Nonlinear dynamic behavior of a 3-DOFs system induced by magnetic levitation

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Abstract – The nonlinear dynamics of a 3-DOFs system induced by magnetic levitation is modelled and investigated. The resulting coupled equations of motion include the nonlinear magnetic stiffness and the electromagnetic damping. They are solved numerically in the frequency domain by using the harmonic balance method (HBM) coupled with the asymptotic numerical continuation technique (ANM). Several numerical simulations have been performed for a specific set of design parameters in order to analyze qualitatively as well as quantitatively the frequency responses of each magnet in terms of oscillation amplitudes and bifurcation topologies. It is shown that the nonlinearity can be tuned to modify the dynamic behavior of the proposed system and thus, it can be functionalized for energy harvesting applications.

Keywords: Nonlinear dynamics / Magnetic levitation / Harmonic balance method / Asymptotic numerical method.

1 Introduction

Magnetic levitation is an exciting technology used in several applications such as transportation, aerospace, automotive and civil engineering. For small levitated devices, the generated magnetic forces are nonlinear and there is a real need to model the nonlinear dynamics of such systems. For instance, Mann et al. [1] showed experimentally and analytically how magnetic levitation could be used in order to extend the bandwidth of vibration energy harvesters through a hardening response. In this paper, we investigate the nonlinear dynamics of a three DOFs system induced by magnetic levitation. The equations of motion have been derived and include the electro-magnetic damping and the magnetic nonlinearity. The nonlinear equation of motion have been solved numerically using the harmonic balance method [2] coupled with asymptotic numerical method [3]. For a specific set of design parameters, the frequency responses of the three magnets have been plotted and can be tuned in terms of oscillation amplitudes and bifurcation topologies with respect to the nonlinearity.

2 Design and system modelling

Inspired by [1], an extension of magnetic levitation for a multi-degree of freedom system is proposed. The considered device shown in Figure 1 is composed of five magnets: M_1 and M_5 are fixed while M_2, M_3 and M_4 are subjected to magnetic levitation

forces. All magnets are placed vertically in such a way that all opposed surfaces have the same pole and wire-wound copper coils are wrapped horizontally around the separation distance between each two adjacent magnets. The 3-DOFs system is submitted to a harmonic base motion: $Y(t) = Y_0 Cos\Omega t$.
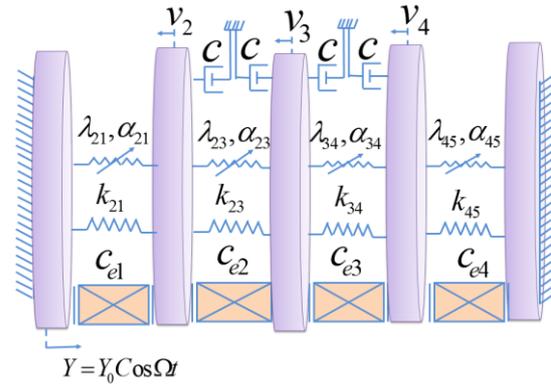

Figure 1. Equivalent scheme of three coupled levitated magnets

The coupled nonlinear equation of motion can be expressed as follows:

$$
\begin{cases}
\begin{pmatrix}
M_2\ddot{v}_2 + (c + c_{e1} + c_{e2})\dot{v}_2 \\
-c_{e3}\dot{v}_3 + k_{21}v_2 + \alpha_{21}v_2^2 + k_{23}(v_2 - v_3) \\
+\alpha_{23}(v_2 - v_3)^2 + \lambda_{23}(v_2 - v_3)^3 + \lambda_{21}v_2^3
\end{pmatrix} = -M_2\ddot{Y} \\[2em]
\begin{pmatrix}
M_3\ddot{v}_3 + (c + c_{e2} + c_{e3})\dot{v}_3 - c_{e4}\dot{v}_4 \\
-c_{e2}\dot{v}_5 + k_{34}(v_3 - v_4) + k_{32}(v_3 - v_2) \\
+\alpha_{34}(v_3 - v_4)^2 + \alpha_{32}(v_3 - v_2)^2 \\
+\lambda_{34}(v_3 - v_4)^3 + \lambda_{32}(v_3 - v_2)^3
\end{pmatrix} = -M_3\ddot{Y} \\[2em]
\begin{pmatrix}
M_4\ddot{v}_4 + (c + c_{e3} + c_{e4})\dot{v}_4 \\
-c_{e3}\dot{v}_3 + k_{45}v_4 + k_{43}(v_4 - v_3) \\
+\alpha_{45}v_4^2 + \alpha_{43}(v_4 - v_3)^2 \\
+\lambda_{43}(v_4 - v_3)^3 + \lambda_{45}v_4^3
\end{pmatrix} = -M_4\ddot{Y}
\end{cases}
$$

$$(1)$$

The parameters of Equation (1) are defined in Table 1. The magnetic transduction is ensured by four coils. The oscillations of the

movable magnets cause magnetic field variations in the separation zones, which provides an induced current (Lenz's Law).

3 Results and discussion

The numerical simulations have been performed for a specific set of design parameters, listed in Table 1.

Table 1. Physical and geometric parameters of the proposed design

Variable	Characteristic	
	Name	*Value*
c_{e1}	Electro-magnetic damping (Ns/m)	0.303
c_{e2}	Electro-magnetic damping (Ns/m)	0.303
c_{e3}	Electro-magnetic damping (Ns/m)	0.758
c_{e4}	Electro-magnetic damping (Ns/m)	3.03
ζ	Mechanical damping ratio	0.115
M_1	Mass (kg)	0.11
M_2	Mass (kg)	0.07
M_3	Mass (kg)	0.06
M_4	Mass (kg)	0.02
M_5	Mass (kg)	0.01
k_{21}	Stiffness (N/m)	201
$k_{23}{=}k_{32}$	Stiffness (N/m)	110
$k_{34}{=}k_{43}$	Stiffness (N/m)	31.0
k_{45}	Stiffness (N/m)	5.0
α_{21}	Nonlinear stiffness (N/m^2)	2.0e4
$\alpha_{23}{=}\alpha_{32}$	Nonlinear stiffness (N/m^2)	1.1e4
$\alpha_{34}{=}\alpha_{43}$	Nonlinear stiffness (N/m^2)	0.31e4
α_{45}	Nonlinear stiffness (N/m^2)	0.05e4
λ_{21}	Nonlinear stiffness (N/m^3)	1.8e6
$\lambda_{23}{=}\lambda_{32}$	Nonlinear stiffness (N/m^3)	9.7e5
$\lambda_{34}{=}\lambda_{43}$	Nonlinear stiffness (N/m^3)	2.87e5
λ_{45}	Nonlinear stiffness (N/m^3)	0.46e5
Y_0	Base motion Amplitude (mm)	8.2

Figure 2 shows the frequency responses of the three magnets. They are characterized by a hardening-type behavior and an unstable branch delimited by two bifurcation points. An extension of the frequency bandwidth is remarkable which is due to the nonlinear

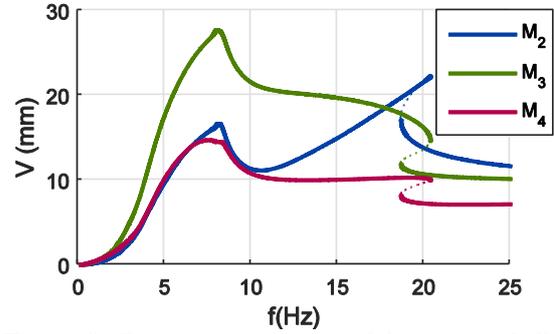coupling. Hence, the magnetic nonlinearity can be functionalized for energy harvesting applications.

Figure 2. Frequency responses of the system. Solid and dashed lines denote respectively stable and unstable branches

4 Conclusion

The nonlinear dynamics of 3-DOFs induced by magnetic levitation has been modelled including the main sources of nonlinearity. The three coupled nonlinear equations of motion have been solved numerically for a specific set of design parameters. We demonstrated that the extension of the frequency bandwidth of each magnet response can be done by tuning the nonlinearities. Future work will include the optimization problem in term of performance enhancement of a multi degrees of freedom vibration energy harvester.

5 References

[1] B. P. Mann and N. D. Sims, Energy harvesting from the nonlinear oscillations of magnetic levitation, Journal of sound and vibration, **319**, 515–530, 2009.

[2] B. Cochelin and C. Vergez, A high order purely frequency-based harmonic balance formulation for continuation of periodic solutions, Journal of Sound and Vibration, **324**, 243–262, 2009.

[3] R. Arquier, S. Karkar, A. Lazarus, O. Thomas, C. Vergez, and B. Cochelin , Manlab: an interactive path-following and bifurcation analysis software /http://manlab.lma.cnrs-mrs.fr/s.