Chapter 5
Nonlinear 2-DOFs Vibration Energy Harvester Based on Magnetic Levitation

I. Abed, N. Kacem, M.L. Bouazizi, and N. Bouhaddi

Abstract The nonlinear dynamics of a two-degree-of-freedom (2-DOFs) vibrating energy harvester (VEH) based on magnetic levitation is modeled and investigated. The equations of motion have been derived while taking into account the magnetic nonlinearity and the electro-magnetic damping. The associated linear eigenvalue problem has been analyzed and optimality conditions have been expressed in term of distance minimization between the two eigenfrequencies of the considered system. The resulting optimal design parameters have been substituted into the coupled nonlinear equations of motion which have been numerically solved. It is shown that the performances of a classical single degree of freedom VEH can be significantly enhanced up to 270 % in term of power density, up to 34 % in term of frequency bandwidth and up to 10 % in term of resonance frequency attenuation.

Keywords Energy harvesting • Nonlinear dynamics • Magnetic levitation • Electro-magnetic damping • Frequency bandwidth

5.1 Introduction

Over recent years, many research activities are oriented to the development of some harvester devices to produce inexhaustible electric energy in their local environment. The purpose in using Vibration energy harvesters (VEHs) is related to the reduction of power requirement as well as the communication in an autonomous way and to the disposition of the monitor, the measure, and the process data in a hostile environment. On this concept, VEHs can be used in many fields such as environmental monitors, wireless sensors and medical implants [1–3]. In order to make VEHs usable, there are several types of transduction, where the most common transduction modes are piezoelectric [4], electrostatic, magnetostrictive and electromagnetic [5] and each of them presents advantages and disadvantages.

The linear VEHs has a low bandwidth and designed to collect power in the ambient dominant frequency. When the excitation and resonance frequency of the system do not coincide, linear VEHs are known to under perform. So, to increase the collected power and the bandwidth frequency, researches are oriented towards the study of nonlinear systems. For instance, Daqaq et al. [6] reported softening frequency response characteristics in a parametrically forced piezoelectric device with structural nonlinearities. Mann et al. [7] showed analytically and experimentally how magnetic levitation could be used to extend device bandwidth through a hardening response. Nevertheless, this extension is limited by dry friction dissipation phenomenon. Mahmoudi et al. [8] proposed an alternative to overcome this issue by guiding the moving magnet vertically

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in an elastic way by means of sandwich beams, combined with an hybrid piezoelectric and electromagnetic transductions. Although their numerical results demonstrated high performances, the electronic circuit may become very complex.

In this paper, we propose a design of a VEH based on the magnetic levitation of two coupled magnets. The equations of motion have been derived and include the magnetic nonlinearity and the electro-magnetic damping. While respecting the optimality conditions expressed in term of distance minimization between the two eigenfrequencies of the associated linear problem, the coupled nonlinear equations of motion have been solved numerically using the harmonic balance method (HBM) [9] coupled with the asymptotic numerical continuation technique (ANM) [10]. We prove numerically that the nonlinear coupling between the magnets permits the performance enhancement of a classical single degree of freedom VEH in terms of frequency attenuation, power density and frequency bandwidth.

5.2 Design and System Modeling

5.2.1 Proposed Device

Inspired by [5, 7], an extension of magnetic levitation for a multi-degree of freedom vibration energy harvester is proposed. As a proof of concept, the considered device shown in Fig. 5.1 is composed of four magnets M_i where $i \in [1, 4]$. M_1 and M_4 are fixed with respect to a Teflon tube inside which M_2 and M_3 are subjected to magnetic levitation forces. All magnets are placed vertically in such a way that all opposed surfaces have the same pole and wire-wound copper coils are wrapped horizontally around the separation distance between each two adjacent magnets.

Three reference frames have been applied in order to describe the electromagnetic induction model represented in Fig. 5.1. The first reference frame is fixed in space and is used to describe the motion amplitude Y_0 and excitation frequency Ω of the outer housing. The second and third reference frames (designed as x_2 and x_3) describe the motion of the moving magnets M_2 and M_3 which are subjected to restoring forces expressed as follows:

$$\mathbf{F}^{m}_{(2)} = \left(-\frac{M_2 g {d_0}^2}{(M_1 Q_1 - M_3 Q_3)}\left(\frac{M_1 Q_1}{(d_0 - v_2)^2} - \frac{M_3 Q_3}{(d_0 + v_2 - v_3)^2}\right) + M_2 g\right)\overrightarrow{\mathbf{x_2}} \quad (5.1)$$

$$\mathbf{F}^{m}_{(3)} = \left(-\frac{M_3 g {d_0}^2}{(M_2 Q_2 - M_4 Q_4)}\left(\frac{M_3 Q_3}{(d_0 + v_2 - v_3)^2} - \frac{M_4 Q_4}{(d_0 + v_3)^2}\right) + M_3 g\right)\overrightarrow{\mathbf{x_3}} \quad (5.2)$$

When the device is subjected to an external mechanical vibration, the moving magnets oscillate around their equilibrium positions and a current is induced in each coil as shown in Fig. 5.2, resulting in the following electrical damping forces:

$$F^{e}_{(2)} = ce_1 \dot{v}_2 - ce_2 (\dot{v}_3 - \dot{v}_2) \quad (5.3)$$

$$F^{e}_{(3)} = ce_3 \dot{v}_3 - ce_2 (\dot{v}_2 - \dot{v}_3) \quad (5.4)$$

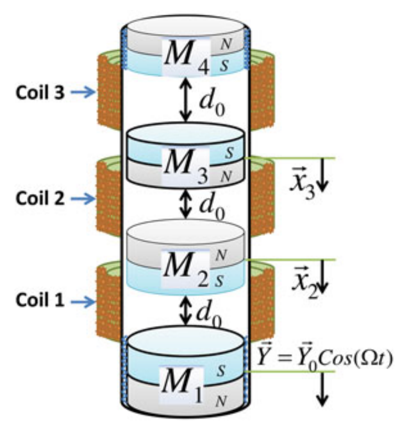

Fig. 5.1 A schematic diagram of the two degree of freedom vibration energy harvester based on magnetic levitation

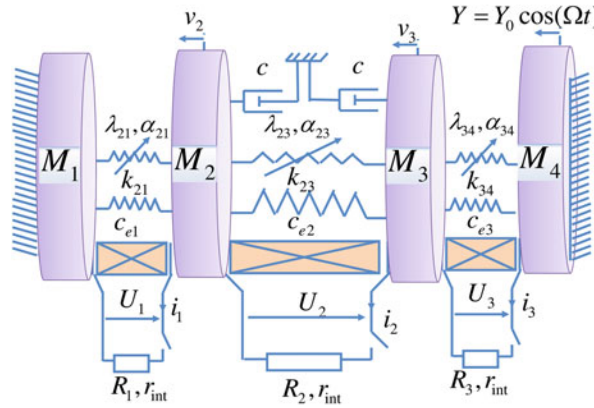

Fig. 5.2 Equivalent electro-mechanical system of the two degree of freedom vibration energy harvester based on magnetic levitation

The electrical damping can be expressed as a function of the internal resistance of the coil, the resistance of external load and the electromechanical coupling coefficient.

$$ce_j = \frac{{\alpha_j}^2}{R_j + r_{int}} \tag{5.5}$$

where $\alpha_j = N_j Bl$, with B the average magnetic field strength, N_j is the number of coil turns and l is the coil length.

Expanding the non-linear magnetic forces written in Eq. (5.2) in Taylor series up to the third order and summing all forces applied to each moving magnet in the vertical direction, we obtain the following governing equations of motion for the mechanical system:

$$\begin{cases} \begin{pmatrix} M_2\ddot{v}_2 + (c + ce_1 + ce_2)\dot{v}_2 - ce_2\dot{v}_3 + k_{23}(v_2 - v_3) + k_{21}(v_2) \\ +\alpha_{23}(v_2 - v_3)^2 - \alpha_{21}(v_2)^2 + \lambda_{23}(v_2 - v_3)^3 + \lambda_{21}(v_2)^3 \end{pmatrix} = -M_2\ddot{Y} \\ \begin{pmatrix} M_3\ddot{v}_3 + (c + ce_2 + ce_3)\dot{v}_3 - ce_2\dot{v}_2 \\ +k_{34}(v_3) + k_{32}(v_3 - v_2) + \alpha_{34}(v_3)^2 \\ -\alpha_{32}(v_3 - v_2)^2 + \lambda_{34}(v_3)^3 + \lambda_{32}(v_3 - v_2)^3 \end{pmatrix} = -M_3\ddot{Y} \end{cases} \tag{5.6}$$

If the magnetic intensities (Q) are equal, $\forall j \in [2, 3]$ the linear and nonlinear stiffnesses can be written as:

$$\begin{array}{c} k_{j(j+1)} = \frac{2gM_jM_{j+1}}{(M_{j-1}-M_{j+1})d_0};\, k_{j(j-1)} = \frac{2gM_jM_{j-1}}{(M_{j-1}-M_{j+1})d_0} \\ \alpha_{j(j+1)} = \frac{3}{2d_0}k_{j(j+1)};\, \alpha_{j(j-1)} = \frac{3}{2d_0}k_{j(j-1)} \\ \lambda_{j(j+1)} = \frac{2}{{d_0}^2}k_{j(j+1)};\, \lambda_{j(j-1)} = \frac{2}{{d_0}^2}k_{j(j-1)} \end{array} \tag{5.7}$$

5.3 Optimality Conditions and Average Power

Firstly, the associated linear problem is written in its matrix form. Then, in order to guaranty the symmetry of the rigidity matrix, we assume that $k_{23} = k_{32}$ which results in the following relation between the magnets in term of mass:

$$M_4 + M_1 = M_2 + M_3 \tag{5.8}$$

While respecting Eq. (5.8), the eigen frequencies can be written as follows:

$${\Omega_1}^2, {\Omega_2}^2 = \left(\frac{1}{2}\left\{\frac{(k_{23}+k_{21})M_3+(k_{34}+k_{32})M_2}{M_2M_3}\right\} \pm \frac{1}{2}\sqrt{\begin{bmatrix} \left\{\frac{(k_{23}+k_{21})M_3+(k_{34}+k_{32})M_2}{M_2M_3}\right\}^2 \\ -4\left\{\frac{(k_{23}+k_{21})(k_{34}+k_{32})-k_{23}k_{32}}{M_2M_3}\right\} \end{bmatrix}} \right) \tag{5.9}$$

In order to enhance the performances of the considered VEH in term of bandwidth, we minimize the distance separating the naturel frequencies expressed in Eq. (5.9). Doing so, we obtain the following optimality conditions:

$$\begin{cases} M_2 = M_1 - M_4 \\ M_3 = 2M_4 \\ M_1 > 2M_4 \end{cases} \tag{5.10}$$

The optimality conditions (5.10) have been substituted in the system of nonlinear equations (5.6) which have been solved numerically using the harmonic balance method coupled with the asymptotic numerical continuation technique. Thus, the nonlinear frequency responses can be plotted in term of velocity that will be used to determine the average power delivered to the electrical load.

Indeed, the magnetic transduction is ensured by three coils. The oscillations of the movable magnets cause magnetic field variations in the separation zones, which provides an induced current (Lenz's Law). The induced current can be expressed as a vibration velocity function $\dot{v}_j(t) = V_j \Omega \sin(\Omega t); \forall j \in [2, 3]$. Then, the average power delivered to the electrical load can be written as follows:

$$P_m = \frac{1}{T}\int_0^T P(t)dt = \frac{\Omega^2}{2}\left(\sum_{j=1}^{3}\left(ce_j\left(V_{j+1} - V_j\right)^2\right)\right) \tag{5.11}$$

5.4 Results and Discussion

5.4.1 Optimal Design Parameters

In order to enhance the bandwidth of the proposed device, the distance between the eigenfrequencies written in Eq. (5.9) is plotted with respect to M_1 and M_4 as shown in Fig. 5.3. Remarkably, Eq. (5.10) are not verified for regions 1 and 2 which are separated by an optimal zone for which $f_2 - f_1 < 4.3\ Hz$. Inside this region, a set of design parameters, listed in Table 5.1, is chosen.

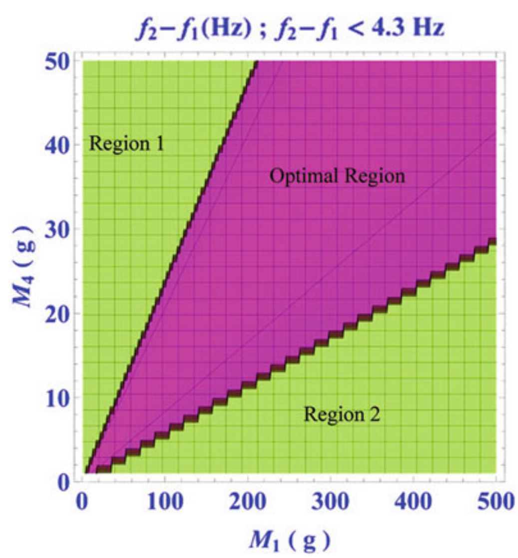

Fig. 5.3 Variation of the distance separating the eigenfrequencies of the proposed VEH with respect to M_1 and M_4. Inside the optimal zone, $f_2 - f_1 < 4.3\ Hz$

Table 5.1 Physical and geometric properties of the proposed 2-DOFs vibration energy harvester

r_{int}	Internal resistance (Ω)	188
l	Total coil length (m)	0.015
d_0	Separation distance (m)	0.015
B	Residual magnetic flux density (T)	1.18
c	Mechanical damping [7] (Ns/m)	0.116
ρ_m	Magnet density (kg/m^2)	7800
$R_j, j \in [1, 3]$	External load resistance (Ω)	10^4
$N_j, j \in [1, 3]$	The coil number (turns)	2000
M_1	Mass (kg)	0.125
M_2	Mass (kg)	0.0195
M_3	Mass (kg)	0.02
M_4	Mass (kg)	0.01

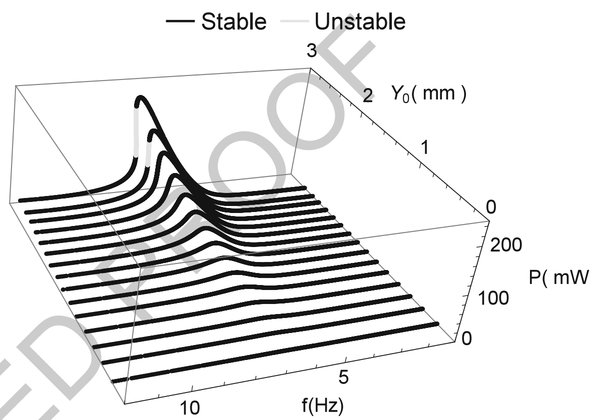

Fig. 5.4 Variation of the frequency responses in term of harvested power with respect to the excitation amplitude for the proposed device. *Black and gray lines* denote respectively stable and unstable branches

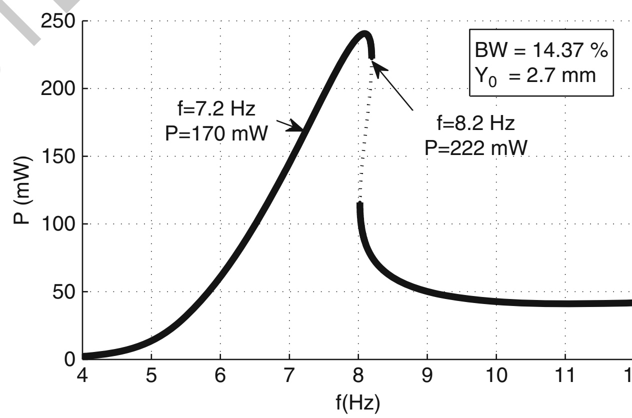

Fig. 5.5 A typical frequency response of the proposed device showing a hardening behavior for an excitation amplitude $Y_0 = 2.7\,mm$, the frequency bandwidth is $BW = 14.4\,\%$. *Solid and dashed lines* denote respectively stable and unstable branches

5.4.2 *Bandwidth Enhancement*

Several numerical simulations have been performed for the set of design parameters listed in Table 5.1. Figure 5.4 displays the evolution of the frequency response in term of harvested power for different values of amplitude excitation Y_0 up to $3\,mm$. It is shown that bistability takes place for large excitation amplitudes ($Y_0 > 2\,mm$) for which the dynamic response has two solutions for a given frequency inside a range separated by two bifurcation points.

Hence, one can take advantage of the nonlinear spring hardening effect in order to enlarge the frequency bandwidth of the VEH. Interestingly, Fig. 5.5 shows that the frequency bandwidth of the proposed device can reach $14.4\,\%$ for an excitation amplitude $Y_0 = 2.7\,mm$.

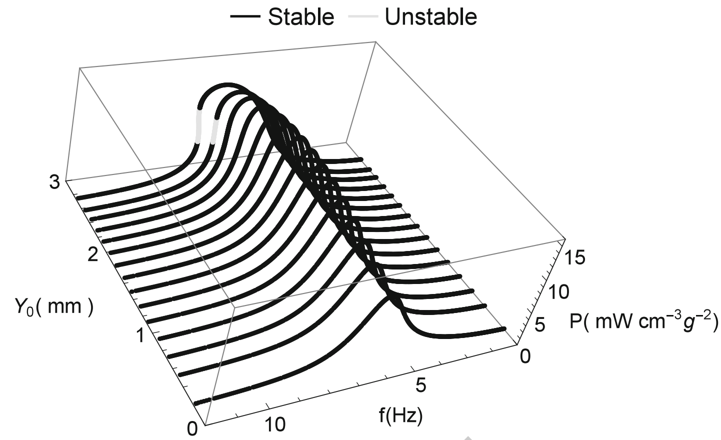

Fig. 5.6 Variation of the frequency responses in term of normalized power density with respect to the excitation amplitude for the proposed device. *Black and gray lines* denote respectively stable and unstable branches

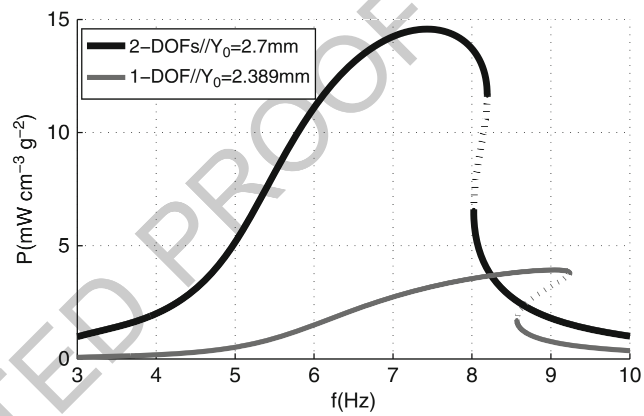

Fig. 5.7 Comparison of the frequency responses in term of normalized power density between the proposed device (2-DOFs) and the single degree of freedom VEH presented in [7]

Table 5.2 Performances of 1-DOF and 2-DOFs vibration energy harvesters based on magnetic levitation

Model	Resonance frequency (Hz)	Frequency bandwidth (%)	Normalized power ($mWcm^{-3}g^{-2}$)	Power (mW)
1-DOF	6.96	10.5	3.93	35.38
2-DOFs	6.60	14.4	14.57	240.50

5.4.3 *Normalized Power Density Enhancement*

Since the harvested power varies with respect to the excitation amplitude and in order to compare the proposed device with the one investigated in [7], in terms of performances, we propose the use of the normalized power density as a kind of energetic efficiency. The latter varies slightly with respect to Y_0 which is proved in Fig. 5.6 showing that the maximum of frequency curves is almost constant independently of the excitation amplitude.

Figure 5.7 shows that the energetic efficiency of a single degree of freedom VEH based on magnetic levitation can be enhanced up to 270 % by coupling two moving magnets. Moreover, the bandwidth can be boosted up to 34 % and the frequency attenuation is about 10 % as listed in Table 5.2.

5.5 Conclusion

The non-linear dynamics of a two-degree-of-freedom vibrating energy harvester (VEH) was modeled including the main sources of non-linearities. The associated linear eigen problem was analytically solved and optimality conditions were derived in term of distance minimization between the two eigenfrequencies. The resulting coupled nonlinear equations of motion were solved numerically for the optimal design, using the harmonic balance method coupled with the asymptotic numerical continuation technique.

Several numerical simulations have been performed to highlight the performance of the proposed device. Particularly, the power density, the bandwidth and the frequency attenuation can be boosted up to 270 %, 34 % and 10 % respectively compared to the case of a single degree of freedom magnetic levitation based VEH, thanks to the nonlinear coupling between the magnets. Future work will include the extension of the proposed concept to large arrays of coupled levitated magnets.

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