# Nonlinear dynamic behavior of a 3-DOFs system induced by magnetic levitation

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**Abstract –** The nonlinear dynamics of a 3-DOFs system induced by magnetic levitation is modelled and investigated. The resulting coupled equations of motion include the nonlinear magnetic stiffness and the electromagnetic damping. They are solved numerically in the frequency domain by using the harmonic balance method (HBM) coupled with the asymptotic numerical continuation technique (ANM). Several numerical simulations have been performed for a specific set of design parameters in order to analyze qualitatively as well as quantitatively the frequency responses of each magnet in terms of oscillation amplitudes and bifurcation topologies. It is shown that the nonlinearity can be tuned to modify the dynamic behavior of the proposed system and thus, it can be functionalized for energy harvesting applications.

**Keywords**: Nonlinear dynamics / Magnetic levitation / Harmonic balance method / Asymptotic numerical method.

# 1 Introduction

Magnetic levitation is an exciting technology used in several applications such as transportation, aerospace, automotive and civil engineering. For small levitated devices, the generated magnetic forces are nonlinear and there is a real need to model the nonlinear dynamics of such systems. For instance, Mann et al. [1] showed experimentally and analytically how magnetic levitation could be used in order to extend the bandwidth of vibration energy harvesters through a hardening response. In this paper, we investigate the nonlinear dynamics of a three DOFs system induced by magnetic levitation. The equations of motion have been derived and include the electro-magnetic damping and the magnetic nonlinearity. The nonlinear equation of motion have been solved numerically using the harmonic balance method [2] coupled with asymptotic numerical method [3]. For a specific set of design parameters, the frequency responses of the three magnets have been plotted and can be tuned in terms of oscillation amplitudes and bifurcation topologies with respect to the nonlinearity.

# 2 Design and system modelling

Inspired by [1], an extension of magnetic levitation for a multi-degree of freedom system is proposed. The considered device shown in Figure 1 is composed of five magnets: $M_1$ and $M_5$ are fixed while $M_2$ , $M_3$ and $M_4$ are subjected to magnetic levitation forces. All magnets are placed vertically in such a way that all opposed surfaces have the same pole and wire-wound copper coils are wrapped horizontally around the separation distance between each two adjacent magnets. The 3-DOFs system is submitted to a harmonic base motion: $Y(t) = Y_0 Cos\Omega t$ .

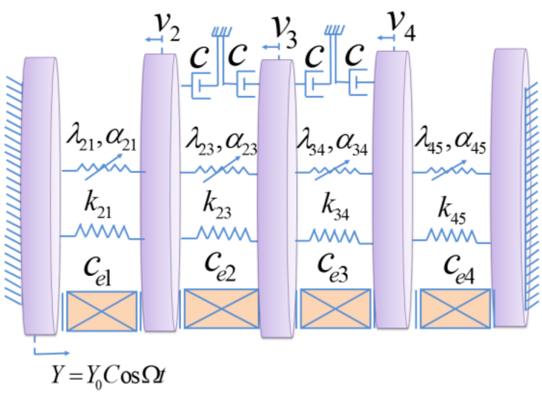


**Figure 1.** Equivalent scheme of three coupled levitated magnets

The coupled nonlinear equation of motion can be expressed as follows:

$$\begin{cases} \begin{pmatrix} M_2\ddot{v}_2 + (c + c_{e1} + c_{e2})\dot{v}_2 \\ -c_{e3}\dot{v}_3 + k_{21}v_2 + \alpha_{21}v_2^{\,2} + k_{23}(v_2 - v_3) \\ +\alpha_{23}(v_2 - v_3)^2 + \lambda_{23}(v_2 - v_3)^3 + \lambda_{21}v_2^{\,3} \end{pmatrix} = -M_2\ddot{Y} \\ \begin{pmatrix} M_3\ddot{v}_3 + (c + c_{e2} + c_{e3})\dot{v}_3 - c_{e4}\dot{v}_4 \\ -c_{e2}\dot{v}_5 + k_{34}(v_3 - v_4) + k_{32}(v_3 - v_2) \\ +\alpha_{34}(v_3 - v_4)^2 + \alpha_{32}(v_3 - v_2)^2 \\ +\lambda_{34}(v_3 - v_4)^3 + \lambda_{32}(v_3 - v_2)^3 \end{pmatrix} = -M_3\ddot{Y} \\ \begin{pmatrix} M_4\ddot{v}_4 + (c + c_{e3} + c_{e4})\dot{v}_4 \\ -c_{e3}\dot{v}_3 + k_{45}v_4 + k_{43}(v_4 - v_3) \\ +\alpha_{45}v_4^{\,2} + \alpha_{43}(v_4 - v_3)^2 \\ +\lambda_{43}(v_4 - v_3)^3 + \lambda_{45}v_4^{\,3} \end{pmatrix} = -M_4\ddot{Y} \end{cases} \tag{1}$$

The parameters of Equation (1) are defined in Table 1. The magnetic transduction is ensured by four coils. The oscillations of the