}_\theta$ are incremental changes from N_0 and are lateral loads due to the applied transverse load p_0. These forces are functions of r. The placement of these expressions into (7) , (12) and (14) yields

$$(r \tilde{N}_r)_{,r} - \tilde{N}_\theta = 0 \tag{16}$$

$$w_{,rrr} + \frac{1}{r} w_{,rr} - \frac{1}{r^2} w_{,r} - \frac{N_0 + \tilde{N}_r}{D} w_{,r} = \frac{p_0 r}{2D} \tag{17}$$

$$\tilde{N}_{\theta,r} + \frac{\tilde{N}_\theta - \tilde{N}_r}{r} + \frac{Eh}{2r}(w_{,r})^2 \tag{18}$$

It is convenient to transform these three equations to a dimensionless form, in which the following dimensionless variables are introduced

$$W = w/h \; ; \; U=u/h \; ; \quad \xi = r/a \; ; \; \theta = \frac{dW}{d\xi} = W' = \frac{a}{h} w_{,r} \; ; \; S_r = \frac{\tilde{N}_r a^2}{E h^3} \; ; \; S_\theta = \frac{\tilde{N}_\theta a^2}{E h^3}$$

We note $(\;)' = \frac{d}{d\xi}$ and introduce the dimensionless curvature term $\Psi = \theta' = \frac{a^2}{h} w_{,rr}$,the nondimensional tension parameter $k = a\sqrt{\frac{N_0}{D}} = \frac{a}{h}\sqrt{\frac{N_0 12(1-\nu^2)}{Eh}}$ and the nondimensional loading parameter $P = \frac{p_0 a^4}{E h^4}$. W is the nondimensional deflection of the plate, ξ the nondimensional radius or the normalized radial distance and θ the slope of the nondimensional defection. It is easy to show that $w_{,r} = \theta h/a$; $w_{,rr} = \theta' h/a^2$ and $w_{,rrr} = \theta'' h/a^2$. We obtain then the three nondimensional equations

$$\xi S'_r + (S_r - S_\theta) = 0 \tag{19}$$

$$\xi^2 \theta'' + \xi \theta' - [1+\xi^2 (k^2 + S_r E h^3/D)] \theta = 6P(1- \nu^2) \xi^3 \tag{20}$$

$$\xi S'_\theta + (S_\theta - S_r) + \frac{\theta^2}{2} = 0 \tag{21}$$

Our objective is to solve equation (20) to obtain the transverse deflection $w(r)$ of the plate, the curvature, the radial stress and the tangential stress.

2.2 Linear problem and an analytical solution

To have an informative insight, we assume that the non-dimensional midplane force S_r is small so that the nonlinear term $S_r \theta$ is negligible. Thus, a non-homogenous linear differential equation for the non-dimensional slope is obtained

$$\xi^2 \theta'' + \xi \theta' - (1+k^2\xi^2)\theta = 6P(1-\nu^2)\xi^3 \tag{22}$$

This equation is a modified Bessel equation[1] possessing the general equation

$$\theta(\xi) = A I_1(k\xi) + B K_1(k\xi) - 6P(1-\nu^2)\xi/k^2 \tag{23}$$

where $I_1(k\xi)$ *and* $B K_1(k\xi)$ are modified Bessel functions of the first kind and second kind respectively. Imposing the boundary conditions : at $\xi = 0$, $\theta = 0$ and at $\xi = 1$, $\theta = 0$, we obtain the constants of integration $A = 6P(1-\nu^2)/I_1(k)k^2$ and B=0. The solution for $\theta(\xi)$ is then

$$\theta(\xi) = \frac{6P(1-\nu^2)}{k^2}\left[\frac{I_1(k\xi)}{I_1(k)} - \xi\right] \tag{24}$$

The curvature term is then

$$\Psi(\xi) = \frac{d\theta}{d\xi} = \frac{6P(1-\nu^2)}{k^2}\left[\frac{k I_0(k\xi)}{I_1(k)} - \frac{1}{\xi}\frac{I_1(k\xi)}{I_1(k)} - 1\right] \tag{25}$$

The corresponding nondimensional transverse deflection is obtained by integrating (24)

$$W(\xi) = \frac{6P(1-\nu^2)}{k^2}\left[\frac{(I_0(k\xi) - I_0(k))}{k I_1(k)} + \frac{1-\xi^2}{2}\right] \tag{26}$$

and the transverse deflection of the plate is

$$w(r) = \frac{6Ph(1-\nu^2)}{k^2}\left[\frac{(I_0(kr/a) - I_0(k))}{k I_1(k)} + \frac{a^2 - r^2}{2a^2}\right] \tag{27}$$

The radial stress σ_r and the tangential stress σ_θ in the plate are found from the definitions of the stress resultants

$$\sigma_r = \frac{N_r}{h} + \frac{12 M_r z}{h^3} = \frac{N_r}{h} + \frac{6 p_0 a^2}{h^3 k^2}\left[1 + \nu - \frac{k I_0(kr/a)}{I_1(k)} - \frac{a(\nu - 1) I_1(kr/a)}{r I_1(k)}\right] z \tag{28}$$

$$\sigma_\theta = \frac{N_\theta}{h} + \frac{12 M_\theta z}{h^3} = \frac{N_\theta}{h} + \frac{6 p_0 a^2}{h^3 k^2}\left[1 + \nu - \frac{k \nu I_0(kr/a)}{I_1(k)} + \frac{a(\nu - 1) I_1(kr/a)}{r I_1(k)}\right] z \tag{29}$$

Two limit cases are now considered: the pure plate case where $k=0$ and the pure membrane case where $k \to \infty$.

For the pure plate case equation (22) becomes an ordinary differential equation and the solution for $\theta(\xi)$ is

$$\theta(\xi) = -\frac{3}{4} P(1-\nu^2)\,\xi(1-\xi^2) \tag{30}$$

We deduce respectively the curvature term, the nondimensional deflection and the transverse deflection of the plate

$$\Psi(\xi)= -\frac{3}{4} P\,(1- \nu^{2})\,(1-3\,\xi^{2}) \tag{31}$$

$$W(\xi)= \frac{3}{16} P\,(1- \nu^{2})\,(1- \xi^{2})^2 \tag{32}$$

$$w(r) = \frac{3}{16} h\, P\,(1- \nu^{2})\,(1-(\frac{r}{a})^{2})^2 \tag{33}$$

For the pure membrane case we obtain from (22)

$$\theta(\xi) = -\frac{6P\,(1-\nu^2)\,\xi}{k^2} \tag{34}$$

and for large values of k , using the property $\lim I_n(u) = \frac{e^u}{\sqrt{2\pi u}}$ as $u \to \infty$, the curvature is approximated from (25) as

$$\Psi(\xi) = -\frac{6P(1-\nu^2)}{k^2}\left[\,(1-k\xi)\,\frac{e^{-k(1-\xi)}}{\xi^{3/2}} +1\,\right] \tag{35}$$

The nondimensional vertical deflection and the transverse deflection of the pure membrane are obtained from (34) by integration

$$W(\xi) = \frac{3}{k^2} P\,(1- \nu^{2})\,(1- \xi^{2}) \tag{36}$$

$$w(r)= \frac{3\,h}{k^2} P\,(1- \nu^{2})\,(1- (\frac{r}{a})^{2}) \tag{37}$$

In Figure 2 the center deflection of the disk (the plate or the membrane), normalized by the transverse load, is plotted against the initial nondimensional tension parameter k. The effect of initial in-plane tension is shown in this figure where two asymptotes are present. The horizontal part of the curve means the center deflection is in a linear proportion to the applied transverse load ($W(0) \propto P$ see also equation (32)) and is not a function of the initial tension, thus it reflects a plate behavior. The inclined straight line indicates a nonlinear variation of the center deflection with the tension parameter k ($W(0)/P \propto 1/k^2$ see also equation (36)), hence, a membrane behavior is revealed due to the nonlinearity of $W(0)/P$. In this Figure 2 there appears to be a transition from pure plate behavior to pure membrane behavior in the region from $k \approx 1$ to $k \approx 20$. For $k<1$ we have a plate behavior and for k>20 membrane behavior dominates the majority of the clamped plate.

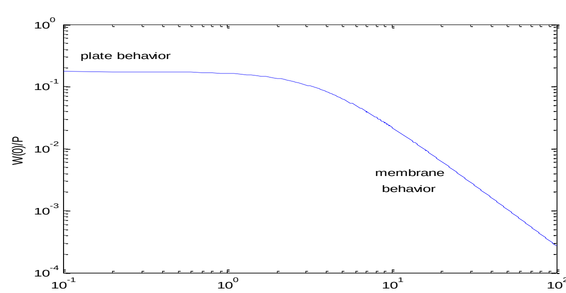

Figure 2. Center deflection normalized by the loading parameter as a function of the tension parameter

In order to have a thorough insight to the effects of initial tension upon the related geometrical responses, normalized deflection shapes are plotted in Figure 3. As the tension parameter k increases a sharp change in the curvature near the edge ($\xi \approx 1$) appears, in order to accommodate the zero slope boundary condition.

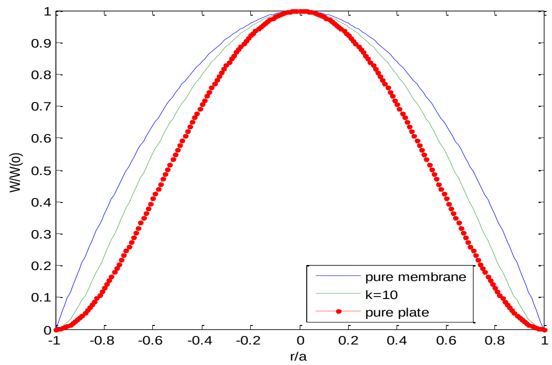

Figure 3. Normalized deflection as a function of normalized radial distance

Figure 4 shows the normalized slope $\theta /W(0)$ of the nondimensional deflection plotted versus the nondimensional or normalized radius $\xi = r/a$. The variation of the normalized deflection slope for a clamped circular plate depends strongly upon the initial tension. For a simply supported plate the dependance tends to be significant only up to a moderate initial in-plane tension load. Comparatively, a clamped end has effectively stiffened the plate aong the edge in response to the change of the initial tension. As the parameter tension k becomes relatively large ($k>50$) a membrane behaviour appears at the edge-zone. In the case of a pure palte it appears an important edge-zone an the extend of the edge-zone gets smaller as the tension parameter k increases.

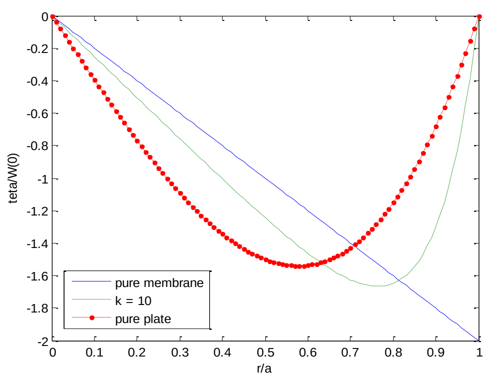

Figure 4. Normalized deflection slope as a function of normalized radial distance

The main contribution to the stress and strain fields come from the initial in–plane tension load and the bending forces, the later of which are dominated by the curvature term $\Psi(\xi)$ given in (25). Figure 5 shows the normalized deflection curvature term $\Psi(\xi)/P$ plotted versus nondimensional radius ξ . The large value of the nondimensional tension parameter k gives the curvature term (35) and for large k the maximum curvature term at the edge is $\Psi(1) = 6P (1- \nu^2)/k$ which is greater than the curvature term $\Psi(0) = -6P (1- \nu^2)/k$ occurring at the center of the circular plate and over most of the remaining portion of the plate. Physically, this indicates the presence of a stress concentration near the clamped plate boundary.

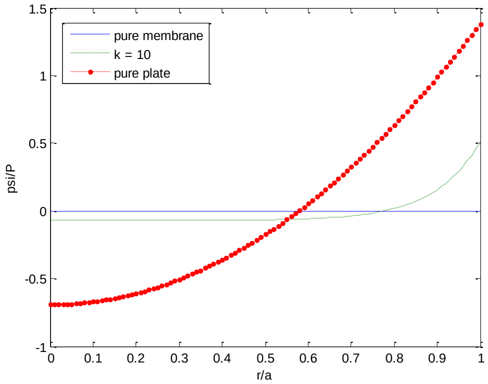

Figure 5. Normalized deflection curvature as a function of normalized radial distance

The nonlinear behavior of the plate is represented by the term $S_r\theta$ in (20) and by the term $\theta^2/2$ in (21). Using equation (20) and combining (19) and (21) we obtain a two coupled second order equations in the variables θ and S_r

$$2\xi^2\,\theta'' + \xi\,\theta' - [\,1+\xi^2\,(k^2 + S_r E\,h^3/D)]\;\theta = 6P(1-\nu^2)\;\xi^3 \tag{38}$$

$$2\xi^2\,S''_r + 4\xi\,S'_r + \theta^2 = 0 \tag{39}$$

This system can be solved numerically and is under investigation.

3. ANALYTICAL DETERMINATION OF THE PULL-IN VOLTAGE

Due to the presence of residual stress and a significantly large deflection of the diaphragm compared to its thickness, the developed strain energy in the middle of the diaphragm causes a stretch of the diaphragm middle surface. Figure 6 shows the principle of deflection of a diaphragm.

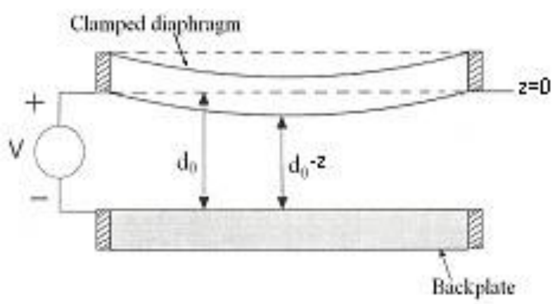

Figure 6. A schematic of electrostatic actuation

From equation (27) we deduce the uniform transverse pressure applied at the center of the plate

$$p_0 = \frac{2\,Dk^2\;w(0)}{a^4}\left[\frac{1}{2} - \frac{I_0(k)}{k\,I_1(k)}\right]^{-1} \tag{40}$$

We assume that the pull-in occurs when the deflection of the movable plate is one-third of the original air gap d_0 and the pull-in electrostatic pression is equal to the uniform pressure load[8]

$$\frac{5\,\varepsilon_0\,V_{pi}^2}{6\,d_0^2} = \frac{2\,Dk^2\,d_0}{3\,a^4}\left[\frac{1}{2} - \frac{I_0(k)}{k\,I_1(k)}\right]^{-1} \tag{41}$$

where V_{pi} is the pull-in voltage and ε_0 is the permittivity in free space. We obtain then the expression for the pull-in voltage for a rigidly clamped circular plate

$$V_{pi} = \frac{2\,k\,d_0}{a^2}\sqrt{\frac{D\,d_0}{5\,\varepsilon_0}\left[\frac{1}{2} - \frac{I_0(k)}{k\,I_1(k)}\right]^{-1}} \tag{42}$$

In the case of a pure plate the uniform transverse pressure is obtained from (33)

$$p_0 = \frac{64\,D\,w(0)}{a^4} \tag{43}$$

We have then at one-third of the original air gap

$$\frac{5\,\varepsilon_0\,V_{pi}^2}{6\,d_0^2} = \frac{64\,D\,d_0}{3\,a^4} \Rightarrow V_{pi} = \frac{8\,d_0}{a^2}\sqrt{\frac{2\,D\,d_0}{5\,\varepsilon_0}} \tag{44}$$

In the case of a pure membrane the uniform transverse pressure is obtained from (37)

$$p_0 = \frac{4\,D\,k^2\,w(0)}{a^4} \tag{45}$$

$$\frac{5\,\varepsilon_0\,V_{pi}^2}{6\,d_0^2} = \frac{4\,D\,k^2\,h}{3\,a^4} \Rightarrow V_{pi} = \frac{k\,d_0}{a^2}\sqrt{\frac{8\,D\,d_0}{5\,\varepsilon_0}} \tag{46}$$

To illustrate the above model of pull-in evaluation a clamped circular diaphragm of Young's modulus E = 169 GPa, Poisson's ratio ν = 0,28, thickness h = 0,8 μm, airgap thickness d_0 = 3,5 μm, permittivity in free space ε_0 = 8,5x10^{-12} F.m^{-1} an residual in-plane stress $\sigma_0 = N_0/h$ = 20 MPa is considered. Figure 7 shows the pull-in voltage for a plate having the previously device parameters and for a pure membrane as a function of radius. It is evident that there is negligible difference between the pull-in voltage evaluated using a pure membrane model and given by equation (46) and the pull-in voltage our circular clamped plate.

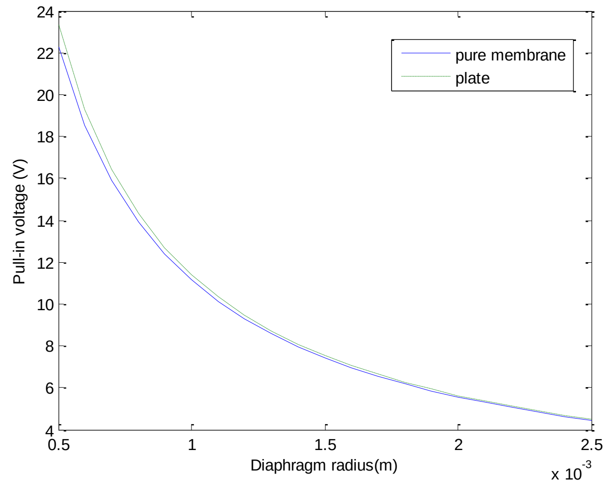

Figure 7. Pull-in voltage for a pure membrane and for a plate

4. CONCLUSION

The deflections of a clamped circular plate under a uniform transverse load and in-plane tension loads have been studied in the communication. The transition from plate behavior to membrane behavior has been described. A new relatively simple closed-form model to evaluate the pull-in voltage associated with a rigidly clamped circular diaphragm subject to an electrostatic force has been presented and numerical results have been obtained showing the pure membrane behavior and plate behavior.

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