A Post-Prognostics Decision Approach to Optimize the Commitment of Fuel Cell Systems in Stationary Applications

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Abstract

The use of fuel cells appears to be of growing interest as a potential alternative to conventional power systems. Fuel cell systems suffer however from insufficient durability and their lifetime may be improved. Prognostics results in the form of Remaining Useful Life are proposed to be used in a Prognostics and Health Management (PHM) framework to maximize the global useful life of a multi-stack fuel cell system under service constraint. The post-prognostics decision approach makes use of convex optimization to define the contribution of each stack over time to a global needed power output. A Mirror-Prox for Saddle Points method is proposed to cope with the assignment problem. Resolution method is detailed and promising simulation results are provided.

1 Introduction and related work

Due to the decline of fossil fuel resources, energy issues are moving to the forefront and the search for new energy solutions is on the rise. In this context, the use of fuel cells appears to be of growing interest as a potential alternative to conventional power systems [1]. Fuel cells are expected to be used in stationary applications, but also in transportation and portable power applications [2]. Durability of fuel cells are however not consistent with such applications. In fact, their lifetime reaches between 1500 and 3000 hours, whereas 5000 hours are required for transportation applications and up to 100000 hours for stationary ones. An important challenge highlighted by Borup et al. [2] consists then in improving the performance, reliability and lifetime of fuel cells.

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As pointed out by Jouin et al. [1], prognostics techniques of Prognostics and Health Management (PHM) domain can help increasing fuel cells lifetime. In particular, the study of the degradation allows to evaluate the current system health state and then to estimate its Remaining Useful Life (RUL). In their state-of-the-art [1], Jouin et al. have pointed out that researches in PHM dealing with fuel cells have been mainly focused on data acquisition and data processing. Less attention has been paid to condition assessment and diagnostics and very few works address prognostics and decision making. Papers taking into account the decision part propose furthermore only corrective actions (see [3] and [4]). For this kind of decision, physical parameters are controlled to master each fuel cell operating conditions as accurately as possible. These internal parameters include inlet and outlet gas flows, pressures and temperatures, single cell and stacks voltages and current. This real-time control (from nanoseconds to seconds) is necessary to compensate the natural fluctuation of these parameters and to avoid too early irreversible degradations. At each time it allows also to set the operating current to meet the needs in power for each fuel cell. In this paper, decision making process is considered at a larger scale of time (hours to weeks). In the considered PHM framework, decision comes within the scope of Prognostic Decision Making (PDM), which aims at choosing an appropriate system configuration [5]. Basically, the addressed problem is to provide the power output value for each fuel cell as a function of time, on the basis of a global power demand. Target application considered here is based on stationary power generation for domestic usage, also known as micro combined heat and power (micro-CHP).

In order to deliver suitable power outputs, fuel cells are used in the form of stacks, composed of many individual connected cells. Each stack is supposed to be independent, but the multi-stack fuel cell system has to deliver a given global power output based on a need of energy. At each time, the total provided power output is the sum of each output of the stacks that are currently running. Each fuel cell stack is able to deliver an output that can vary continuously and take any value within a given interval. The optimization problem consists then in determining the appropriate output for each fuel cell stack during the whole production horizon. All the stacks are not supposed to be running at each time if the target output can be reached by using only a subset of them. All of the stacks are not always available, their end of life being reached or not. Considering a global needed power output, the multi-stack system useful life depends not only on each stack useful life, but also on both the schedule and the operating condition settings that define the contribution of each stack over time. The same statement applies to batteries in a health management context. Saha et al. [6] have for instance addressed the maximization of the battery charge used while constraining the probability of a battery shut off in flight for electric unmanned aerial vehicles. Predictions on remaining battery life are used to optimize mission plans without exceeding the available battery charge. In a same way, we propose to use prognostics results in the form of RUL to maximize the global useful life of a multi-stack fuel cell system under service constraint.

In a PHM framework, as shown in [7], a platform useful life can be extended by managing the usage of machines thanks to the knowledge of each machine RUL. In this study, machine throughputs have been considered to be in a discrete domain. Complexity results have been proposed in [8] for different instances of the optimization problem. The problem can be solved in polynomial time under some restrictive assumptions, while it turns out to be NP-complete in the general case. Optimal solutions can be found in limited time only for small size instances considering a very limited number of machines, very few throughput values and short production horizons by solving an integer linear program. For larger problems, many polynomial heuristics have been provided in [7] and [8] to cope with the problem of maximizing a platform useful life under service constraint. Efficiency of these heuristics have been assessed through exhaustive simulations. In this paper, the model for machines is more complex to fit the fuel cells behavior. The approach described before does not match in the focused fuel cell management context. The main difference is that the power of a fuel cell stack is controlled continuously. Moreover, each fuel cell lifetime depends on both its current power output and its previous use, in addition to environmental and operating conditions, cell design, assembly and degradation mechanisms [1]. Another specificity of the considered application leads to observe a continuous use of machines during the schedule. Change of output is still allowed, but the number of scheduled shutdowns has to be minimized for each fuel cell stack. Indeed, each stop-and-start of a fuel cell induces damages [2].

The proposed approach is based on the convex programming paradigm. In order to solve the considered type of problem in the framework of convex optimization, a penalized optimization problem which incorporates ℓ_1 norms is introduced, leading to solutions with sparse first derivative, while uniformly controlling the slopes of considered functions. This non-smooth penalized approach is the subject of extensive research in the machine learning, computational statistics and signal processing communities [9]. The ℓ_1 penalization approach has been recently advertised for the search of the sparsest solution of an under-determined system of linear equations [9] and for the piecewise affine approximation [10]. These ideas have lead to very important discoveries in the fields of mathematics and computer science in relation to the frontier between P and NP [11]. Using the ℓ_1 penalization approach, the considered scheduling problem is proposed to be addressed via optimizing a composite function subject to several constraints due to fuel cell intrinsic characteristics.

2 Problem statement

2.1 Application framework

A fuel cell system is composed of a set $\mathscr{M}$ of M fuel cell stacks M_m ($1 \leqslant m \leqslant M$) with $\mathscr{M} = \{M_1, \ldots, M_M\}$. All the stacks, composed of many individual connected single cells, are supposed to be always supplied with all reactants required for fuel cell internal chemical reactions leading to the power conversion. Resulting outcome is an instant power output $P_m(t)$ for each stack M_m. Stacks are supposed to be able to deliver outputs that can vary continuously within a given power output interval such that $0 \leqslant P_{min,m} \leqslant P_m(t) \leqslant P_{max,m}(t)$ for all $m \in \{1, \ldots, M\}$, with $P_{max,m}(t)$ typically decreasing over time. Up to now, the impact of variable operating conditions on fuel cell lifetime is not well-known and prognostics methods are not consistent with dynamic conditions. In fact, fuel cells being multi-physics and multi-scale systems [1], the study of their degradation process is very difficult. Thus a simplified trend depicting the behavior of fuel cell stacks controlled with variable operating conditions is proposed in Figure 1 and used to determine at each time the range of available power outputs and their associated RUL. This representation respects main properties of fuel cells: power amplitude is decreasing over time and RUL values depend on previous usages. Considering this, two periods of time using two different outputs for the same fuel cell stack can not always be permuted in a schedule.

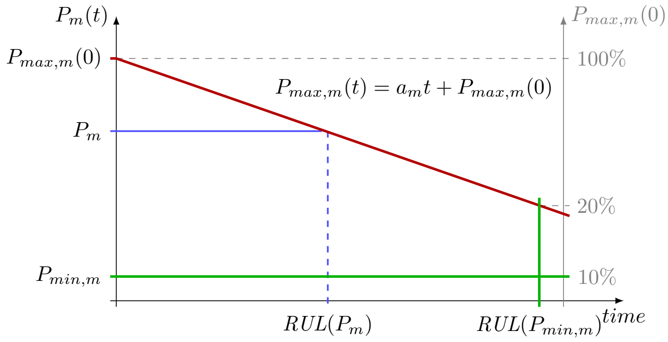


Figure 1: Evolution over time of the range of available outputs for a fuel cell stack M_m

Fuel cell stacks can be used simultaneously and independently from each other. At each time, the global outcome $P_{tot}(t)$ corresponds to the sum of each stack power contribution. During the whole production horizon T, this global output has to reach a given load demand $\sigma = \sigma(t)$, based on the energy requirement of the considered application. At any time t such that $0 \leqslant t \leqslant T$, the required service level $\sigma(t)$ is supposed to be lower or equal to the maximal global power output that the considered set of fuel cell stacks is able to deliver considering each stack initial health state (at time $t = 0$).

2.2 Optimization problem

Considering a multi-stack fuel cell system such as defined in the application framework (Section 2.1), the purpose is to exhibit a commitment strategy by defining at each time the contribution of each fuel cell stack to the global output so as to reach the power demand $\sigma(t)$ as long as possible. In the considered stationary power generation framework, this demand is supposed to be piecewise constant over time. Overproduction is tolerated if it allows to extend the global system useful life, but should be avoided as far as possible. Actually, storage of fuel cell stacks outcomes being not considered in this paper, overproduction is lost. All the fuel cell stacks are not supposed to be in use at any time if a subset of them is enough to reach the demand. Since fuel cells suffer from wear and tear, some stacks can also be not available if their end of life (EOL) has been reached. Moreover, when a stack has been started up, a continuous use is all the same observed as far as possible until its end of life. Change of output is still allowed, but scheduled shutdowns are avoided. Stopping and restarting a fuel cell can indeed induce damages [2]. Available power outputs for each stack are determined using the fuel cell behavior representation described in Figure 1. The issue is then to find the appropriate assignment of stacks over time during the whole production horizon.

The combinatorial optimization approach proposed in previous works dealing with a similar optimization problem (see [7] and [8]) and considering time discretization allowed to find optimal solutions in limited time only for small problem instances. However, in the application framework considered here, the horizon and/or the number of machines may be quite large. Considering this, an approach making use of a time discretization may be intractable and relaxations or heuristics may be mandatory to cope with large instances of the optimization problem. In order to avoid this, we propose to change completely the paradigm used to search for solutions and to reason on the scale of the whole production horizon. The contribution of each fuel cell stack during its whole lifetime is considered in this case as a whole and its evolution over time is determined using convex optimization. A new formulation of the scheduling problem, that leads to a convex programming problem taking into account fuel cell properties and associated constraints described in Section 2.1, is then proposed in Section 3.

3 Mathematical formulation

The use of convex optimization is proposed to cope with the optimization problem, so that solutions can be found in polynomial time even if large scale problems are considered. Some mathematical notations are first defined to formulate the problem as the solution of a convex program. An ℓ_1 penalization approach is then proposed to obtain polynomially shaped vectors which can be interpreted as the discretized version of a polynomial function of time. The main interest in using such penalties is that they are well structured convex and thus amenable to efficient methods of convex optimization. This ℓ_1 penalization approach leading to a non-smooth objective function, the use of Nesterov's smoothing technique is finally proposed. This first-order method for convex optimization, based on an optimal scheme for smooth optimization of a differentiable and convex function, allows to improve the convergence properties of the proposed resolution method detailed in Section 4.

3.1 Notations

Let $f_m(t)$, $t = 0, \ldots, T$, be the contribution of fuel cell stack M_m to the global output. For each stack, this contribution can vary over the time span $[0, \ldots, T]$, with T the production horizon. Let us assume that each $f_m(t)$ satisfies:

$$f_m(t) \geqslant 0 \quad \forall\, t \in [0, \ldots, T] \tag{1}$$

The constraint associated to the required service level $\sigma(t)$ can be expressed as follows:

$$\sum_{m=1}^{M} f_m(t) \geqslant \sigma(t) \quad \forall\, t \in [0, \ldots, T] \tag{2}$$

The following upper bound is also imposed on each fuel cell stack contribution over time:

$$f_m(t) \leqslant f_{\max,m}(t) \quad \forall\, m = 1, \ldots, M,\ \forall\, t \in [0, \ldots, T] \tag{3}$$

This upper bound corresponds to the maximal reachable output, which typically declines during the use of a fuel cell stack M_m.

A certain consumption rate constraint is finally set for each stack M_m and may be written as:

$$\sum_{t=0}^{T} \Phi(f_m(t)) \leqslant 1 \tag{4}$$

with Φ a convex function. These consumption constraints express each stack limited lifetime. In accordance with the representation of fuel cells behavior proposed in Figure 1, the consumption rate considered for a stack M_m is the following:

$$\Phi(f_m(t)) = \frac{a_m}{f_m(t) - f_{max,m}(0)} \tag{5}$$

with a_m the speed associated to the maximal power output decrease (see Figure 1).

3.2 ℓ_1 penalization approach

The purpose is to find the functions $f_m(t)$ ($m = 1, \ldots, M$ and $t = 0, \ldots, T$). The main idea is to use an approach which was recently promoted in signal processing and computational statistics. In [12], Kim, Koh, Boyd and Gorinevsky showed the practical interest of minimizing the ℓ_1-norm for obtaining sparsity in the context of function modeling over time. More precisely, they showed through multiple experiments that minimizing the ℓ_1-norm of the finite differences of a vector leads, under very mild conditions, to a vector which is piecewise constant. The same idea can be used to obtain polynomially shaped (of any order) vectors which can be interpreted as the discretized version of a polynomial function of time. ℓ_1 penalization is then used to impose a small number of jumps for each function f_m ($m = 1, \ldots, M$). Let $\Delta : \mathbb{R}^{T+1} \mapsto \mathbb{R}^T$ denote the operator which takes the successive differences, i.e.:

$$f = \begin{bmatrix} f(0) \\ \vdots \\ f(T) \end{bmatrix} \qquad \Delta f = \begin{bmatrix} f(1) - f(0) \\ \vdots \\ f(T) - f(T-1) \end{bmatrix} \tag{6}$$

Under the constraints described in previous section, many power output values may be feasible for a given horizon T. Optimizing the horizon may provide solutions which have unreasonable shapes with respect to the physical properties of the machines. In particular, all the machines may not be involved at any time. Thus, we expect to see working windows occuring in the solution. This is modelled by the sparsity of f_m and its first derivative. The solution may also be very oscillatory. Such bad features could be overcome by imposing the sparsity of the various components to be less than prescribed by certain physical constraints. A bad news is that sparsity is not convex and leads to NP-hard feasibility problems. A simple solution can be incorporated into the approach. The idea is to replace sparsity by a convex surrogate. Such surrogates have proved to be very efficient in the signal processing field [9, 12]. In most cases, the corresponding relaxation sums up to minimize the ℓ_1-norm of the quantity whose sparsity is to be controlled.

Using the ℓ_1 penalization approach, one obtains that the considered problem can be addressed via optimizing the following composite function:

$$\phi(F) = \sum_{m=1}^{M} \lambda_m \|\Delta f_m\|_1 \tag{7}$$

subject to the constraints (1), (2), (3) and (4). $\|\Delta f_m\|_1$ is a penalty for imposing a certain sparsity on Δf_m, used to minimize the discontinuity of the final solution (small number of jumps). In order to enforce that f_m equal zero more often than would lead the previous objective, one can propose the following objective function:

$$\mathcal{F}(F) = \lambda_0 \|F\|_1 + \sum_{m=1}^{M} \lambda_m \|\Delta f_m\|_1 \tag{8}$$

3.3 Nesterov smoothing technique

ℓ_1 penalization approach proposed in previous section ensures that the objective function taken into account is convex. This objective function is however non-smooth. Improvement of the convergence properties of the resolution method proposed in Section 4 for the minimization of the objective function can be achieved in the scheme proposed by Nesterov in [13] for non-smooth convex optimization. Nesterov's smoothing technique makes use of smooth approximations of non-differentiable functions. Becker et al. [14] have used this technique for ℓ_1 norms as needed for the LASSO and also for the objective function considered in the present paper (see Equation (8)).

In this framework, the minimization problem is proposed to be recast as a saddle-point problem. The considered objective function, say g, being convex but non-smooth, may be written as a maximization. Minimization can then be written as:

$$\min_{x \in Q_p} (g(x)) \;\text{ with }\; g(x) = \max_{u \in Q_d} < u, Wx > \tag{9}$$

with Q_d the dual feasible set, supposed to be convex. The corresponding saddle-point problem is then the following:

$$\min_{x \in Q_p} \max_{u \in Q_d} < u, Wx > \tag{10}$$

Substitution of the non-smooth objective function by the following smooth approximation is proposed by Nesterov [13]:

$$g_\mu(x) = \max_{u \in Q_d} < u, Wx > -\mu p_d(u) \tag{11}$$

with $p_d(u)$ a prox-function for Q_d, that is, $p_d(u)$ is continuous and strongly convex on Q_d. In the algorithm NESTA (shorthand for Nesterov's algorithm), based on the smoothing technique introduced by Nesterov, Becker et al. [14] propose a convenient choice for the prox-function: $p_d(u) = \frac{1}{2}\|u\|_2^2$. In the following, it will be defined as $p_d(u) = \|u\|_2^2$.

Considering that each ℓ_1 norm in the objective function defined in Equation (8) is of the form $\|x\|_1 = \max_{u \in Q_d} < u, x >$, where the dual feasible set is the ℓ_∞ ball ($Q_d = \{u \, s.t. \, \|u\|_\infty \leqslant 1\}$), a natural smooth approximation to the ℓ_1 norm is then:

$$g_\mu(x) \approx \max_{\|u\|_\infty \leqslant 1} < u, F > -\mu \|u\|_2^2 \tag{12}$$

Then, the objective function previously defined in Equation (8) can be approximated by:

$$\begin{aligned} \mathcal{F}(F) \approx & \lambda_0 \max_{\|u\|_\infty \leqslant 1} < u, F > \\ & + \sum_{m=1}^{M} (\lambda_m \max_{\|u_m\|_\infty \leqslant 1} < u_m, \Delta f_m > \end{aligned} \tag{13}$$

subject to the constraints (1), (2), (3) and (4).

4 Resolution method

4.1 Intermediate definitions

Consider first the entropy function $h(x) = \sum_{i=1}^{d} x_i \ln(x_i)$. The gradient of this function is then:

$$\nabla h(x) = \begin{bmatrix} \ln(x_1) + 1 \\ \vdots \\ \ln(x_d) + 1 \end{bmatrix} \tag{14}$$

Let us then introduce the following functions which adequately describe the constraints defined in Equations (1), (2), (3) and (4):

$$\psi_0(F) = \sum_{m=1}^{M} f_m(t) - \sigma(t) \quad \forall \, t = 0, \ldots, T \tag{15}$$

$$\psi_{1,m}(F) = f_m \quad \forall \, m = 1, \ldots, M \tag{16}$$

$$\psi_{2,m}(F) = f_{\max,m} - f_m \quad \forall \, m = 1, \ldots, M \tag{17}$$

$$\psi_{3,m}(F) = 1 - \sum_{t=0}^{T} \Phi(f_m(t)) \quad \forall \, m = 1, \ldots, M \tag{18}$$

with $\psi_0 : \mathbb{R}^{M(T+1)} \mapsto \mathbb{R}^{T+1}$, $\psi_{1,m} : \mathbb{R}^{M(T+1)} \mapsto \mathbb{R}^{T+1}$, $\psi_{2,m} : \mathbb{R}^{M(T+1)} \mapsto \mathbb{R}^{T+1}$, $\psi_{3,m} : \mathbb{R}^{M(T+1)} \mapsto \mathbb{R}^{M}$ and

$$F = [f_1(0), f_2(0), \ldots, f_M(0), \ldots, f_1(T), \ldots, f_M(T)] \, .$$

4.2 Mirror-Prox for Saddle Points (MP-SP)

A Mirror Prox method is proposed to cope with the problem of minimizing the objective function detailed in Equation (8). The Mirror Prox algorithm is a variant of the Mirror Descent algorithm, which has first been proposed by

Nemirovskii and Yudin [15] for convex programming. It has been extensively studied recently and several relationships have been discovered between the Mirror Descent scheme and Bregman-proximal methods. We refer the interested reader to [16] for a detailed and very pedagogical description of Mirror Descent algorithms.

4.2.1 Main ideas

Assume for a moment that the objective function $\mathcal{F}$ is differentiable. The problem can be described as follows:

$$\underset{F\in\mathbb{R}^{2M(T+1)}}{\operatorname{argmin}} \left(\|F\|_1 + \phi(F)\right) \tag{19}$$

such that $\psi_0(F) \geq 0$ and $\psi_{K,m}(F) \geq 0,\ \ \forall\ K = 1,\dots,3$ and $\ \forall\ m = 1,\dots,M$. Let us denote the constraint set by $\mathcal{C}$. The standard projected gradient algorithm is of the form

$$F^{(l+1)} \quad = \quad P_{\mathcal{C}}\left(F^{(l)} - \lambda^{(l)}\ \nabla\mathcal{F}(F^{(l)})\right) \tag{20}$$

where $P_{\mathcal{C}}$ is the projection operator onto the set $\mathcal{C}$.

A mirror function is a convex function whose gradient is one-to-one and has a defining set which may conveniently incorporate simple constraints. Let θ be such a function. Then, the mirror descent iteration is given by:

$$\begin{aligned} \nabla\theta(G^{(l+1)}) &= \nabla\theta(F^{(l)}) - \lambda^{(l)}\ \nabla\mathcal{F}(F^{(l)}) \\ F^{(l+1)} &= P_{\mathcal{C}}(G^{(l+1)}). \end{aligned}$$

As a very useful example, one might consider θ, a mirror map on $R^{M(T+1)}$, defined by:

$$\theta(F) = \sum_{t=0}^{T}\sum_{m=1}^{M} F(m,t)\ln(F(m,t)). \tag{21}$$

In accordance with the gradient of the entropy function defined in Equation (14), we have then:

$$\nabla\theta(F) = \ln(F) + 1 \tag{22}$$

4.2.2 The Mirror-Prox for Saddle Points algorithm

The main difficulty in the Mirror Prox scheme is that projecting onto the constraint set $\mathcal{C}$ might not be so easy. In order to overcome this problem, one possibility is to consider an algorithm which solves the primal-dual saddle point problem for the Lagrange function. An other possibility, which is addressed in this paper, is to include the constraints in the objective function. In this case, satisfaction of each constraint is ensured by the optimization of an associated function. For this purpose, one can define the Lagrange function as follows:

$$L(F,u) = \mathcal{F}(F) + \mathcal{C}_{dem}(F,\sigma) + \mathcal{C}_{slope}(F,f_{max}) \tag{23}$$

and find a saddle point of this function, with

$$\mathcal{C}_{dem}(F,\sigma) = \lambda_{dem} \sum_{t=0}^{T} \frac{1}{t+1} exp\Big(- \gamma\big(\sum_{m=1}^{M} f_m(t) - \sigma(t)\big)\Big) \tag{24}$$

the function associated to the constraints (15), with λ_{dem} and $\gamma \in \mathbb{R}$ and

$$\mathcal{C}_{slope}(F, f_{max}) = \lambda_{slope} \sum_{t=0}^{T-1} \sum_{m=1}^{M} exp\big(\beta\big(f_{max,m}(t+1) - f_{max,m}(t) + \alpha_m f_m(t)\big)\big) \tag{25}$$

the function associated to the constraints (15) with λ_{slope}, β and $\alpha_m \in \mathbb{R}\ \forall\ m = 1, \ldots, M$.

One can propose the following Mirror Descent scheme:

$$\begin{aligned}\nabla\theta_{\lambda_0,\lambda_1,\lambda_{dem},\lambda_{slope}}(F^{(l+1)}, u^{(l+1)}) = &\\ \nabla\theta_{\lambda_0,\lambda_1,\lambda_{dem},\lambda_{slope}}(F^{(l)}, u^{(l)}) &\\ - \eta \left(\begin{bmatrix} \nabla_F L(F^{(l)}, u^{(l)}) \\ -\nabla_u L(F^{(l)}, u^{(l)}) \end{bmatrix} \right) &\end{aligned} \tag{26}$$

where

$$\begin{aligned}\theta_{\lambda_0,\lambda_1,\lambda_{dem},\lambda_{slope}}(F, u) = \lambda_0 \max_{\|u\|_\infty \leqslant 1} < u, F > &\\ + \sum_{m=1}^{M} (\lambda_m \max_{\|u_m\|_\infty \leqslant 1} < u_m, \Delta f_m > &\\ - \lambda_{dem}\gamma \sum_{t=0}^{T} \frac{1}{t+1} exp\Big(- \gamma\big(\sum_{m=1}^{M} f_m(t) - \sigma(t)\big)\Big) &\\ + \lambda_{slope}\beta\alpha_m \sum_{t=0}^{T-1} \sum_{m=1}^{M} exp\big(\beta\big(f_{max,m}(t+1) &\\ - f_{max,m}(t) + \alpha_m f_m(t)\big)\big) &\end{aligned} \tag{27}$$

Computational details can be found in the companion research report [17].

4.3 Improvement of the method convergence rate

Consideration of the proposed Mirror-Prox scheme allows to reach better convergence speed than the one that would have been obtained with a Mirror-Descent scheme. In order to accelerate the convergence of the method, some projection steps are moreover included. After each iteration of the Mirror-Prox, update of each $f_{max,m}(t)$ value is first performed as a function of $F_m(t)$ (see Equation (28)).

$$\begin{gathered} f_{max,m}(t) = f_{max,m}(t-1) + \alpha'_m g(F_m(t-1)) \\ \forall\, t = 1, \ldots, T,\ \ \forall\, m = 1, \ldots, M \\ \text{with } \alpha' \in \mathbb{R}^M \text{ and } g : \mathbb{R}^M \mapsto \mathbb{R}^M \end{gathered} \tag{28}$$

Then, for each time for which the global contribution do not reach the demand level, contributions of stacks that are used are increased to the maximal output reachable if possible, until the needed global output is reached. The new configuration is stored in an intermediate variable, which is used to guide the evolution of F in the Mirror-Prox scheme through a gradient step (see details in the companion research report [17]).

5 Simulation results

The resolution method proposed in previous section has been evaluated through simulations on random problem instances. After a description of the problem generation and some remarks on the method configuration, efficiency of the approach is discussed.

5.1 Problem generation

Random problem configurations have been generated using a simulator and configured with many parameters including the number of stacks in the considered multi-stack fuel cell system, M, and intrinsic fuel cell characteristics. The latter have been defined on the basis of fuel cell manufacturer specifications and considering a maximal lifetime $RUL_{max,m} = RUL(P_{min,m}) = 1500$ hours $\pm 20\%$. Power values taken into account are the following: $P_{max,m}(0) = 500\,W \pm 5\%$ and $P_{min,m} = 0.15.P_{max,m}(0)$.

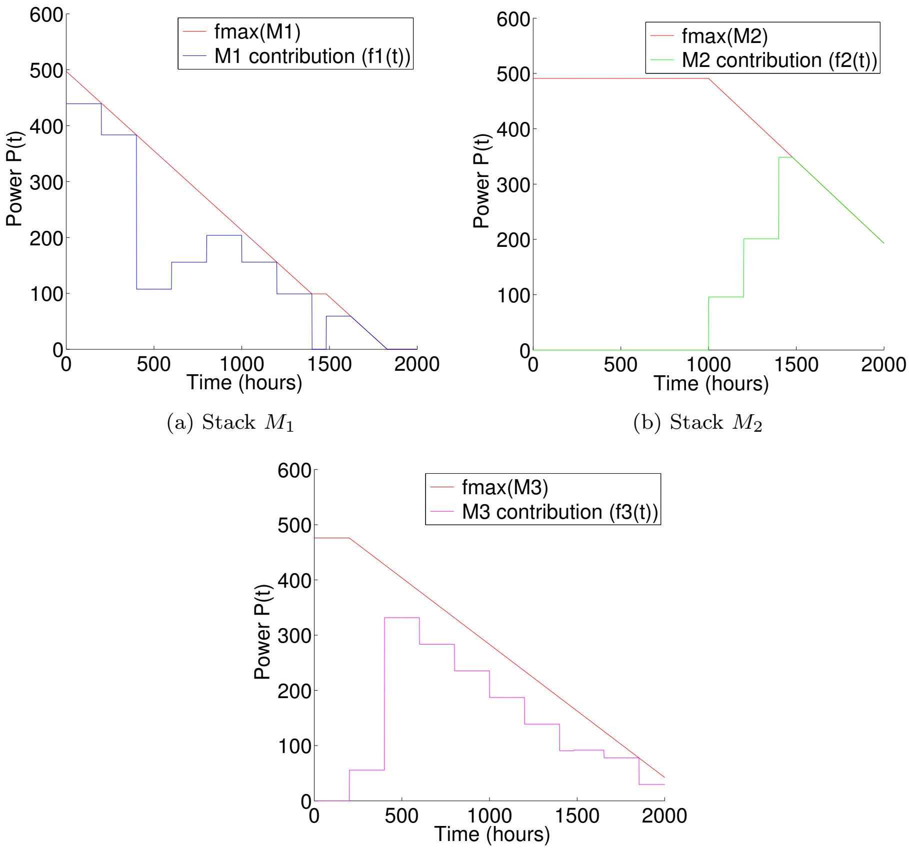
For the results presented hereafter, the power demand has been assumed to be constant during the whole scheduling horizon: $\sigma(t) = \sigma$. Without any lost of generality, only one demand value has then been associated to each problem configuration.

5.2 Method configuration

Some mathematical parameters introduced by the optimization methods need to be determined to insure the quality of solutions. Tuning of the relaxation parameters $\lambda_{1,m}$, $\lambda_{2,m}$ and $\lambda_{2'}$ $(m = 1, \ldots, M)$ has been done by choosing them independently of m and by trying several values out until an appropriate shape is obtained.

Different weights have been associated to the problem constraints and determined depending on their significance level for the solution. A great weight has then been associated to the constraint $\psi_0(F)$ to favor the reaching of the load demand. Times for which the load demand is not reached being gathered at the end of schedules obtained with the resolution method, this insures the horizon maximization.

Consideration of projections on the sets of constraints defined in Section 4.3 allows to improve the convergence speed, but a minimal number of iterations is all the same necessary to give the method time to converge. Quality of solutions from the point of view of the reached production horizon globally increases with the number of iterations and stabilizes starting from a certain value. This value has been determined on the basis of several tests and used to define the minimal needed number of iterations, which limits computation times while insuring solutions optimality.



(a) Stack M_1

(b) Stack M_2

(c) Stack M_3

Figure 2: Comparison of each stack contribution over time to the corresponding maximal power output

5.3 Results

A schedule obtained with the proposed Mirror-Prox method associated to projections on the sets of constraints for a system composed of 3 fuel cell stacks is proposed in Figure 3. Contribution of each stack to the global power output is detailed in Figure 2 and compared to the corresponding maximal power output reachable. One can see in Figure 3 that the power demand is satisfied at the beginning of the schedule. The production horizon is limited by each fuel cell stack lifetime, but depends also on the scheduling process. One can also notice in Figure 2 that the evolution of the maximal power output of each machine depends on its commitment over time. In fact, when a machine is not used, the maximal power output remains constant. This delayed evolution of $P_{max,m}$ for each M_m is ensured by both Equations (25) and (28) and allows to optimize the use of the set of machines. In fact, at each time, no unnecessary machine is used.

The resolution method introduced in this paper allows then to propose a schedule of a multi-stack fuel cell system that defines the commitment of each

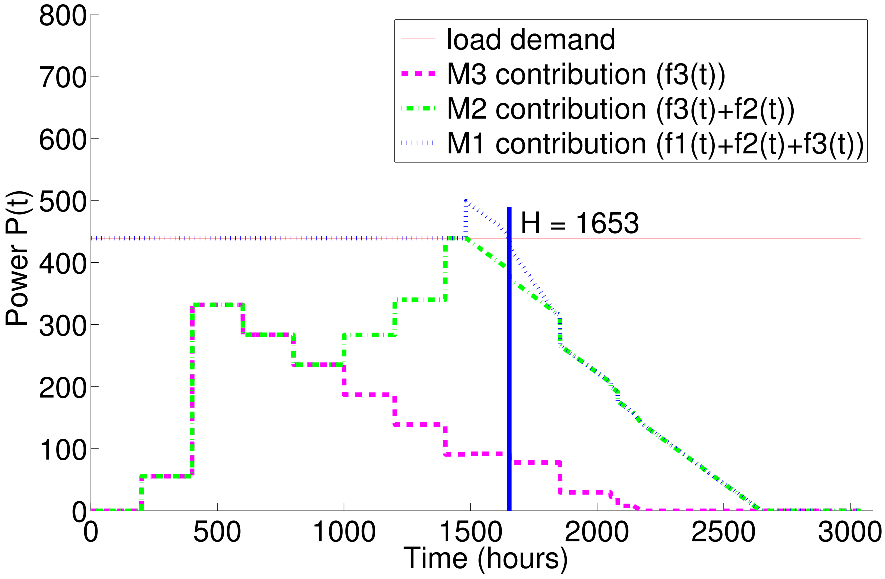


Figure 3: Schedule obtained for a set of 3 fuel cell stacks

stack to reach a certain load demand. The proposed Mirror-Prox method gives furthermore better solutions than strategies that would share out stacks contributions following very basic rules. One can consider a first basic strategy $S1$, that divides the load demand equally between the M stacks. In this case, all the stacks have to provide the same power, which equals $\sigma \,/\, M$, and the horizon is limited by the stack which RUL is minimal for the selected power output. Two other strategies can be based on particular weightings. Let be $S2$ the strategy that considers a weighting on maximal power output reachable at time t_0. Contribution of each stack to the load demand is then defined as a percentage defined as follows: $P_{max,m} \,/\, P_{max,tot}$, with $P_{max,tot} = \sum_{m=1}^{M} P_{max,m}$. In this case, the higher the maximal power output is, the higher is the contribution of the corresponding stack to the global output. Let be $S3$ the strategy that considers a weighting on the potential, defined by the area under the curve depicting the evolution of $P_{max,m}(t)$. In a same way as for strategy $S2$, contribution of stacks with high potential are the most significant. Considering the same multi-stack system with which results provided in Figures 2 and 3 have been obtained, horizon reached by strategy $S1$ is $H_1 = 1155$ hours. In a same way, $H_2 = 1160$ hours and $H_3 = 1193$ hours. Horizon reached with the proposed strategy based on convex optimization is longer ($H = 1653$ hours) Then, in comparison with basic strategies, the strategy detailed in Section 4 defines better stack commitment allowing to extend the multi-stack system production horizon. Even better horizons could be reached by relaxing the constraint defined by equation 15, which ensures that the demand is reached at each time during the production horizon. In fact, solutions provided by the convex optimization program may include periods of time during which the global outcome falls below the load demand. But this does not mean that the demand can not be reached anymore afterward. Allowing a certain underproduction rate can be consistent in a hybrid global system where the missing power output can be punctually provided by a secondary energy supplier. As an example based on the case considered for results provided in Figures 2 and 3, allowing an under-production rate of 10% ($0.1 \times \sigma$) permits to extend the system lifetime from 1653 to 1703 hours.

6 Conclusion and future work

A management of fuel cell systems has been proposed in a PHM framework. Decision coming within the scope of Prognostic Decision Making has been addressed considering longer timeframes than those proposed so far in the literature on fuel cells. The use of convex programming has been proposed to cope with the scheduling problem of multi-stack fuel cell systems under service constraint. A mathematical formulation of the problem involving ℓ_1 penalizations and smoothing techniques have been used to control the shape of the solutions. The minimization of the objective function under constraints has been addressed through a Mirror-Prox scheme.

The model taken into account to describe fuel cell systems particular behavior has been highly simplified. In fact, the transformation of real properties to mathematical functions has been restrained to convex functions to allow the use of convex programming. All the fuel cell properties are then for the moment not observed by the solutions obtained with the proposed approach, but this first study is promising. It shows indeed that a global resolution on the scale of the whole production horizon can be used to define the commitment of machines over time with the horizon maximization as objective.

As future work, enhancement of the considered mathematical formulation will be addressed to suit the associated model to a realistic evolution over time of fuel cell characteristics. The method being relatively fast and scalable, extensive simulations will also be performed to assess its efficiency when considering a huge number of machines.

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