Chapter 24
Uncertainty Propagation Combining Robust Condensation and Generalized Polynomial Chaos Expansion

K. Chikhaoui, N. Kacem, N. Bouhaddi, and M. Guedri

Abstract Among probabilistic uncertainty propagation methods, the generalized Polynomial Chaos Expansion (gPCE) has recently shown a growing emphasis. The numerical cost of the non-intrusive regression method used to compute the gPCE coefficient depends on the successive Latin Hypercube Sampling (LHS) evaluations, especially for large size FE models, large number of uncertain parameters, presence of nonlinearities and when using iterative techniques to compute the dynamic responses. To overcome this issue, the regression technique is coupled with a robust condensation method adapted to the Craig-Bampton component mode synthesis approach leading to computational cost reduction without significant loss of accuracy. The performance of the proposed method and its comparison to the LHS simulation are illustrated by computing the time response of a structure composed of several coupled-beams containing localized nonlinearities and stochastic design parameters.

Keywords Robustness • Uncertainty • Generalized polynomial chaos expansion • Component mode synthesis • Meta-model

24.1 Introduction

In structural mechanics and in practically all branches of industry, when parametric uncertainty is incorporated in the mathematical model, its propagation is needed to evaluate how the randomness of input parameters affects the computed outputs and to handle, consequently, more and more realistic behaviors. In a probabilistic framework, statistical approaches such as the well-known Latin Hypercube Sampling (LHS) [1] use a great number of samples of random variables for reasonable accuracy and therefore require a prohibitive computation time. On the other side, among other non statistical approaches, the generalized polynomial chaos expansion (gPCE) [2] has shown a growing emphasis in recent years due to its simple implementation and high performance. It combines polynomial basis vectors and deterministic coefficients, which can be computed by means of intrusive or non-intrusive approaches [3]. In the context of the latter, more advantageous than the former since it considers the original FE model as a black box, among other existing methods [4], regression technique permits to estimate the gPCE coefficients by minimizing the difference between the referential LHS response and the approximation, corresponding to a set of random variables chosen among combinations of Hermite polynomial roots.

Nevertheless, the most expensive part of its implementation lies in the successive LHS evaluations. To overcome this issue, we focus, in this paper, on coupling the non-intrusive regression technique with a robust condensation method adapted to the Craig-Bampton component mode synthesis (CBCMS). In fact, the prohibitive cost of prediction is due to: the computational time of the full model using direct analysis, the computation of reduction basis for each random sample while propagating uncertainties and the number of iterations necessary to obtain accurate approximations using iterative techniques in presence of nonlinearities. Consequently, model condensation must be applied. Furthermore, the use of the CBCMS [5, 6] allows considering some substructures, which are uncertain and/or containing localized nonlinearities, being condensed independently of the others. Nevertheless, the main issue lies on forming a robust enriched condensed Craig-Bampton Transformation (CBT) [6, 7] which takes into account uncertainties and localized nonlinearities.

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In the literature, some works focus on coupling uncertainty propagation methods and condensation techniques in order to attain a robust design. In [8], the authors implemented the CMS taking into account stochastic aspect to compute frequency linear structures responses using stochastic spectral FE method (SSFEM). In [9], a reduced-order model (ROM) is integrated into SSFEM, using a basis spanned by displacements and derivatives of displacements, and implemented to optimize the shape of a linear shell structure. Afonso et al. [10] implemented Monte Carlo (MC) method and Probabilistic Collocation Method (PCM) based on reduced-order modeling (ROM) approach incorporated via proper orthogonal decomposition method (POD).

The main originality of this paper lies on combining gPCE and robust CBCMS in order to obtain a metamodel, which allows computing stationary temporal solution of large size stochastic periodic structures containing local nonlinearities, with high accuracy and low computational cost.

24.2 Theoretical Backgrounds

24.2.1 Generalized Polynomial Chaos Expansion (gPCE)

Thanks to the generalization of the PC approach [2], several types of random variables and orthogonal polynomials are taken into account to develop its expansion. In this section, a compact form of the gPCE formulation is presented [2–4]. The gPCE of second order random variable is a decomposition, truncated by retaining only terms of the polynomials with degree up to p, of the form

$$Y = \sum_{\alpha=0}^{P} y_\alpha \Phi_\alpha (\xi) = y^T \mathbf{\Phi} (\xi),$$ (24.1)

where y_α are the unknown deterministic coefficients, Φ_α the multivariate polynomials of d independent random variables $\xi = \{\xi_i (\theta)\}_{i=1}^{d}$ and α the multidimensional index such as $|\alpha| = \sum_{i=1}^{d} \alpha_i$.

The number of terms retained in Eq. 24.1 is defined as

$$P + 1 = (d + p)! / d! p!.$$ (24.2)

Solving the gPCE consists on computing the coefficients y_α. Hence, the non-intrusive regression method is implemented, in its standard form, minimizing the difference between the gPCE approximate solution and the exact one as follow

$$\widetilde{y} = \underset{y}{Arg\ min} \frac{1}{N} \sum_{n=1}^{N} \left[\{Y^{(n)}\} - y^T \Phi \left(\xi^{(n)}\right) \right]^2.$$ (24.3)

The solution considered as exact is a set of LHS responses $\{Y^{(n)} = y\left(\xi^{(n)}\right), n = 1, \ldots, N\}$ computed corresponding to a set of random variables $\Xi = \{\xi^{(n)}\}_{n=1}^{N}$ called experimental design (ED).

The final obtained approximate solution of Eq. 24.3 is of the form

$$\widetilde{y} = \left(\mathbf{\Phi}^T \mathbf{\Phi}\right)^{-1} \mathbf{\Phi}^T \{y\} = \mathbf{\Phi}^+ \{y\}$$ (24.4)

where $\mathbf{\Phi}_{nj} \equiv \left(\Phi_j \left(\xi^{(n)}\right)\right)_{\substack{n = 1, \ldots, N \\ j = 0, \ldots, P}}$ is called the data matrix.

A necessary condition for the numerical stability of the regression approximation is choosing an ED of size $N \geq P + 1$ which ensures the well-conditioning of the matrix $(\mathbf{\Phi}^T \mathbf{\Phi})$ which will be inverted. The selection of the ED is carried out using two different methods; according to the first one, the ED is chosen randomly with respect to the probability distribution of the random variables, the second method consists on selecting the ED, in a deterministic way, among Hermite polynomial roots [3]. In fact, the roots of the Hermite polynomial of degree $p + 1$ are at first computed, then all their possible combinations $(p + 1)^d$ are calculated and finally these roots combinations are classified such that the following variable

$$\zeta_N \left(\xi^{(n)} \right) = 2\pi^{-d/2} exp \left(-\frac{1}{2} \xi^{(n)2} \right) \tag{24.5}$$

is maximized or $\xi^{(n)2}$ minimized. The roots combinations retained to create the ED are subject to another selection [4]. 68
Indeed, to ensure that the invertible matrix $(\mathbf{\Phi}^T \mathbf{\Phi})$ is well-conditioned, a condition number κ defined as 69

$$\kappa = \left(\mathbf{\Phi}^T \mathbf{\Phi} \right)^{-1} . \mathbf{\Phi}^T \mathbf{\Phi} \tag{24.6}$$

must be minimized, where . is the 1-norm of the matrix. To the smallest value of κ corresponds a number N of roots 70
combinations which verify Eq. 24.5 and thus create the ED needed for gPCE coefficients. 71

Once obtained, the estimated coefficients give the final gPCE as a metamodel of the form 72

$$\widetilde{Y} = \widetilde{\mathcal{M}} \left(X \left(\xi \right) \right) = \sum_{\alpha=0}^{P} \widetilde{y}_\alpha \Phi_\alpha \left(\xi \right), \tag{24.7}$$

In uncertainty analysis, some statistical quantities have to be calculated. The mean and the variance are respectively given 73
by $\widetilde{\mu}_Y = \widetilde{y}_0$ and $\widetilde{\sigma}_Y^2 = \displaystyle\sum_{\alpha=0}^{P} \widetilde{y}_\alpha^2.$ 74

Note that the N successive deterministic FE evaluations needed for LHS method is the most expensive part of the gPCE 75
implementation especially for large size FE models, large number of uncertain parameters, presence of nonlinearities and 76
when using iterative techniques to compute dynamic structure responses. To overcome this issue, the regression technique is 77
coupled with a robust condensation method adapted to the CBCMS. 78

24.2.2 Model Condensation 79

In nonlinear dynamics, a mechanical system can generally be represented in the time domain by the following differential 80
equation 81

$$\begin{cases} [M] \{\ddot{y}\} + [B] \{\dot{y}\} + \{f_{int}\} = \{f_{ext}\} \\ \{y\} (t_0) = \{y\}_0, \{\dot{y}\} (t_0) = \{\dot{y}\}_0 : \quad \text{initial conditions} \end{cases} \tag{24.8}$$

where $[M]$ and $[B]$ stand for the mass and damping matrices of the system, $\{f_{ext}\}$ the exciting force and $\{f_{int}\} = $ 82
$([K] + \{f_{NL}\} (\{y\}, \{\dot{y}\})) \{y\}$ the internal force vector, $[K]$ is the stiffness matrix. 83

Modeling complex structures requires large size FE models for satisfying accuracy. To overcome the high computational 84
cost of the analysis, a reduced order model has to be designed using only few normal modes with respect to the frequency 85
range of interest. Therefore, the projection of the time response on an adequate condensation basis, considering the variable 86
transformation $\{y(t)\} = [T] \{q(t)\}$ to generalized coordinates, is needed. Hence, the equation of motion Eq. 24.8 becomes 87

$$[T]^T [M] [T] \{\ddot{q}(t)\} + [T]^T [B] [T] \{\dot{q}(t)\} + [T]^T \{f_{int}\}_r = [T]^T \{f_{ext}\}, \tag{24.9}$$

where the index r denotes the term reduced, $[T]^T [M] [T] = [M]_r$, $[T]^T [B] [T] = [B]_r$, in this case, the internal force vector 88
is expressed as 89

$$\{f_{int}\}_r = [\text{T}]^T ([\text{K}] + \{f_{NL}\} ([\text{T}] \{q\}, [\text{T}] \{\dot{q}\})) [\text{T}] \{q\}. \tag{24.10}$$

The time solution of the Eq. 24.9 can be approximated by using the Newmark nonlinear time integration scheme, which 90
allows expressing the displacement and the velocity at the instant t_{n+1} as a function of their expression at t_n. 91

The implementation of the Newmark method permits to write the equation of motion Eq. 24.9 in an incremental form 92

$$\left([K_{eff}]_r \right)_n \Delta \{q\}_n = \Delta \left(\{f_{eff}\}_r \right)_n \tag{24.11}$$

where $([K_{eff}]_r)_n$ is the instantaneous (effective) stiffness matrix function of the tangent stiffness matrix $([K_T]_r)_n = \partial(\{f_{int}\}_r)_n/\partial [T]\{q\}_n$ obtained by applying the Newton iterative scheme.

24.2.3 Robust Craig-Bampton Component Mode Synthesis Method (CBCMS)

Dividing the complete structure into several components (substructures) is interesting in the case of presence of uncertainties and localized nonlinearities in large size complex structures. It permits to apply the adequate condensation technique to each component independently of the others.

In deterministic linear case, the reduced model is obtained using standard CBCMS [5, 6] of the linear system in its blocked interface configuration. In this case, the CBT is defined, for a substructure k, as

$$\{y\}^k = \left\{ \begin{array}{c} y_j \\ y_i \end{array} \right\}^k = \left[\begin{array}{cc} I_{jj} & 0 \\ \psi_{ij} & \varphi_i \end{array} \right]^k \left\{ \begin{array}{c} q_j \\ q_c \end{array} \right\}^k = [T_{CB}]^k \{q\}^k \tag{24.12}$$

where $\{y_i\}^k$ are the interior coordinates transformed to the blocked junctions modal coordinates $\{q_c\}^k$, $\{y_j\}^k$ are the junction coordinates denoted as constraint coordinates $\{q_j\}^k$, $[\psi_{ij}] = -K_{ii}^{-1}K_{ij}$ is the static subbasis which contains the constraint modes, I_{jj} is the identity matrix, and $[\varphi_i]$ is the dynamic one containing the truncated normal modes basis at blocked interfaces $(\{y_j\} = 0)$ of the corresponding component. Nevertheless, using standard CBT requires computing more and even all normal modes for accurate results, which leads to a prohibitive computation cost.

In stochastic case with localized nonlinearities, the standard CBT $[T_{CB}]$ cannot satisfy the required accuracy and robustness of the model. Therefore, adding a complementary basis $[\Delta T]$ is necessary in order to form an enriched CBT (ECBT) $[T_{ECB}]$. The obtained basis, for each sub-structure, is thus of the form

$$[T_{ECB}]^k = \left[T_{CB} \vdots \Delta T \right]^k. \tag{24.13}$$

The complementary basis is a set of static residuals calculated according to the type of enrichment.

For each component k, to enrich the basis by taking into account stochastic aspect, the residual vectors are static responses with correspondence to a set of residual forces $[F_S]$ presenting the stochastic effects. These forces are generated depending on the stochastic zones of the mass and stiffness matrices [8]. The first complementary subbasis $[\Delta T_S]^k$ is thereafter obtained and added to the standard Craig-Bampton one.

To take into account localized nonlinearity effects, another type of complementary subbasis $[\Delta T_{NL}]^k$ has to be created [7] as a set of static responses corresponding to unit residual force vectors $\{F_i\}^k$ with respect to each nonlinear degree of freedom (dof) i for each component k.

The CBT can also be enriched if the external loading effect is considered [6]. Thus, an additional subbasis $[\Delta T_E]^k$ must be computed using a set of unit static loadings $\{F_E\}^k$ imposed on internal excited substructure dofs.

Consequently, the final enriched basis has the following form

$$[T_{ECB}]^k = \left[T_{CB} \vdots \Delta T_S \vdots \Delta T_{NL} \vdots \Delta T_E \right]^k = \left[\begin{array}{cccc} I_{jj} & 0 \vdots & 0 \vdots & 0 \vdots & 0 \\ \psi_{ij} & \varphi_i \vdots & \Delta T_S \vdots & \Delta T_{NL} \vdots & \Delta T_E \end{array} \right]^k \tag{24.14}$$

Note that a singular value decomposition (SVD) is needed to ensure linear independence of the vectors forming each complementary subbasis and also carried out on $[T_{ECB}]^k$ to ensure the linear independence of the subbases and thus the well-conditioning of the ECBT. The synthesis of the complete structure requires finally the assembly of the different CBT matrices according to the hypothesis of continuity of displacements at junction dofs.

24.2.4 Robust Metamodel Combining CBCMS and gPCE

The CBCMS and the gPCE methods were previously independently presented. The main aim of this paper is to couple them in order to replace large size dynamical structures with considerably condensed and sufficiently accurate metamodel. Indeed, we propose to compute the succession of N deterministic responses of LHS simulations $\{Y^{(n)} = y(\xi^{(n)}),\ n = 1, \ldots, N\,\}$,
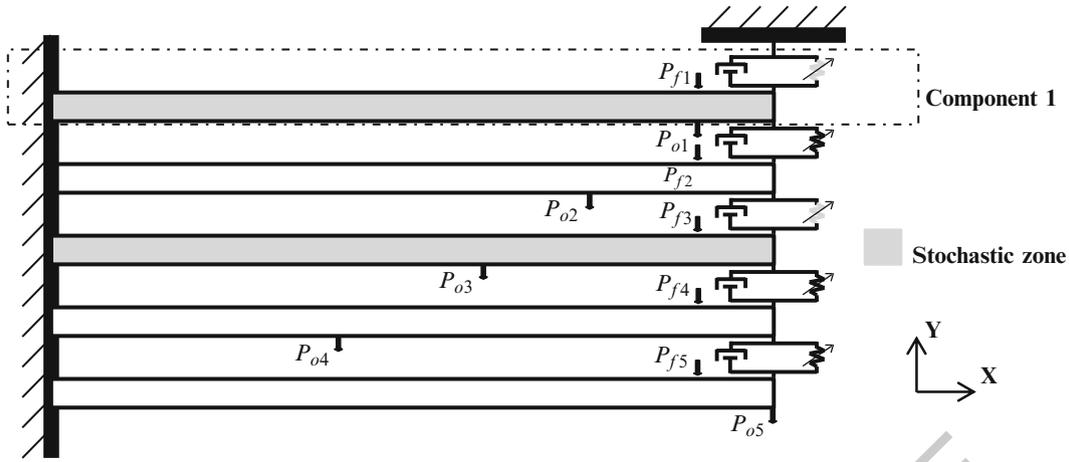
Fig. 24.1 Coupled beams structure

needed for the gPCE regression method implementation (Eq. 24.4), using the ECBT Eq. 24.14. The notion of robustness 128
consists, in our study, on satisfying two different criterions; the accuracy of the responses and the gain in terms of 129
computational time. To verify the first criterion, a set of temporal statistic moments $\mathcal{M}_i$, also called times indicators or 130
energy criterions, has to be calculated in order to quantify the response accuracy in terms of amplitude and periodicity errors. 131
These moments are expressed as [11] 132

$$\mathcal{M}_i = \int\limits_{-\infty}^{+\infty} (t - t_s)^i \, y(t)^2 dt \tag{24.15}$$

where $y(t)$ is the temporal response, i the order of the moment and t_s the temporal shift chosen in our case as $t_s = 0$. 133

The total energy of the response $E = \mathcal{M}_0$ permits the verification of the accuracy in term of amplitude. $\mathcal{M}_1/\mathcal{M}_0 = T$ 134
and $\mathcal{M}_2/\mathcal{M}_0 - (\mathcal{M}_1/\mathcal{M}_0)^2 = D^2$ are respectively the central time and the root mean square duration computed to verify 135
the accuracy of the response in term of periodicity. 136

A numerical application is presented in Sect. 24.3 in order to illustrate the main features of the proposed robust metamodel 137
designed to analyze the dynamic behavior in presence of uncertainties and localized nonlinearities. 138

24.3   Numerical Application 139

24.3.1   Proposed Structure and Validation Process 140

The proposed academic structure, Fig. (24.1), is composed of five identical beams loaded in pure flexion. The discretization 141
of each beam into 20 elements (two dofs per node: v_y, θ_z) leads to a 200 dofs FE model. The beams are of rectangular 142
section with $b = 3 \times 10^{-2}m$ and $h = 1 \times 10^{-2}m$, length $L_b = 5 \times 10^{-1}m$, Young modulus $E_0 = 2.1 \times 10^{11}Pa$, 143
density $\rho_0 = 7800 \, kg.m^{-3}$ and Poisson's ratio $\nu = 0.3$ and are submitted each one to a localized excitation force (N) 144
$P_f = 10 \times cos\,(2\pi f_1 t)$, where $f_1 = 82.96 \, rad.s^{-1}$ is the first eigenfrequency, according to the v_y dof (Y direction). The 145
five beams are coupled using five local dampers such as $c = 10^2 N.s.m^{-1}$, five linear springs $k_0 = 10^6 N.m^{-1}$ and nonlinear 146
ones $k_{NL} = 10^{11} N.m^{-1}$. Several observation points $P_{oj} \; (j = 1, 2, \ldots)$ are considered to evaluate the efficiency of the 147
proposed metamodel. 148

The stationary temporal response evaluation is done in the time interval [0–0.5 s], divided into steps of $10^{-4}s$, in which 149
the stationary regime is already attained. 150

To apply the CBCMS method, the complete structure is divided into substructures. In fact, being in the case of periodic 151
structure, we propose to consider each set of coupling elements (localized damper, linear and nonlinear springs) and a beam 152
as a substructure (Fig. 24.1). The first and the third components are considered as two stochastic zones in which the Young 153
modulus of the beam and the linear coupling stiffness are supposed to be uncertain parameters such as 154

Table 24.1 Model size and associated temporal moments

Methods		Model (dofs)	E		T		D²	
			$E \times 10^4$ (m².s)	Error (%)	$T \times 10^2$ (s)	Error (%)	$D^2 \times 10^3$ (s)	Error (%)
LHS (A)	LHS-REF	200	15.93	0.00	49.93	0.00	82.03	0.00
	LHS-ECBT	24	15.78	0.94	49.92	0.02	82.01	0.02
REG (B)	REG-2-REF	200	15.75	1.13	49.84	0.18	82.26	0.28
	REG-4-REF	200	15.88	0.31	49.86	0.14	82.19	0.19
	REG-2-ECBT	24	15.45	3.01	49.82	0.22	82.31	0.34
	REG-4-ECBT	24	15.57	2.26	49.85	0.16	82.23	0.24

t3.1
t3.2

$$E = E_0 \left(1 + \sigma_E \xi_E\right) \ \text{ and } k = k_0 \left(1 + \sigma_k \xi_k\right) \tag{24.16}$$

where ξ_E and ξ_k are two random variables of respectively uniform and lognormal probability distributions and $\sigma_E = \sigma_k = $ 10% are the considered dispersions. The effect of the stochastic aspect of the uncertain parameters is shown using the MAC (Modal Assurance Criterion) [6] matrix, which compares the normal modes of the deterministic model to the means of the modes of the stochastic model computed with correspondence to each random variable.

The proposed metamodel results are discussed with respect to the responses considered as reference, computed using LHS method using 1,000 samples of random variables. In fact, the process of evaluation of the metamodel efficiency is as follow

(A) Implementing the 1,000 samples LHS method on: 1- the complete model (projection on complete eigenvectors basis) denoted LHS-REF, 2- the reduced model (projection on the ECBT matrix) denoted LHS-ECBT;
(B) Implementing the regression gPCE: 1- on the complete model for two gPCE orders (2 and 4) denoted respectively REG-2-REF and REG-4-REF, 2- combined with the ECBT (proposed metamodel) also for two gPCE orders, denoted respectively REG-2-ECBT and REG-4-ECBT.

24.4 Results and Discussion

The ECBT matrix contains $n_j = 8$ junctions dofs, $n_i = 8$ retained normal modes and $n_e = n_{eS} + n_{eNL} + n_{eE} = 2 + 5 + 1 = 8$ enriching static residual vectors, where n_{eS} corresponds to the stochastic enrichment, n_{eNL} to the nonlinear one and n_{eE} to the external force enrichment. Hence, the size of the ECBT transformation matrix is 200×24, which signify that the reduction ratio of the full problem is of 88 % (200 dofs).

For a second order gPCE, the ED is of size 31 random variables combinations chosen with correspondence to the conditions defined in Eqs. 24.5 and 24.6 among 81 Hermite polynomial roots combinations $((p + 1)^d = 3^4 = 81)$ and then transformed with respect to the probability distributions (Table 24.1). For the fourth order, 84 random variables are needed among 625 possible combinations.

Figure 24.2 shows the stochastic effect on the model normal modes through the MAC matrix. Figures 24.3 and 24.4 illustrates the comparison between the means of stochastic velocities obtained by implementing the different methods with correspondence to the above proposed process (A and B) at two chosen observation points P_{o1} and P_{o4}.

The MAC matrix, presented in Fig. 24.2, illustrates the stochastic effect on the eight first modes retained for ECBT. Indeed, the level of uncertainty is not high but sufficient to show the interest of the stochastic enrichment. Consequently, the level of nonlinearities is chosen to be low in order to ensure the contribution of each type of enrichment in the ECBT. In fact, high level of nonlinearities leads to a negligible stochastic enrichment contribution in the ECBT and thus enriching using only nonlinear effect is sufficient.

The obtained results of different implemented models are compared in terms of accuracy and time consuming by means of the CPU computation time and the temporal moments E, T and D^2 (Tables 24.1 and 24.2). In fact, Table 24.1 recalls the size of the problems to be solved and gives the values of the associated temporal moments and Table 24.2 contains different computational time values.

Fig. 24.2 MAC matrix comparing the deterministic normal modes and the means of the stochastic ones
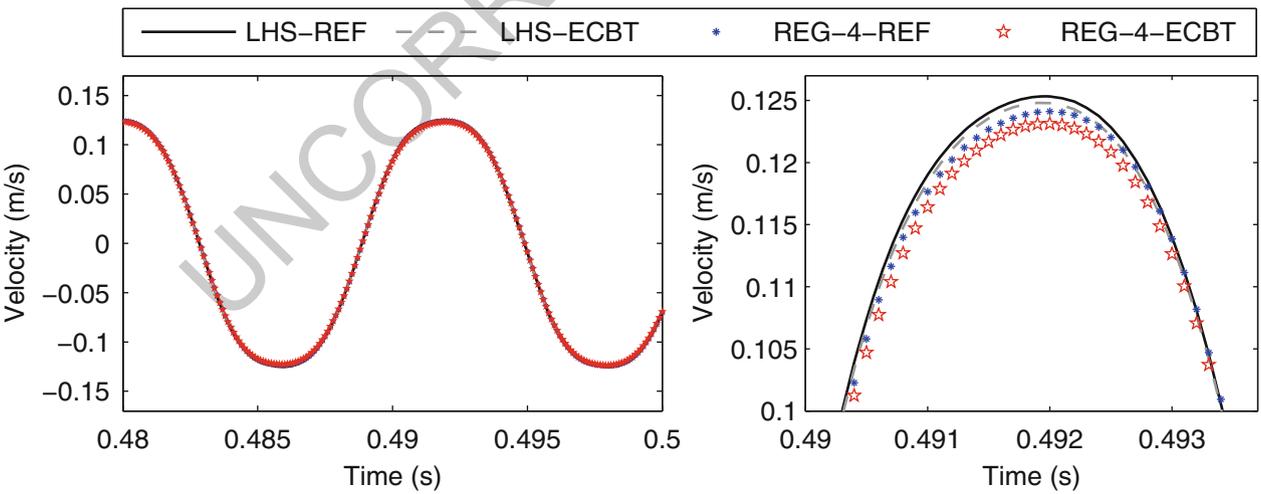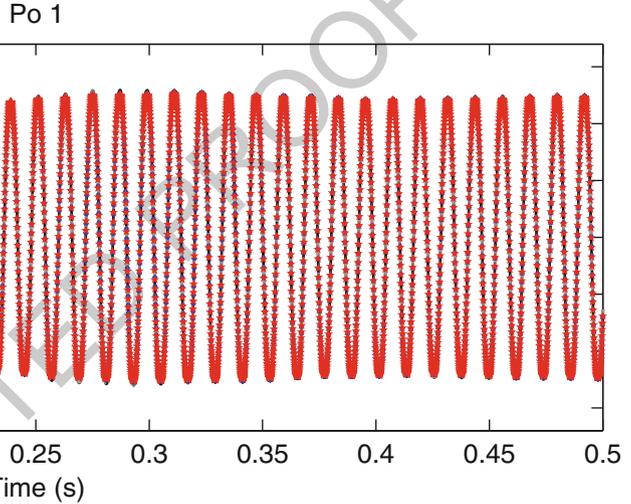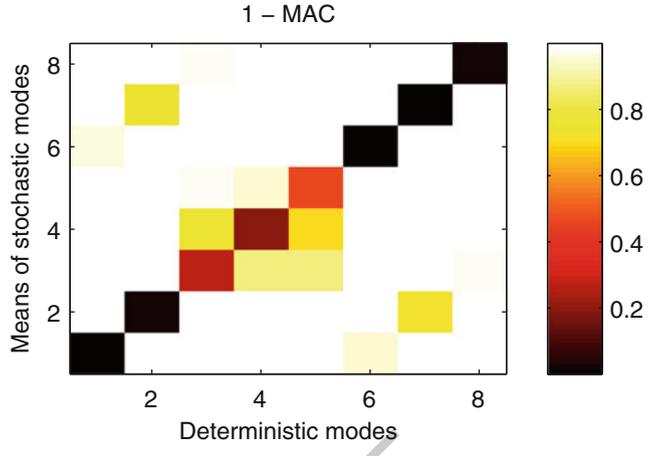
Fig. 24.3 Mean of the stochastic velocity at the observation point P_{o1} computed on the reference (complete) model (REF) and the condensed one (ECBT) using LHS and fourth order gPCE methods

The results displayed in Figs. 24.3 and 24.4 show that the proposed metamodel can replace the original one without a significant loss of accuracy with respect to the reference results computed by LHS method on the full structure. Note that the higher the order of the gPCE is, the more accurate the results are, the longer the required computational time is, as shown in
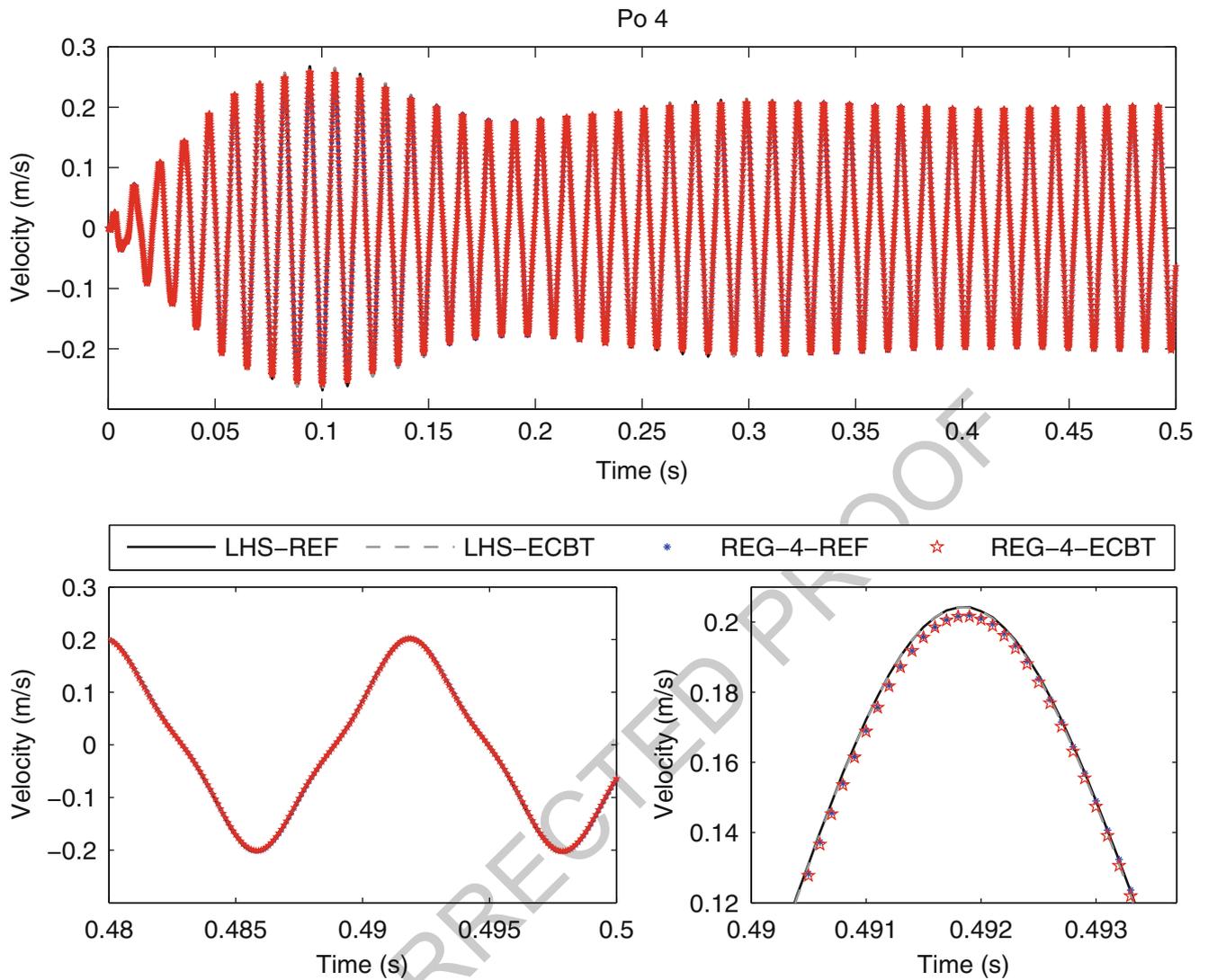
Fig. 24.4 Mean of the stochastic velocity at the observation point P_{o4} computed on the reference (complete) model (REF) and the condensed one (ECBT) using LHS and fourth order gPCE methods

Table 24.2 Computational CPU time

	Methods	CPU (min)	CPU (%)
LHS (A)	LHS-REF	6049.10	100
	LHS-ECBT	4405.13	72.82
REG (B)	REG-2-REF	201.19	3.33
	REG-4-REF	463.99	7.67
	REG-2-ECBT	155.40	2.57
	REG-4-ECBT	399.61	6.61

t5.1
t5.2

Table 24.2. Only the fourth order gPCE approximations are presented, the second order gPCE results are discussed through their temporal moments presented in Table 24.1. The ECBT efficiency can be at first evaluated comparing the two first curves (Figs. 24.3 and 24.4) and the two first lines of Table 24.1, which show that the amplitude and periodicity errors between the results of the LHS method on full and condensed model are very small.

Table 24.2 shows that the computation time can be reduced by 93.4 % when implementing the proposed metamodel, and it increases when the gPCE order is higher. Note that despite the fact that the example is not sufficiently representative in size and uncertain parameter number to show the efficiency of the CBCMS, the gain of nearly 30 % attained when implementing

ECBT with LHS or gPCE reflects more than 30 h of time reduction (1 % of gain is 61 min). For larger size examples, and   198
also when using properly optimized algorithms, the robust CBCMS contribution should be more illustrated.   199

24.5   Concluding Remarks   200

In this paper, a robust metamodel is proposed in order to approximate structure behaviors in presence of uncertainties and   201
localized nonlinearities. This metamodel is obtained by the combination between the parametric uncertainty propagation   202
method and a robust CMS approach. The efficiency of the metamodel was successfully evaluated on the temporal response   203
approximations of the coupled beams structure with respect to the reference LHS method results, thanks to different verified   204
robustness criterions. The reduction in terms of time consuming and model size using the comparison of the computation   205
times and temporal moments prove the efficiency and the usefulness of the proposed metamodel for the robustness analysis   206
of large size nonlinear stochastic model.   207

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