Uncertainties Propagation through Robust Reduced Model

Khaoula Chikhaoui[1,2], Noureddine Bouhaddi[1], Najib Kacem[1], Mohamed Guedri[2], and Mohamed Soula[2]

[1] FEMTO-ST Institute, UMR 6174, Applied Mechanics Department,
University of Franche-Comté, 24 Chemin de l'Epitaphe 25000 Besançon France
{khaoula.chikhaoui,noureddine.bouhaddi,
najib.kacem}@femto-st.fr
[2] National High School of Engineers of Tunis (ENSIT), University of Tunis,
5 Avenue Taha Hussein, BP 56, Bâb Manara, Tunis, Tunisia
mohamed.guedri@isetn.rnu.tn, soulamed2003@yahoo.fr

Abstract. Designing large-scale systems in which parametric uncertainties and localized nonlinearities are incorporated requires the implementation of both uncertainty propagation and robust model condensation methods. In this context, we propose to propagate uncertainties through a model, which combines the statistical Latin Hypercube Sampling (LHS) technique and a robust condensation method. The latter is based on the enrichment of a truncated eigenvectors bases using static residuals taking into account parametric uncertainty and localized nonlinearity effects. The efficiency, in terms of accuracy and time consuming, of the proposed method is evaluated on the nonlinear time response of a 2D frame structure.

Keywords: uncertainties, localized nonlinearities, robustness, model reduction.

1 Introduction

In a probabilistic framework, the well-known statistical LHS method (Helton and Davis 2003) allows propagating parametric uncertainties with high level of accuracy. It derives from the Monte Carlo Method (MC) (Rubinstein 1981) and converges faster than the latter since it distributes the sample points more evenly across intervals of equal probability. Nevertheless, the main issue of such method lies on the prohibitive cost of its implementation. The latter depends essentially on the great number of samples of random variables required for best accuracy. To overcome this issue, especially in the case of large-scale systems and iterative dynamic resolution procedures, it is inevitable to use reduced order models. In this context, the standard reduction methods are no longer efficient for designing

M. Chouchane et al. (eds.), *Design and Modeling of Mechanical Systems - II*,
Lecture Notes in Mechanical Engineering, DOI: 10.1007/978-3-319-17527-0_53

models, which incorporate both uncertainties and localized nonlinearities, since standard truncated eigenvectors bases of linear associated models do not contain any information about the latter aspects. Therefore, in order to form robust reduced models, we propose to enrich the standard bases by adding static residual vectors, which take into account the stochastic aspect and the localized nonlinearity effect.

In the literature, Balmès (Balmès 1996) and Masson (Masson et al. 2006) introduced the concepts of evaluating the static contribution of the neglected eigenvectors resulting in a set of additional vectors completing the original Ritz basis to evaluate frequency response of a modified structure. Segalman (Segalman 2007) added to the linear modes a set of basis functions to capture the nonlinearities to reduce the order of dynamical systems with localized nonlinearities. In (Bouazizi et al. 2006), a method based on the equivalent linearization method is applied to predict dynamic responses of structures affected by structural modifications and localized nonlinearities using a reduced basis enriched by static residual vectors.

Several works focus on coupling uncertainty propagation and model reduction methods. Guedri (Guedri et al. 2006) implemented the residuals enrichment technique to take into account uncertainties in the computation of the frequency responses of linear structures using stochastic spectral FE method (SSFEM). In (Maute et al. 2009), a reduced-order model (ROM) is integrated into SSFEM, using a basis spanned by displacements and derivatives of displacements, and implemented to optimize the shape of a linear shell structure.

In this work, the efficiency of the combination between the LHS method and the robust condensation technique, in terms of accuracy and computational time gain, is evaluated on the time response approximation of a frame structure, which contains localized nonlinearities and stochastic design parameters.

2 Robust Reduced Model Dedicated to the Propagation of Uncertainties

The nonlinear dynamic behavior of a mechanical system can generally be represented by the differential equation

$$[M]\{\ddot{y}\}+[B]\{\dot{y}\}+\{f_{\text{int}}\}=\{f_{ext}\} \tag{1}$$

Where $[K]$, $[M]$ and $[B]$ stand for the stiffness, mass and damping matrices of the system, $\{f_{int}\} = ([K]+\{f_{NL}\}(\{y\},\{y\square\}))\{y\}$ for the internal forces vector and $\{f_{ext}\}$, and for the exciting forces vector.

Resolving the governing equation (1), in time domain, on the full finite element model (FEM) requires high numerical cost, especially when using nonlinear Newmark time integration scheme (Géradin and Rixen 1997). Hence, a reduced order model has to be implemented. Standard condensation techniques are based on the use of truncated eigenvectors bases $[T] = [\varphi_r]$ of the associated linear system, the index r is relative to the reduced term. Therefore, using such bases, in a stochastic case with localized nonlinearities, cannot satisfy the required accuracy.

To overcome this issue, the enrichment of $[\varphi_r]$ by adding a complementary subbasis $[\Delta T]$ is necessary in order to construct a robust basis $[T] = [\varphi_r \perp \Delta T]$.

$[\Delta T]$ is a set of static residual vectors calculated according to the type of enrichment. To take into account localized nonlinear effects, the static residuals must be computed (Bouazizi et al. 2006), according to the following form

$$\left[\Delta T_{NL}\right]_i = \left[K_0\right]^{-1}\left\{F_i\right\},\ i = 1,\ldots,m \tag{2}$$

where $\{F_i\}$ is the residual force vectors containing unit values in nonlinear degrees of freedom and zeros otherwise, m is the total number of nonlinear degrees of freedom (dofs).

In addition, to take into account stochastic aspect (Guedri et al. 2006), another type of residual vectors

$$\left[\Delta T_S\right] = \left[R\right]\left[F_{S_z}\right] \tag{3}$$

must be computed, where $[R] = [K_0]^{-1}-[\varphi_r][\Lambda_r]^{-1}[\varphi_r]^{t}$ is the static residual flexibility matrix and $[F_{Sz}]$ is a set of force vectors representing the stochastic effects for each stochastic zone (z), $[K_0]$ is the deterministic stiffness matrix and $[\Lambda_r]$ is the spectral one (containing only retained eigenvalues).

Then, the singular value decomposition (SVD) and the normalization of the additional residuals similarly to the standard basis $[\varphi_r]$ are inevitable in order to ensure, respectively, the linear independence (the well-conditioning) and the orthogonality of the different vectors. Consequently, the enriched basis (EB) has the following form

$$\left[T\right] = \left[\varphi_r \ \vdots\ \Delta T_{NL} \ \vdots\ \Delta T_S\right] \tag{4}$$

Projecting the time response on this basis, such as $\{y\} = [T]\{q\}$ permits to express the equation of motion (1), at time step $n+1$, in the reduced following form

$$\left[M\right]_r \left\{\ddot{q}\right\}_{n+1} + \left[B\right]_r \left\{\dot{q}\right\}_{n+1} + \left(\left\{f_{\text{int}}\right\}_r\right)_{n+1} = \left(\left\{f_{ext}\right\}_r\right)_{n+1} \tag{5}$$

where $[M]_{\rm r} = [T]^t\,[M]\,[T]$, $[B]_{\rm r} = [T]^t\,[B]\,[T]$, $\{f_{ext}\}_{\rm r} = [T]^t\,\{f_{ext}\}$, and $\{f_{int}\}_{\rm r} = [T]^t([K]+\{f_{NL}\}([T]\{q\},[T]\{\dot{q}\}))[T]\{q\}$ are the reduced matrices and vectors.

Approximating the solution using the Newmark nonlinear time integration scheme and the iterative Newton-Raphson technique consists on minimizing the residual vector

$$R_{n+1} = \left[M\right]_r \left\{\ddot{q}\right\}_{n+1} + \left[B\right]_r \left\{\dot{q}\right\}_{n+1} + \left(\left\{f_{\text{int}}\right\}_r\right)_{n+1} - \left(\left\{f_{ext}\right\}_r\right)_{n+1} \tag{6}$$

The equation (6) can be approximated, for the iteration k, as:

$$\left(\left[K_{eff}\right]_r\right)_{n+1}^k \Delta\{q\}^k = -R_{n+1}^k \tag{7}$$

where $\left(\left[K_{eff}\right]_r\right)_{n+1}^k = \left[\frac{\partial R}{\partial\{q\}}\right]_{\{q\}_{n+1}^k}$ is the effective stiffness matrix function of the tangent stiffness matrix, updated at each iteration,

$$\left(\left[K_T\right]_r\right)_{n+1} = [T]^t\left(\left[\frac{\partial\{f_{\text{int}}\}_{n+1}}{\partial\{y\}_{n+1}}\right]_{[T]\{q\}_{n+1}^k}\right)[T]. \tag{8}$$

The statistical LHS method (Helton and Davis 2003) of uncertainty propagation consists on computing N successive deterministic responses $\{Y^{(n)} = y(\xi^{(n)}), n=1, \ldots, N\}$ according to a set of N samples of random variables.

Statistical moments can then be computed, such as the mean and the variance (1^{st} and 2^{nd} moment).

The proposed method in this paper consists to combine the LHS uncertainty propagation approach and model reduction by projecting the responses on the robust enriched basis defined above.

3 Numerical Example

In order to valid the proposed method and to illustrate their numerical performances, a 2D frame model is proposed (Fig.1). The mechanical and geometrical properties of the structure are given by: the width b=3.10^{-2} m and the thickness h_0=5.10^{-2} m of the rectangular section, the length of the beams L=1.5 m, the Young modulus E_0=210 GPa and the density ρ_0=7800 $kg.m^{-3}$. We suppose that the damping is proportional so that the modal damping is η=0.03 and two localized nonlinear Duffing springs of stiffness k_{NL} = 10^{20} $N.m^{-3}$ are linked to the frame structure. Two localized forces with equal amplitude $F(N)$=10^{3}cos($2\pi f_2 t$) excite the second eigenmode (f_2=78.8 $rad.s^{-1}$) of the frame structure. Using two-dimensional beam finite elements (three dofs per node: u_x, v_y, θ_z) to discretize the 2D structure leads to 160 finite elements and a full model of 474 dofs.

Three zones (Fig.1) are supposed to be stochastic: the Young modulus and the density, in the vertical beams, and the thickness in the upper horizontal beam. Their randomness is modeled as

$$E = E_0\left(1+\sigma_E\xi_E\right); \rho = \rho_0\left(1+\sigma_\rho\xi_\rho\right); h = h_0\left(1+\sigma_h\xi_h\right) \tag{9}$$

where ξ_E , ξ_ρ and ξ_h are random variables of respectively lognormal, lognormal and exponential probability distributions and σ_E = σ_ρ = σ_h = 0.2 are the considered dispersions.

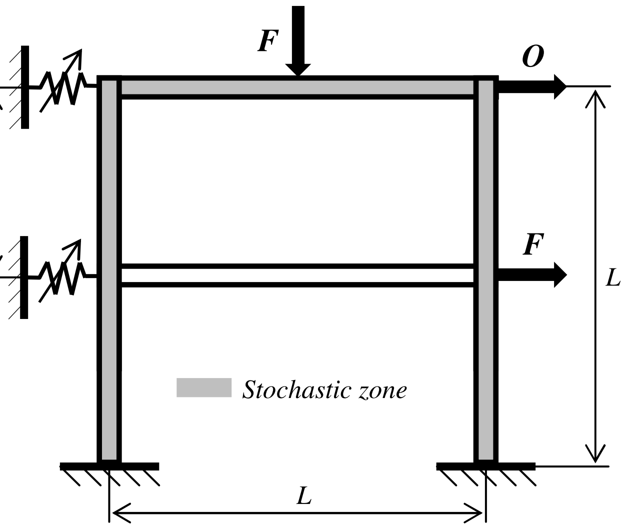

Fig. 1 Frame structure with stochastic parameters and localized nonlinearities

To evaluate the efficiency of the proposed method in nonlinear dynamic analysis, with uncertainties and localized nonlinearities, two different criteria have to be satisfied. The time indicators (Hemez and Doebling 2003)

$$M_i = \int_{-\infty}^{+\infty} (t-t_s)^i \, y(t)^2 \, dt, \tag{10}$$

where i the order of the moment and t_s the temporal shift chosen in our case as $t_s=0$ allow quantifying the accuracy of the response in terms of amplitude using the total energy $E=M_0$ and periodicity using the central time and the root mean square duration defined respectively by $T=M_1/M_0$ and $D^2=(M_2/M_0)-(M_1/M_0)^2$.

To evaluate the time consuming, the Central Processing Unit (CPU) time is computed for each implemented method.

The obtained results combining the LHS method, for 1000 samples, and the robust reduced model (LHS-EB) are compared to the reference responses using LHS method, for 1000 sample on the full model (LHS-REF). Several observation dofs are considered; the results below correspond to a chosen dof (Fig. 1, dof *O*).

Note that high uncertainty and nonlinearity levels are chosen in order to evaluate the accuracy of the enriched basis with respect to the standard one.

The comparison between the deterministic modes and the mean of the stochastic ones through the MAC (Modal Assurance Criterion) matrix, Fig. 2, and the superposition of the deterministic response and the mean of the stochastic one, Fig. 3, illustrate the effect of the uncertainties. In fact, the randomness of the chosen uncertain input parameters affects both the stiffness and the mass matrices.

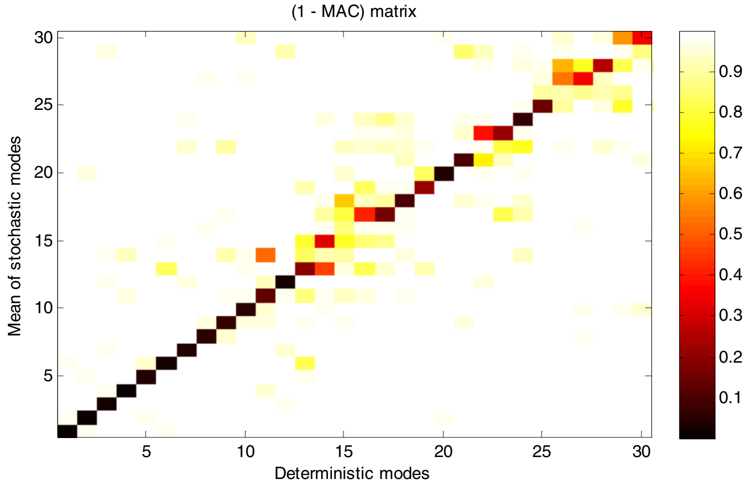

Fig. 2 MAC matrix comparing the deterministic modes and the mean of the stochastic modes

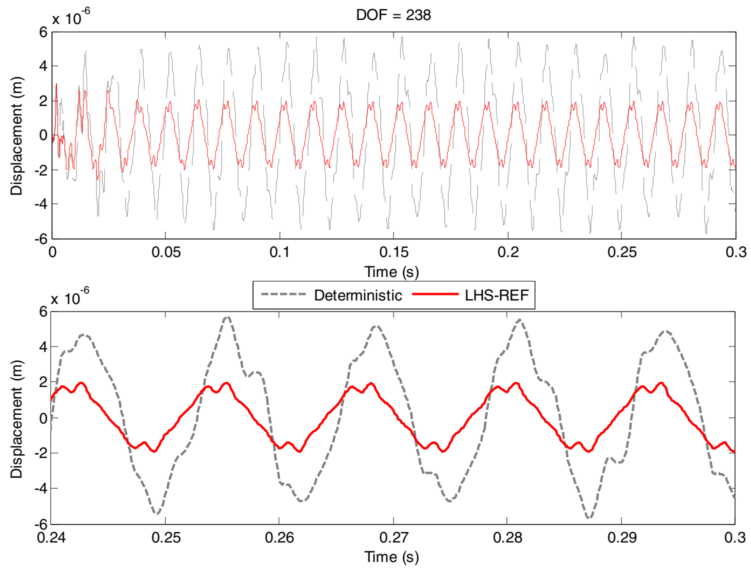

Fig. 3 Mean of the stochastic displacement computed with the full model (LHS-REF) compared to the deterministic model

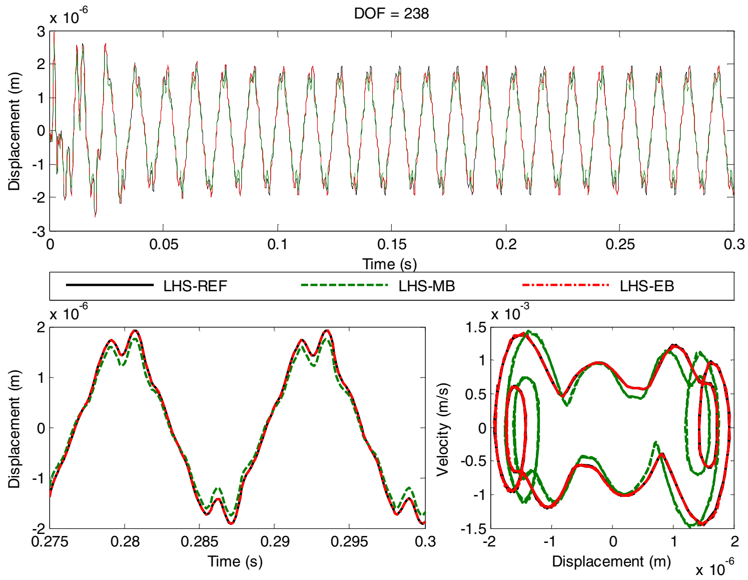

Fig. 4 Mean of the stochastic displacements and phase diagrams computed using the EB (LHS-EB), the standard truncated eigenvectors basis (LHS-MB) and the full model (LHS-REF)

Fig.4 shows the performance of the robust model (57 vectors in the enriched basis EB) by its comparison to the standard basis of same size, with respect to the reference responses obtained with the full model. The standard modal basis (MB) cannot represent the model behavior accurately while the enriched basis (EB) allows it.

The model size, the time indicators (mean values of all dof reponses) and the CPU time are listed in Table 1. The proposed method allows a reduction size ratio of 87.9 % and a computational time gain of 52.7 % with very small errors on accuracy.

Table 1 Model size and evaluation criteria

Method	Model size (dof)	Errors on Time indicators (%)			CPU time (%)
		E	T	D^2	
LHS-REF	474	0.00	0.00	0.00	100
LHS-MB	57	0.70	0.10	0.05	39.9
LHS-EB	57	0.00	0.00	0.00	47.3

4 Conclusion

Combining uncertainties propagation and robust model reduction permits to approximate the dynamic behavior of mechanical structures containing stochastic design parameters and localized nonlinearities with a low computing time and without a significant loss of accuracy. The robustness of the model reduction method is achieved with the enrichment of the standard truncated modal basis using static residuals, which permits to take into account both the stochastic and the localized nonlinearity effects. Future work will include the extension of the proposed methodology to complex mechanical structures.

References

Helton, J.C., Davis, F.J.: Latin hypercube sampling and the propagation of uncertainty in analyses of complex systems. Reliability Engineering and System Safety 81, 23–69 (2003)

Rubinstein, R.-Y.: Simulation and the Monte Carlo methods. John Wiley & Sons (1981)

Balmès, E.: Parametric families of reduced finite element models. Theory and applications. Mechanical Systems and Signal Processing 10(4), 381–394 (1996)

Masson, G., Ait Brik, B., Cogan, S., Bouhaddi, N.: Component mode synthesis CMS based on an enriched Ritz approach for efficient structural optimization. Journal of Sound and Vibration 296(4-5), 845–860 (2006)

Segalman, D.J.: Model reduction of systems with localized nonlinearities. ASME Journal of Computational and Nonlinear Dynamics 2(3), 249–266 (2007)

Bouazizi, M.L., Guedri, M., Bouhaddi, N.: Robust component modal synthesis method adapted to the survey of the dynamic behavior of structures with localized nonlinearities. Mechanical Systems and Signal Processing 20, 131–157 (2006)

Guedri, M., Bouhaddi, N., Majed, R.: Reduction of the stochastic finite element models using a robust dynamic condensation method. Journal of Sound and Vibration 297, 123–145 (2006)

Maute, K., Weickum, G., Eldred, M.: A reduced-order stochastic finite element approach for design optimization under uncertainty. Structural Safety 31, 450–459 (2009)

Gérardin, M., Rixen, D.: Mechanical Vibrations: Theory and Application to Structural Dynamics, 2nd edn. John Wiley & Sons (1997)

Hemez, F.M., Doebling, S.W.: From shock response spectrum to temporal moments and vice-versa. In: International Modal Analysis Conference (IMAC-XXI), Kissirnmee, Florida, February 3-6 (2003)