

Advances in high quality factor optical resonators for optoelectronics

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ABSTRACT

Recent developments and results concerning high quality factor optical resonators and some applications to optoelectronics are described in this paper.

Keywords: Oscillator, optical resonator, high quality factor.

1. INTRODUCTION

Substantial progress has been done in order to obtain optical resonators with high quality factors (Qs). These resonators are particularly interesting for many applications in microwave-photonics, time-frequency metrology and in fundamental physics. [1–4]. For instance, these high-Q optical resonators can be used as high selectivity filters for optical and microwave filtering, all-optical switching and also in low threshold non-linear optics. Recent studies have demonstrated interesting results obtained with crystalline whispering gallery mode resonators (WGMRs), such as MgF₂ (magnesium fluoride), quartz or CaF₂ (calcium fluoride) WGMRs [5–9]. On the other hand, rare-earth doped micro-spheres have been used to set-up very compact laser sources that can generate narrow linewidth emission [10,11]. Another interesting type of high-Q optical resonators is the fiber ring resonator (FRR), which is easy to fabricate and to use, and can feature ultra-high quality factors (Qs above 10¹⁰) [12–14].

The design/fabrication and the characterization of these optical resonators are two first critical steps on the way to get very high intrinsic optical quality factors (Q₀). Afterwards, to be able to use such high-Q resonators, depending on the application, the laser lightwave must be efficiently coupled into and out of the optical resonator. At that time, an optimal coupling state must be found in order to take full advantage of the resonator's high Q_{int}. Afterwards, the laser lightwave must be stabilized onto one optical mode of the resonator. All these steps will be addressed in this chapter.

These resonators are characterized using different methods such as slow and fast frequency sweeping [15–17]. In this chapter we also discuss the characterizations techniques of passive optical resonators and also active optical resonators like Erbium-doped ZBLALiP micro-spheres [18,19] and silica micro-spheres coated by film with molar composition 70SiO₂-30HFO₂ and activated with 0.3% mol Er³⁺ [17, 20] in terms of Q-factor.

We will focus on the different optical Qs' results obtained by using the cavity ring-down (CRD) technique for various optical resonators. Qs up to 10¹⁰ are obtained. We will also talk about the different thermal treatments used to increase the Q-factors of crystalline optical resonators. Afterwards, we will present some examples of applications in the optoelectronic field, such as ultra-stable optoelectronic oscillators (OEOs) used to generate high spectral purity microwave signals.

2. DIFFERENT TYPES OF OPTICAL RESONATORS

In this section, four different types of resonators are discussed. We focus on those presenting the best Qs. These resonators are millimetric-size, like the WGMRs which are of different shapes (ex. micro-spheres, disks, torus etc.), or fibered, like the FRRs that are few meters long or few tens of meters long. This section is mainly inspired from an article published with our colleagues from several laboratories [21].

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2.1. Film coated silica micro-spheres:

Silica micro-spheres are coated by Er³⁺- activated silica-hafnia films. They are designed and realized in Italy at the IFN-CNR [18,19]. Micro-spheres are produced by the following process: melting the tip of a standard optical telecommunication fiber. Typical diameter of the micro-sphere is 140±10 μm determined by a standard optical microscope. The micro-spheres are coated with a film with the molar composition 70SiO₂-30HfO₂ activated with 0.3% mol Er³⁺. The films are coated using a dip coating technique. Figure 1 shows a typical spectrum of the lasing that can be obtained using such resonators.

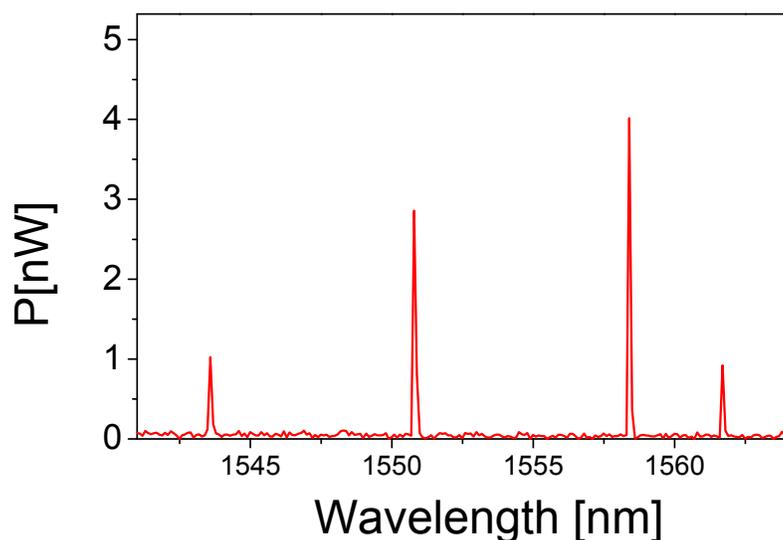


Figure 1. Laser whispering gallery modes of a silica sphere coated by 1 μm film 70SiO₂-30HfO₂ activated with 0.3% mol Er³⁺ ions.

2.2. Erbium doped ZBLALiP micro spheres:

Erbium doped ZBLALiP micro spheres are obtained by fusion in a microwave plasma torch at FOTON [9]. Diameters of the micro spheres are between 50 μm and 150 μm. The spheres are coupled using tapered optical fibers. The method used to measure high Qs consists of a careful analysis of the CRD signal obtained by exciting the resonator with a fast frequency-modulated laser [12], (see Figure 2). The analysis gives a $Q_0 = 1.055 \times 10^{10}$ and an external quality factor $Q_e = 1.063 \times 10^{10}$. The global loaded quality factor ($Q_{Opt}^{-1} = Q_0^{-1} + Q_e^{-1}$) is therefore determined to be equal to 5.294×10^9 .

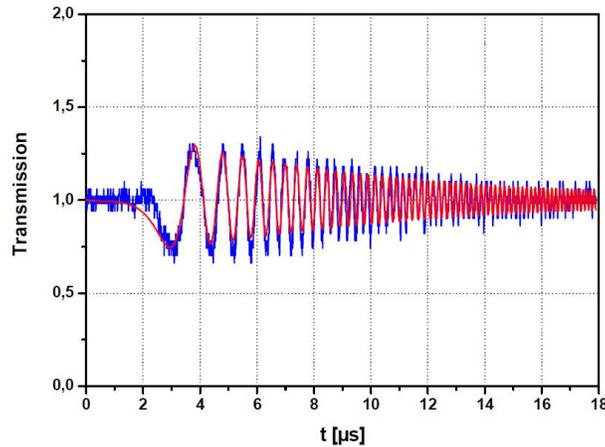


Figure 2. CRD signal obtained at a quasi-critical regime of losses compensation. An intrinsic quality factor $Q_0 = 1.055 \times 10^{10}$ and an external quality factor $Q_e = 1.063 \times 10^{10}$ can be derived.

2.3. Disk WGMRs:

It has been demonstrated that an oscillator referenced on an optical resonator can deliver high spectral purity microwave signal at a frequency equal to a multiple of the resonator's free spectral range (FSR) [3] with taper coupling. In the case of a WGMR, the FSR will be in the order of few GHz to few tens of GHz depending on the WGMR's size. The use of a high Q factor WGM resonator based on magnesium fluoride or on calcium fluoride is suitable for microwave photonics applications, as such resonators can feature very high Qs. 5.5-mm diameter disk-shaped MgF₂-based and CaF₂-based WGMRs are fabricated at FEMTO-ST. Several other materials are also used. During their fabrication, the WGMRs' surface roughness is in the range of few nanometers (around 5 nm). Intrinsic Qs above 3×10^9 have been demonstrated.

2.4. Fiber ring resonators:

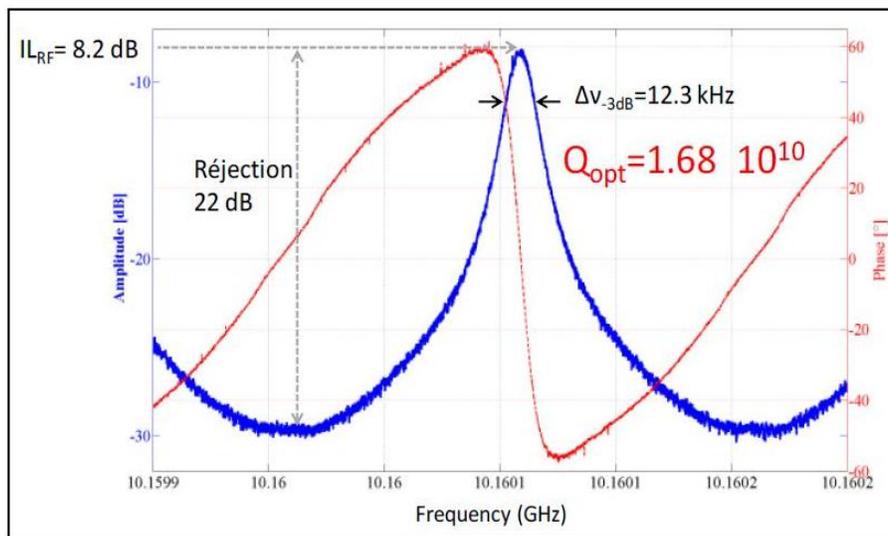


Figure 3. Measurement of the transmission S_{21} amplitude response of a 1000m-long fiber ring resonator, using the Pound Drever Hall (PDH) technique to lock the laser lightwave onto an optical resonance of the resonator [23,24] and a vector network analyzer. The measured loaded Q factor and the insertion loss (IL) are identical to simulated ones. S_{21} phase response is also measured as it can give the same information on the resonator's loaded Q factor.

An elegant alternative to the WGMRs is the ultra-high Q fiber ring resonator. Indeed, Q_s above 10^{10} have been achieved using these relatively compact resonators (few meters or few tens of meters) [13], (see Figure 3). Accordingly, outstanding performances have been demonstrated for OEOs based on FRRs [12,22], (see Figure 4). These FRRs are fabricated by our colleagues at LAAS-CNRS. They particularly have the advantages to be compact, easy to fabricate and to use [13].

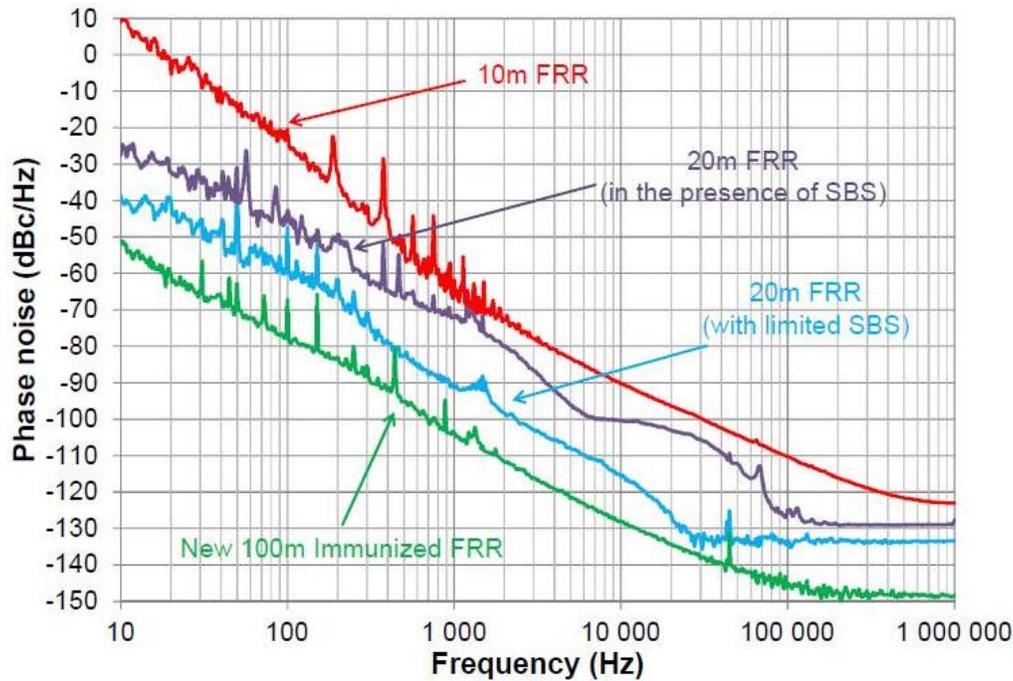


Figure 4. Different phase noise results obtained through the different noise studies made to improve the phase noise of a FRR-based OEO [12]. The phase noise measurements are performed using an Agilent E5052B signal source analyzer. SBS stands for stimulated Brillouin scattering.

On the other hand, the ultra-high Q_{opt} of this FRRs increases the circulating optical power inside the FRR and leads to the generation of different nonlinear optical phenomena at very low thresholds, especially the Rayleigh and Brillouin scatterings [12,25], (see Figure 5). These phenomena can be very useful for different applications, like high power and high spectral purity millimeter-waves generation [21]. Nevertheless, they can be considered as parasitic signals in an OEO setup. Therefore they can degrade its phase noise [12]. That's why both theoretical and experimental studies were performed to be able at the same time to suppress these phenomena and reduce the phase noise in an OEO setup. These studies were performed at Toulouse. They were focused particularly on two optical scattering phenomena: Rayleigh and Brillouin scatterings [12].

The use of a low injected optical power inside the FRR has first allowed a good limitation of these two nonlinear scattering phenomena (see Figure 4). As a result, a remarkable reduction in the OEO phase noise has been demonstrated at 10 Hz offset frequency from a 10 GHz microwave carrier [25]. Afterwards, an OEO based on a new immunized and high-Q FRR has been demonstrated with a very low phase noise level. Indeed, experimental results have demonstrated a significant reduction in the phase noise of the final optimized system, with a -50 dBc/Hz phase noise level at 10 Hz offset frequency from a 10 GHz carrier [12,22], (see the green curve in Figure 4). This result is also comparable to the best existing oscillator of the same type, an active cavity based coupled optoelectronic oscillator.

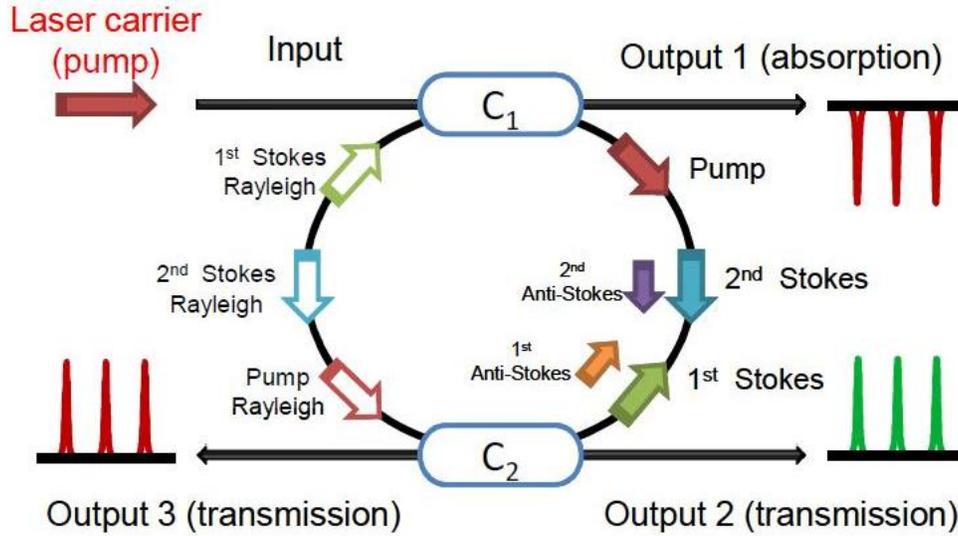


Figure 5. Double directional coupler fiber ring resonator, and generation of nonlinear optical effects inside the 20m-long FRR.

On the other hand, the performance of an OEO based on two different monocrystalline disk-shaped WGMRs (CaF_2 and MgF_2) have been investigated. First, the performance of the two WGMRs have been compared to prove the importance of an accurate laser lightwave coupling into and out of the WGMR when the WGMR is meant to be used as the stability element inside an OEO. This accurate coupling was performed in order to take full advantage of the WGMR's high intrinsic quality factor. Indeed, as Figure 6 shows, the MgF_2 WGMR has a lower intrinsic Q factor than the CaF_2 WGMR. However, due to an accurate optical coupling in the case of the MgF_2 WGMR, a better loaded Q factor has been obtained, as shown in Figure 7(a). Accordingly, better phase noise results have been obtained for the OEO based on the MgF_2 WGMR [see Figure 7(b)]. A phase noise level of -97 dBc/Hz at 10 kHz from a 6 GHz carrier have been demonstrated [26]. On the other hand, it has been shown in [26], that the OEO based on WGMR is a complex system, where each noise contribution can severely degrade the phase noise performance. Also, it has been proven that an inappropriate setting of the parameters of the laser stabilization loop could drastically affect the performance of an optical resonator based OEO.

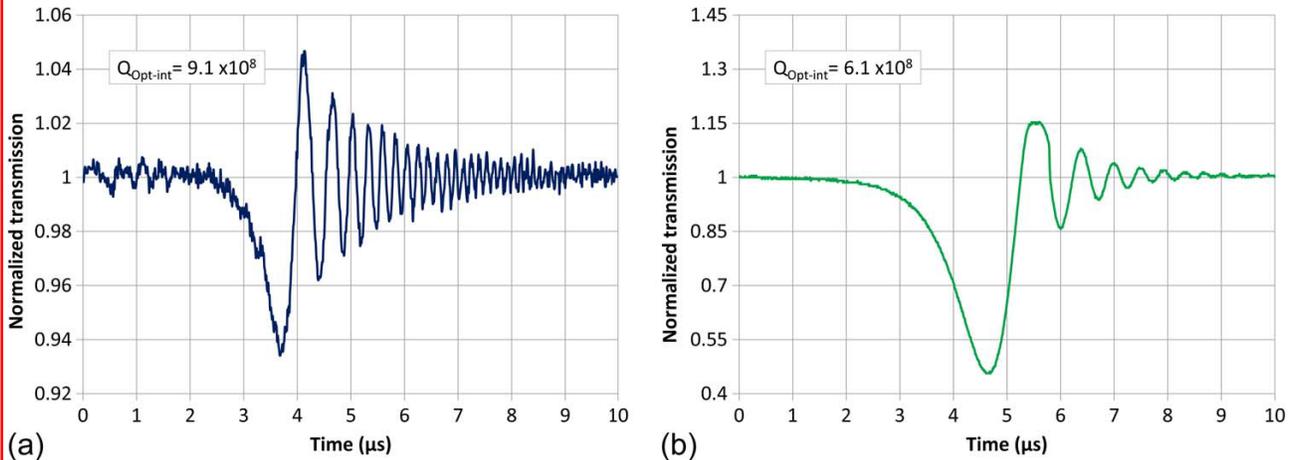


Figure 6. CRD measurement for two selected optical modes respectively excited in (a) the CaF_2 WGMR and (b) the MgF_2 WGMR.

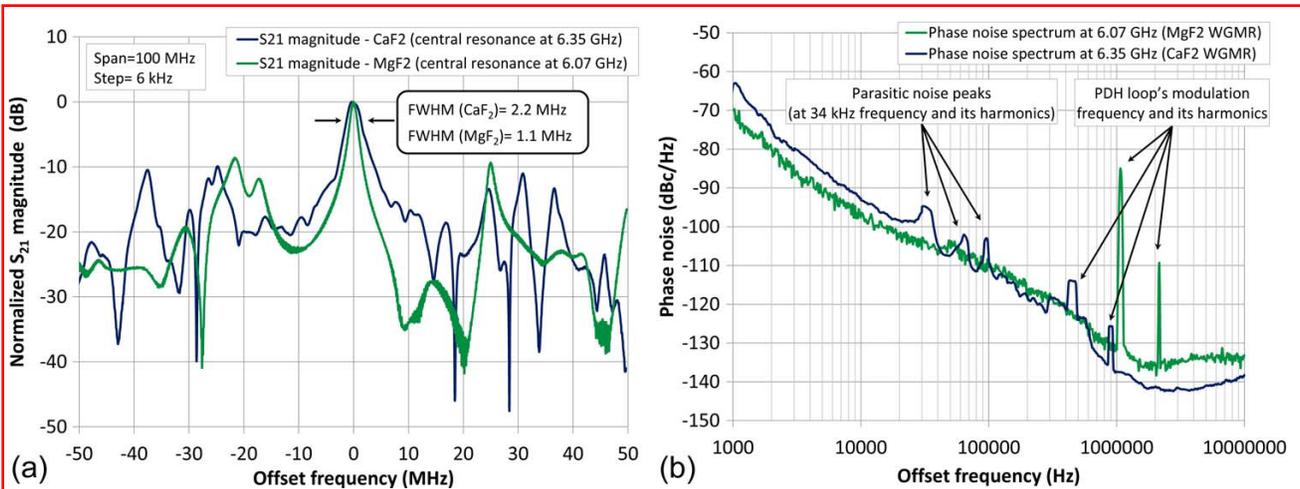


Figure 7. (a) S_{21} coefficient's magnitude measurement focused on the modes at 6.35 GHz in the CaF₂ WGMR and at 6.07 GHz in the MgF₂ WGMR. (b) Phase noise spectra obtained when the CaF₂ WGMR and the MgF₂ WGMR are used to stabilize the oscillation frequency in an OEO setup.

Accordingly, the performance of an optoelectronic oscillator based on different optical stability elements has been investigated. These stability elements were: the MgF₂ WGMR, a 4km long optical delay-line and a combination of both optical stability elements. This has been done in order to prove that, besides its use as a stability element, the high selectivity of a WGMR as a band-pass filter can lead to considerably reduce the spurious modes generated in a delay-line based OEO.

Despite the advantage of using a deal-line (DL) as an energy storage element in an OEO, the DL has still two major limitations that are actually linked: firstly the frequency unconditioned oscillation startup known as oscillation mode-hopping and secondly the presence of spurious modes in the generated signal of the OEO. If we can say it in other words, in an OEO based on a DL, the oscillation can start at a different frequency each time the OEO loop is closed. Frequencies for which the gain and phase conditions know as Barkhausen's criterion, are satisfied [see Fig. 6]. These possible oscillation modes are spaced by c/nL where n is the refractive index of the optical fiber and L is its length. In the 4km long DL, an oscillation is therefore possible every 50 kHz. To prevent such behavior, a very high selectivity and non-flat maximum optical (or microwave) band-pass filter must be added to the OEO's optoelectronic loop in order to select a frequency on which the oscillation should start on. Nevertheless the current optical and microwave band-pass filters are not very selective. Therefore the oscillation mode hopping can be relatively minimized but unfortunately not completely inhibited. Besides that, the oscillation RF spectrum will still be very large. It will contain numerous spurious modes, which are spaced too narrowly to be filtered by such filters. That's why these spurious modes are not convenient for different applications where the stability of the generated signal and its spectral purity are crucial, for example Doppler radars, reference signals distribution, telecommunications, etc. The WGMR represents in such a scenario, a great alternative as a band-pass filter. It is mainly thanks to its very high selectivity. Furthermore, it is considered as a complementary delay element, because of the large delay i. e. its high Q , it can induce to the optical carrier carrying an RF signal inside the OEO loop.

For these reasons, the performances of an OEO based on a combination of 4km-long DL and the MgF₂ WGMR have been investigated [27]. This combined optical stability element is abbreviated in this chapter by DL-WGMR.

As the results in Figure 8 show, the addition of the MgF₂ WGMR in the DL-based OEO (the DL-WGMR based OEO) has led to reduce the generated spurious modes by more than 53 dB for the first-neighboring spur [27].

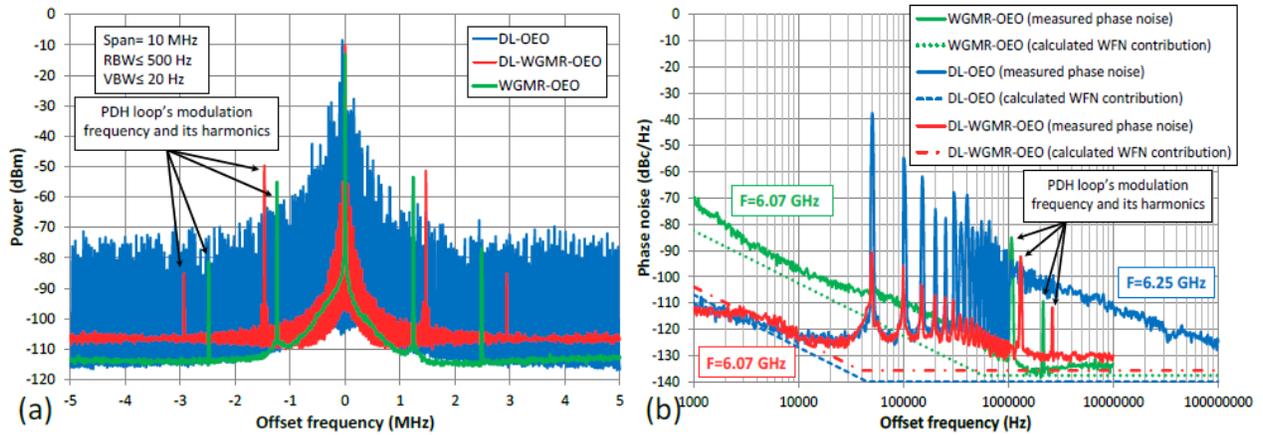


Figure 8. Comparison of the oscillation RF and phase noise spectra of three OEOs based each on a different optical stability element: a MgF_2 WGMR, a 4km long DL and a 4km long DL combined with a MgF_2 WGMR. The spurious modes are highly rejected in the DL-WGMR based OEO thanks to the high selectivity of the MgF_2 WGMR as a band-pass optical filter.

3. TYPICAL PROCESS FOR REALIZING RESONATORS

The diameter of the resonator is chosen in the range 5-6 mm in order to get a FSR in the range of 10 GHz. The manufacturing process is divided in two steps: the grinding and the polishing. The grinding generates the geometry of an optical guide: a rim of 50 micrometer on the circumference of the disk. It is realized using silicon carbide and on a very stable support to minimize the geometry default and increase the speed of grinding. The polishing represents the decreasing in the optical guide roughness. It is realized with a sub-micron diamond powder on felt (see Fig. 9). The hardness of the disk increases the time of each step. This process allows obtaining a very low roughness, 2 nm (see Fig. 10), and thus a very high Q factor [28].

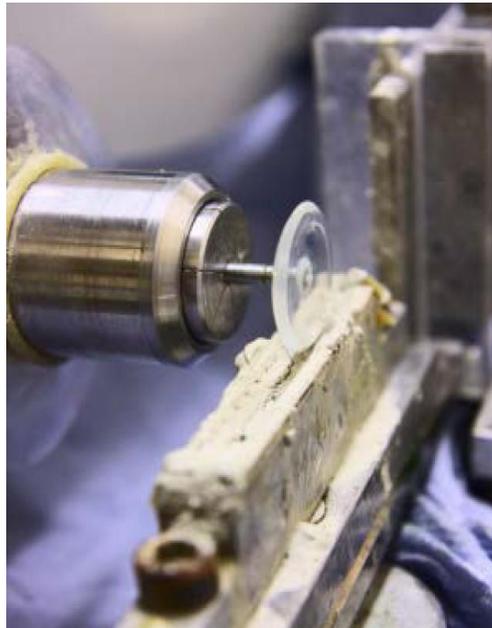


Figure 9. Polishing a mini-resonator.

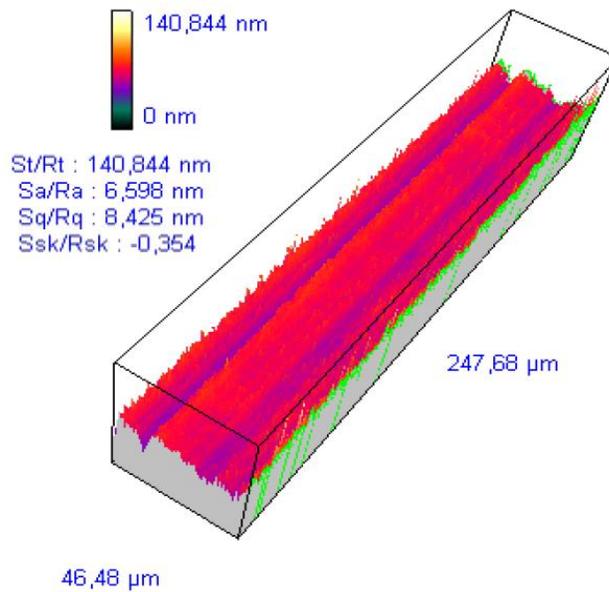


Figure 10. Average surface roughness is 6.5nm

The fabrication process of whispering gallery mode disk resonators is presented with different characterization techniques in order to monitor roughness and optical quality factor. This type of resonators is then used as the reference frequency element to stabilize the oscillation frequency of an optoelectronic oscillator. Experimental results are provided and demonstrate the efficiency of the method. Good coupling (see Fig. 11) method are reported in reference [15].

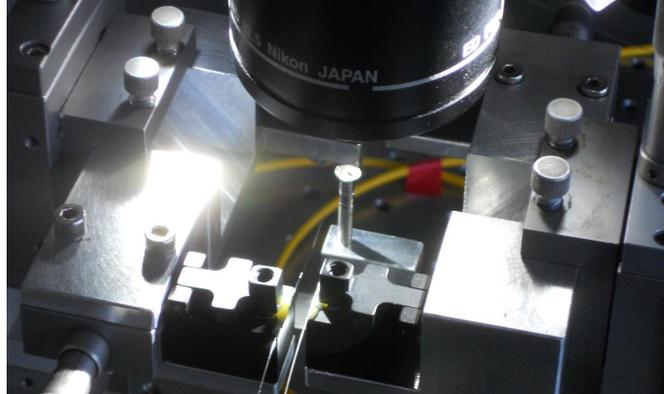


Figure 11. A 5.5 mm diameter disk-resonator coupled in transmission mode to two tapered fibers. The resonator acts like a band-pass filter with a high quality factor

4. CHARACTERIZATION OF RESONATOR QUALITY FACTORS

To characterize the Q-factor of resonators, a simple method was developed and presented in reference [14] to determine simultaneously the main characteristics of passive or active high-Q optical resonators. Such method is mainly based on cavity ring down spectroscopy. In such method, the probe wavelength is rapidly swept across the resonance. This technique helps to obtain the loaded cavity lifetime of passive resonators. The method was tested on Er³⁺ doped fiber

resonators. It was also applied to determine the intrinsic and external Q-factors of an MgF₂ whispering gallery mode (WGM) resonator.

The problem of the coupling between a high Q-factor resonator and its external coupler was improved theoretically and experimentally and reported in reference [15].

5. STATE-OF-THE-ART WGM OPTICAL RESONATORS

In the literature, we found four interesting papers reporting the best performances, to our knowledge, of WGM optical resonators.

5.1. Barium fluoride based optical resonator [29]

A monolithic optical whispering-gallery-mode resonator fabricated with barium fluoride (BaF₂) was demonstrated with an ultra-high quality (Q) factor above 10^9 at 1550 nm. It was measured with both the linewidth and cavity-ring-down methods. It can be underlined that vertical scanning optical profilometry shows that the root mean square surface roughness of 2 nm is achieved for a mm-size disk. It is actually the first time that a one billion Q-factor is achievable by precision polishing despite it is achieved in a relatively soft crystal with a 3 mohs hardness. Complex thermo-optical dynamics can take place in these resonators. Beside usual applications in nonlinear optics and microwave photonics, high-energy particle scintillation detection utilizing monolithic BaF₂ resonators potentially becomes possible.

5.2. Quartz based optical resonator [30]

A quality factor exceeding 5×10^9 was obtained in Besançon, France, in whispering-gallery mode (WGM) resonators fabricated of crystalline quartz. In x-cut resonators, a significant electrical tunability of WGMs was observed. Additionally, an electro-optic modulator with a sub-megahertz passband at 12 GHz was achieved. Some other photonics applications of the crystal quartz WGM resonators in narrow band agile tunable filters, compact narrow linewidth lasers, and microwave and millimeter wave oscillators were discussed in this reference.

5.3. Magnesium fluoride based optical resonator [31]

It is noticeable that generation of a 20 nm wide, 35 GHz repetition rate optical frequency comb was reported in a MgF₂ WGM resonator pumped with 2 mW of 1543 nm light. The high efficiency of comb generation is then associated with the small anomalous group velocity dispersion of the resonator. Growth dynamics of the comb is compared with theoretical predictions that were demonstrated earlier.

Magnesium Fluoride Whispering Gallery Mode Disk-Resonators were also reported in [9] for Microwave Photonics Applications.

5.4. Calcium fluoride based optical resonator [32]

A crystalline CaF₂ optical whispering gallery mode resonator was experimentally characterized by the quality factor $Q > 2 \times 10^{10}$ at $\lambda = 1.319 \mu\text{m}$. The same paper reports the demonstration of ultrahigh-Q resonators of lithium niobate and lithium tantalate ($Q > 2 \times 10^8$), and sapphire ($Q > 10^9$). Fundamental limits of the Q factor were discussed, and application of such resonators for measurement of ultra-small optical losses in various materials were investigated. Thermo-optical instability in whispering gallery modes of crystalline resonators was analyzed.

It is also interesting to read another paper concerning optical resonators with ten million finesse [33].

6. APPLICATIONS TO OPTOELECTRONICS OSCILLATORS

Optical resonators are good candidates for OEOs [3, 34]. The oscillation loop must be stabilized [35].

A complete study was performed on OEO. It is reported in [27]: the phase noise performance of optoelectronic oscillators was investigated for optical energy storage elements in the following configurations listed here:

1. a high Q-factor WGM resonator
2. an optical delay-line
1. a combination of these two elements.

First, the stability properties of these various optical elements were characterized. Then they were systematically compared in the optical and in the microwave frequency domains. The spectral purity of the oscillator is theoretically and experimentally examined for each of these cases. As a conclusion for this section, the generated spurious modes are highly rejected when the resonator is used as both delay and filtering element inside the delay-line based oscillator. Authors of the cited reference [27] demonstrated that a spur rejection by more than 53 dB for the first-neighboring spur.

Phase noise can be characterized [36]. Effect of Laser Coupling and Active Stabilization on the Phase Noise Performance of OEOs Based on Whispering-Gallery-Mode Resonators was investigated in [26]. It was shown that the resonator's intrinsic optical quality factor was not the only parameter to optimize in order to get a good phase-noise performance. The effect of the laser light wave coupling into and out of the optical resonator on the spectral purity of the oscillator was studied. Experimental results were obtained using two ultrahigh-Q disk resonators manufactured with calcium and magnesium fluoride, respectively.

7. CONCLUSION

Several types of resonators and recent performances in terms of high quality factor resonators have been presented in this paper, such as applications to opto-electronics.

8. ACKNOWLEDGEMENTS

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