

Random Generation and Enumeration of Accessible Deterministic Real-time Pushdown Automata

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Abstract

This paper presents a general framework for the uniform random generation of deterministic real-time accessible pushdown automata. A polynomial time algorithm to randomly generate a pushdown automaton having a fixed stack operations total size is proposed. The influence of the accepting condition (empty stack, final state) on the reachability of the generated automata is investigated.

1 Introduction

Finite automata, of any kind, are widely used for their algorithmic properties in many fields of computer science like model-checking, pattern matching and machine learning. Developing new efficient algorithms for finite automata is therefore a challenging problem still addressed by many recent papers. New algorithms are frequently motivated by improvement of worst cases bound. However, several examples, such as sorting algorithms, primality testing or solving linear problems, show that worst case complexity is not always the right way to evaluate the practical performance of an algorithm. When benchmarks are not available, random testing, with a controlled distribution, represents an efficient mean of performance testing. In this context, the problem of uniformly generating finite automata is a challenging problem.

This paper tackles with the problem of the uniform random generation of real-time deterministic pushdown automata. Using classical combinatorial techniques, we expose how to extend existing works on the generation of finite deterministic automata to pushdown automata. More precisely, we show in Section 3 how to uniformly generate and enumerate (in the complete case) accessible real-time deterministic pushdown automata. In Section 4, it is shown that using a rejection algorithm it is possible to efficiently generate pushdown automata that don't accept an empty language. The influence of the accepting condition (final state or empty-stack) on the reachability of the generated pushdown automata is also experimentally studied in Section 4.

Related work. The enumeration of deterministic finite automata has been first investigated in [Vys59] and was applied to several subclasses of deterministic finite automata [Kor78, Kor86, Rob85, Lis06]. The uniform random generation of accessible deterministic complete automata was initially proposed in [Nic00] for two-letter alphabets and the approach was extended to larger alphabets in [CP05]. Better algorithms can be found in [BN07, CN12]. The random generation of possibly incomplete automata is analyzed in [BDN09]. The recent paper [CF11] presents how to use Monte-Carlo approaches to generate deterministic acyclic automata. As far as we know, the only work focusing on the random generation of deterministic transducers is [HNS10]. This work can be applied to the random generation of deterministic real-time pushdown automata that can be possibly incomplete. However the requirement to fix the size of the stack operation on each transition, represents a major restriction. The reader interested in the random generation of deterministic automata is referred to the survey [Nic14].

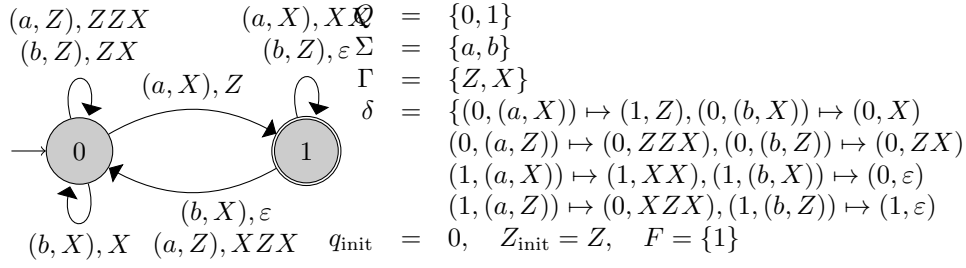


Figure 1: P_{toy} , a complete RDPDA.

2 Formal Background

We assume that the reader is familiar with classical notions on formal languages. For more information on automata theory or on pushdown automata the reader is referred to [HU79] or to [Sak09]. For a general reference on random generation and enumeration of combinatorial structures see [FZC94]. For any word w on an alphabet Σ , $|w|$ denotes its length. The empty word is denoted ε . The cardinal of a finite set X is denoted $|X|$.

Deterministic Finite Automata. A *deterministic finite automaton* on Σ is a tuple $(Q, \Sigma, \delta, q_{\text{init}}, F)$ where Q is a finite set of states, Σ is a finite alphabet, $q_{\text{init}} \in Q$ is the initial state, $F \subseteq Q$ is the set of final states and δ is a partial function from $Q \times \Sigma$ into Q . If δ is not partial, i.e., defined for each $(q, a) \in Q \times \Sigma$, the automaton is said *complete*. A triplet of the form $(q, a, \delta(p, a))$ is called a *transition*. A finite automaton is graphically represented by a labeled finite graph whose vertices are the states of the automaton and edges are the transitions. A deterministic finite automaton is *accessible* if for each state q there exists a path from the initial state to q . Two finite automata $(Q_1, \Sigma, \delta_1, q_{\text{init}1}, F_1)$ and $(Q_2, \Sigma, \delta_2, q_{\text{init}2}, F_2)$ are isomorphic if they are identical up to the state's names, formally if there exists a one-to-one function φ from Q_1 into Q_2 such that (1) $\varphi(q_{\text{init}1}) = q_{\text{init}2}$, (2) $\varphi(F_1) = F_2$, and (3) $\delta_1(q, a) = p$ iff $\delta_2(\varphi(q), a) = \varphi(p)$.

Pushdown Automata. A *real-time deterministic pushdown automaton*, RDPDA for short, is a tuple $(Q, \Sigma, \Gamma, Z_{\text{init}}, \delta, q_{\text{init}}, F)$ where Q is a finite set of states, Σ and Γ are finite disjoint alphabets, $q_{\text{init}} \in Q$ is the initial state, $F \subseteq Q$ is the set of final states, Z_{init} is the initial stack symbol and δ is a partial function from $Q \times (\Sigma \times \Gamma)$ into $Q \times \Gamma^*$. If δ is not partial, i.e., defined for each $(q, (a, X)) \in Q \times (\Sigma \times \Gamma)$, the RDPDA is said to be *complete*. A triplet of the form $(q, (a, X), w, p)$ with $\delta(q, (a, X)) = (p, w)$ is called a *transition* and w is the *output* of the transition. The *output size* of a transition $(q, (a, X), w, p)$ is the length of w . The *output size* of a RDPDA is the sum of the sizes of its transitions. The *underlying automaton* of an RDPDA, is the finite automaton $(Q, \Sigma \times \Gamma, \delta', q_{\text{init}}, F)$, with $\delta'(q, (a, X)) = p$ iff $\delta(q, (a, X)) = (p, w)$ for some $w \in \Gamma^*$. An RDPDA is *accessible* if its underlying automaton is accessible. A transition whose output is ε is called a *pop transition*. An example of a complete accessible RDPDA is depicted in Fig. 1. The related underlying finite automaton is depicted in Fig. 2.

A *configuration* of a RDPDA is an element of $Q \times \Gamma^*$. The *initial configuration* is $(q_{\text{init}}, Z_{\text{init}})$. Two configurations (q_1, w_1) and (q_2, w_2) are *a-consecutive*, denoted $(q_1, w_1) \models_a (q_2, w_2)$ if the following conditions are satisfied:

- $w_1 \neq \varepsilon$, and let $w_1 = w_3 X$ with $X \in \Gamma$,
- $\delta(q_1, (X, a)) = (q_2, w_4)$ and $w_2 = w_3 w_4$.

Two configurations are *consecutive* if there is a letter a such that there are a -consecutive. A state p of a RDPDA is *reachable* if there exists a sequence of consecutive configurations $(p_1, w_1), \dots, (p_n, w_n)$ such that (p_1, w_1) is the initial configuration and $p_n = p$. Moreover, if $w_n = \varepsilon$, p is said to be *reachable with an empty stack*. A RDPDA is *reachable* if all its states are reachable. Consider for instance the RDPDA of Fig. 3, where the initial stack symbol is X . State 3 is not reachable since the transition from 0 to 3 cannot be fired. State 1 is reachable with an empty stack. State

2 is reachable, but not reachable with an empty stack. Note that a reachable state is accessible, but the converse is not true in general: accessibility is a notion defined on the underlying finite automaton.

The configurations $(1, XZ)$ and $(2, XXZX)$ are a -consecutive on the RDPDA depicted in Fig. 1.

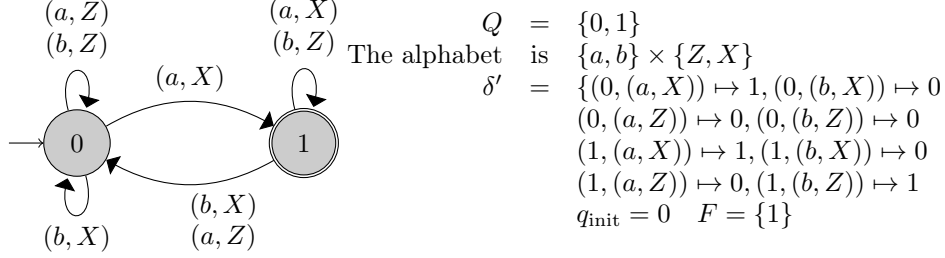


Figure 2: Underlying automaton of P_{toy} .

There are three main kinds of accepting conditions for a word $u = a_1 \dots a_k \in \Sigma^*$ by an RDPDA:

- Under the *empty-stack condition*, u is accepted if there exists configurations c_1, \dots, c_{k+1} such that c_1 is the initial configuration, c_i and c_{i+1} are a_i -consecutive, and c_{k+1} is of the form (q, ε) .
- Under the *final-state condition*, u is accepted if there exists configurations c_1, \dots, c_{k+1} such that c_1 is the initial configuration, c_i and c_{i+1} are a_i -consecutive, and c_{k+1} is of the form (q, w) , with $q \in F$.
- Under the *final-state and empty-stack condition*, u is accepted if there exists configurations c_1, \dots, c_{k+1} such that c_1 is the initial configuration, c_i and c_{i+1} are a_i -consecutive, and c_{k+1} is of the form (q, ε) , with $q \in F$.

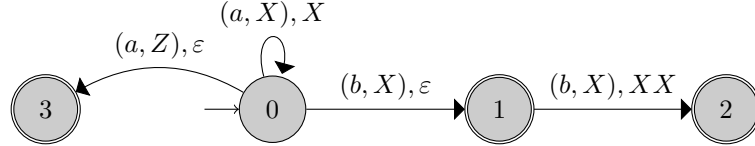


Figure 3: Acceptance conditions.

Consider for instance the RDPDA of Fig. 3, where the initial stack symbol is X . With the empty-stack condition only the word b is accepted, as well as for the empty-stack and final state condition. With the final state condition, the accepted language is $a^*(b + bb)$.

Two RDPDA $(Q_1, \Sigma, \Gamma, \delta_1, q_{\text{init}1}, F_1)$ and $(Q_2, \Sigma, \Gamma, \delta_2, q_{\text{init}2}, F_2)$ are isomorphic if there exists a one-to-one function φ from Q_1 into Q_2 such that (i) $\varphi(q_{\text{init}1}) = q_{\text{init}2}$, and (ii) $\varphi(F_1) = F_2$, and

$$(iii) \quad \delta_1(q, (a, X)) = (p, w) \quad \text{iff} \quad \delta_2(\varphi(q), (a, X)) = (\varphi(p), w).$$

Note that if two RDPDA are isomorphic, then their underlying automata are isomorphic too.

Generating Functions. A *combinatorial class* is a class \mathfrak{C} of objects associated with a size function $|\cdot|$ from \mathfrak{C} into \mathbb{N} such that for any integer n there are finitely many elements of \mathfrak{C} of size n . The ordinary generating function for \mathfrak{C} is $C(z) = \sum_{c \in \mathfrak{C}} z^{|c|}$. The n -th coefficient of $C(z)$ is exactly the number of objects of size n and is denoted $[z^n]C(z)$. The reader is referred to [FS08] for the general methodology of analytic combinatorics, and especially the use of generating functions to count objects. The following result [FS08, Theorem VIII.8] will be useful in this paper.

Theorem 1 Let $C(z)$ be an ordinary generating function satisfying: (1) $C(z)$ is analytic at 0 and have only positive coefficients, (2) $C(0) \neq 0$ and (3) $C(z)$ is aperiodic. Let R be the radius of convergence of $C(z)$ and $T = \lim_{x \rightarrow R^-} x \frac{C'(x)}{C(x)}$. Let $\lambda \in]0, T[$ and ζ be the unique solution of $x \frac{C'(x)}{C(x)} = \lambda$. Then, for $N = \lambda n$ an integer, one has

$$[z^N]C(z)^n = \frac{C(\zeta)^n}{\zeta^{N+1} \sqrt{2\pi n \xi}} (1 + o(1)),$$

where $\xi = \frac{d^2}{dz^2} (\log C(z) - \lambda \log(z)) |_{z=\zeta}$.

In the above theorem, T is called the *spread* of the $C(z)$.

Rejection Algorithms. A rejection algorithm is a probabilistic algorithm to randomly generate an element in a set X , using an algorithm A for generating an element of Y in the simple way: repeat A until it returns an element of X . Such an algorithm is tractable if the expected number of iterations can be kept under control (for instance is fixed): the probability that an element of Y is in X has to be large enough.

Random Generation. The theory of Generating Functions provides an efficient way to randomly and uniformly generate an element of size n of a combinatorial class \mathfrak{C} using a recursive approach [FS08]. It requires a $O(n^2)$ precomputation time and each random sample is obtained in time $O(n \log n)$. Another efficient way to uniformly generate element of \mathfrak{C} is to use Boltzmann samplers [DFLS04]: the random generation of an object with a size about n (approximate sampling) is performed in $O(n)$, while the random generation of an object with a size exactly n is performed in expected time $O(n^2)$ using a rejection algorithm (without precomputation). Boltzmann samplers are quite easy to implement but are restricted to a limited number of combinatorial constructions. They also require the evaluation of some generating functions at some values of the variable.

3 Random Generation and Enumeration of RDPDA

In this section, Σ and Γ are fixed disjoint alphabets of respective cardinals α and β . We denote by $\mathfrak{T}_{s,n,m}$ combinatorial class of the RDPDA (on Σ, Γ) with n states, s transitions and with an output size of m , up to isomorphism. Note that this class is well defined since two isomorphic RDPDA have the same output size. Let $\rho = \alpha\beta$.

3.1 Enumeration of RDPDA

We are interested in the random generation of accessible RDPDA up to isomorphism. The class of all isomorphic classes of accessible automata on $\Sigma \times \Gamma$ with n states and s transitions is denoted $\mathfrak{A}_{s,n}$. Let ψ be the function from $\mathfrak{T}_{s,n,m}$ into $\mathfrak{A}_{s,n}$ mapping RDPDA to their underlying automata. The number of elements of $\psi^{-1}(\mathcal{A})$, where \mathcal{A} is an element of $\mathfrak{A}_{s,n}$, is the number of possible output labelling of the s transitions of \mathcal{A} , which only depends on s and m and is independent of \mathcal{A} . We denote by $c_{s,m}$ this number of labelings. The following proposition is a direct consequence of the above remark.

Proposition 2 One has $|\mathfrak{T}_{s,n,m}| = |\mathfrak{A}_{s,n}| \cdot c_{s,m}$.

Proposition 2 is the base of the enumeration for RDPDA. Let $M(z)$ denote the ordinary generating function of elements of Γ^* . Then, using Proposition 2 and classical constructions on generating functions, one has

$$|\mathfrak{T}_{s,n,m}(\mathcal{M})| = |\mathfrak{A}_{s,n}| \cdot [z^m]M(z)^s. \quad (1)$$

Equation (1) will be exploited for complete finite automata using the following result of [Kor78] – see also [BDN07].

Theorem 3 ([Kor78]) There exists a constant γ_ρ , such that $|\mathfrak{A}_{\rho n, n}| \sim n \gamma_\rho \left\{ \begin{smallmatrix} \rho n \\ n \end{smallmatrix} \right\}$, where $\left\{ \begin{smallmatrix} x \\ y \end{smallmatrix} \right\}$ denotes the Stirling numbers of the second kind.

The case $s = \rho n$ corresponds to complete accessible automata. We will particularly focus throughout this paper on the complete case for a fixed average size λ for the transitions: we assume that there is a fixed $\lambda > 0$ such that $m = \lambda s = \lambda n \rho$. In this context, (1) becomes

$$|\mathfrak{F}_{s,n,m}(\mathcal{M})| \sim n \gamma_\rho \binom{\rho n}{n} [z^{\lambda n \rho}] M(z)^{n \rho}. \quad (2)$$

Since $M(z)$ is the generating function of words on Γ , one has $M(z) = \frac{1}{1-\beta z}$. Therefore (see [FS08]), one has:

$$[z^m] F_s(z) = \beta^m \frac{s(s+1)(s+2)\dots(s+m-1)}{m!}. \quad (3)$$

The generating function F_s satisfies the hypotheses of Theorem 1, with an infinite spread. Therefore for any strictly positive λ and any integer $m = \lambda s$, one has

$$[z^{\lambda s}] F_s(z) = \frac{C(\zeta)^s}{\zeta^{\lambda s+1} \sqrt{2\pi s \xi}} (1 + o(1)), \quad (4)$$

where $C(z) = \frac{1}{1-\beta z}$ and $\zeta = \frac{\lambda}{\beta(\lambda+1)}$ is the unique solution of $x \frac{C'(x)}{C(x)} = \lambda$. It follows that

$$[z^{\lambda s}] F_s(z) = \frac{(\lambda+1)^{(\lambda+1)s+1} \beta^{\lambda s+1}}{\lambda^{\lambda s+1} \sqrt{2\pi s \xi}} (1 + o(1)). \quad (5)$$

In addition

$$\xi = \frac{d^2}{dz^2} (\log C(z) - \lambda \log(z)) |_{z=\zeta} = \frac{(\lambda+1)^3 \beta^2}{\lambda}. \quad (6)$$

Consequently (5) can be rewritten as

$$[z^{\lambda s}] F_s(z) = \frac{(\lambda+1)^{(\lambda+1)s-1/2} \beta^{\lambda s}}{\lambda^{\lambda s+1/2} \sqrt{2\pi s}} (1 + o(1)). \quad (7)$$

The following proposition is a direct consequence of the combination of (7), Proposition 2 and Theorem 3.

Proposition 4 *The number $f_{\lambda,n}$ of complete accessible RDPDA with n states and with an output size of λn , with $\lambda \geq 1$ a fixed rational number, satisfies*

$$f_{\lambda,n} = \gamma_\rho n \binom{\rho n}{n} \frac{(\lambda+1)^{(\lambda+1)n\rho-1/2} \beta^{\lambda n \rho}}{\lambda^{\lambda n \rho+1/2} \sqrt{2\pi n \rho}} (1 + o(1)),$$

Note that $f_{\lambda,n} = |\mathfrak{F}_{n\rho,n,\lambda\rho n}|$. The following result can be easily obtained.

Proposition 5 *For RDPDA in $\mathfrak{F}_{s,n,m}$, the average number of pop transitions is $\frac{s(s-1)}{s+m-1}$.*

PROOF. We introduce the generating function $G_s(z, u) = \left(\frac{1}{1-\beta z} - 1 + u \right)^s$ counting RDPDA where z counts the output size and u the number of pop transitions. Using [FS08, Proposition III.2], the average number of pop transitions is $\frac{[z^m] \frac{\partial}{\partial u} G_s(z, u) |_{u=1}}{[z^m] G_s(z, 1)}$. Since $\frac{\partial}{\partial u} G_s(z, u) = s G_{s-1}(z, u)$, the proposition is a direct consequence of (3). \square

3.2 Random Generation

The random generation is also based on Proposition 2; the general schema for the uniform random generation of an element of $\mathfrak{T}_{s,n,m}$ consists of two steps:

1. Generate uniformly an element \mathcal{A} of $\mathfrak{A}_{s,n}$.
2. Generate the output of the transitions of \mathcal{A} such that the sum of their sizes is m .

The first step can be performed in the general case using [HNS10]. For generating complete deterministic RDPDA – when $s = n\alpha$ – faster algorithms are described in [BDN07] and [CN12]. In the general case, the complexity is $O(n^3)$ and for the complete case, the complexity falls to $O(n^{\frac{3}{2}})$. The second step can be easily done using the classical recursive approach as described in [FZC94] or using Boltzmann samplers.

With a non-optimized Python implementation running on a 2.5GHz personal computer it is possible to generate 100 complete RDPDA with hundreds of states in few minutes.

4 Influence of the Accepting Condition

Accessibility defined for a RDPDA does not mean that the accessible states can be reached by a calculus. Therefore the random generation may produce semantically RDPDA simpler than wanted. One of the requirements may be to generate RDPDA accepting non empty languages. Another requirement is to produce only reachable states. Finally, if the final state-empty stack accepting condition is chosen, it is frequently required that final states are empty stack reachable.

4.1 Emptiness of accepted languages

Proposition 6 *Whatever the selected accepting condition, the probability that an accessible RDPDA with n states, s transitions and an output size of m , accepts a non empty language is greater or equal to $\frac{s-1}{2\beta(s+m-1)}$.*

PROOF. Since the considered RDPDA are accessible, there is at least one outgoing transition from the initial state. We will evaluate the probability that this transition is of the form $(q_{\text{init}}, a, Z_{\text{init}}, \varepsilon, p)$ with p final. There is no condition on a . The probability that the stack symbol is Z_{init} is $\frac{1}{\beta}$ since all letters have the same role. The probability that p is final is $\frac{1}{2}$ (see [CP05, BN07]). By Proposition 5 the probability that this transition is a pop transition is $\frac{s-1}{s+m-1}$. It follows that the probability that the transition has the claimed form is $\frac{s-1}{2\beta(s+m-1)}$. If this transition exists, the RDPDA accepts the word a , proving the proposition. \square \square

By Proposition 6, if $m = \lambda s$, for a fixed λ , then RDPDA accepting non-empty languages can be randomly generated by a rejection algorithm, with an expected constant number of rejects. Experiments show that most of the states are reachable (see Tables 3 and 4).

4.2 Empty-stack Reachability

Proposition 6 shows it is possible to generate complete RDPDA accepting a non-empty language (if m and s are of the same order). However, it doesn't suffice since many states of generated automata can be unreachable. Under the final-state and empty-stack condition of acceptance, a final state that is not reachable with an empty stack is a useless final state, i.e. it cannot be used as a final state to recognize a word – but it can be involved as any state for accepting a word. Using for instance [FWW97], one can decide in polynomial time whether a state is reachable with an empty stack.

Table 1 reports experiments on the average number of empty-stack reachable states. For this experiment, we consider complete and accessible RDPDA with $\alpha = 2$, and $\beta = 2$. Since a state is final with a probability $1/2$, dividing the number by 2 in the table provides the average number of final states reachable with an empty stack. Experiments show that if λ is greater than 1, then the average number of states reachable with an empty stack is quite small. Remind that λ is the

number of states \rightarrow	5	10	15	20	30	40	60	100
$\lambda = 0.5$	3.56	6.14	8.02	11.1	16.26	15	24.7	49.5
$\lambda = 1$	2.6	4.62	4.7	6.16	7.06	7.82	13.85	17.3
$\lambda = 1.5$	2.36	3.05	3.61	3.62	5.2	5.5	5.68	5.8
$\lambda = 2$	2.0	2.6	2.81	3.4	3.02	3.1	3.21	3.89
$\lambda = 3$	1.65	1.8	1.83	1.8	2.26	2.44	2.34	2.6
$\lambda = 5$	1.3	1.41	1.43	1.4	1.42	2.1	1.5	1.5

Table 1: Average number of reachable states with an empty stack, $\alpha = 2$, $\beta = 2$

number of states \rightarrow	5	10	15	20	30	40	60	100
$\lambda = 0.5$	4.05	7.75	11.54	16.09	23.63	30.87	46.83	80.62
$\lambda = 1$	3.09	5.25	8.19	10.78	13.97	22.55	31.23	44.52
$\lambda = 1.5$	2.94	4.44	5.91	8.24	9.06	11.93	18.77	29.58
$\lambda = 2$	2.68	4.44	5.19	6.5	8.53	9.39	12.84	19.91
$\lambda = 3$	2.53	3.29	4.38	4.81	6.37	7.49	8.72	12.18
$\lambda = 5$	2.29	3.31	3.81	4.53	5.09	4.71	5.28	6.42

Table 2: Average number of reachable states (at least 40% of pop transitions); $\alpha = \beta = 2$.

average size of the outputs. For each case, 100 complete RDPDA have been generated. Clearly the random generation of complete accessible RDPDA based on the sizes of the output will not produce enough pop-transitions to empty the stack. Adding a criterion on a minimal number k of pop-transitions may be a solution that can be achieved in the following way :

1. Choose uniformly k transitions of the underlying finite automata that will be pop-transitions.
2. Decorate the $s - k$ other transitions with strings for a total size of m .

The experiments reported in Table 1 has also been done for this procedure to compare the number of rejects. We choose the case $\alpha = \beta = 2$ again with at least 40% of pop-transitions. The results are reported in Table 2.

Imposing a minimal number of pop transitions improves the efficiency (relative to the number of reachable state) for small values of λ . However it is not sufficient when $\lambda \geq 1$.

4.3 Reachability (with no stack condition)

If we are now interested in the random generation with the *final state* condition, regardless of the stack, it is interesting to know the number of reachable states (which is on average twice the number of final reachable states). Using [FWW97], the average number of reachable states have been assessed experimentally. Results are reported in Table 3 for $\alpha = 2$ and $\beta = 2$ and in Table 4 for $\alpha = 3$ and $\beta = 5$.

For the random generation of complete accessible RDPDA with a final state accepting condition, our framework seems to be suitable: most of the states are reachable. For the two other accepting conditions, the value of λ has to be small. Otherwise, there will be too few pop transitions to clean

number of states \rightarrow	10	20	30	40	50	60	80	100
$\lambda = 1$	8.29	14.89	21.5	26.93	32.48	35.55	44.4	52.86
$\lambda = 2$	8.73	16.35	25.3	33.45	39.56	47.99	62.32	81.02
$\lambda = 3$	8.84	17.67	27.14	36.19	45.7	54.69	73.35	89.26
$\lambda = 5$	9.23	18.3	28.06	37.47	47.61	56.7	76.11	95.15

Table 3: Average number of reachable states, $\alpha = 2$ and $\beta = 2$.

number of states \rightarrow	10	20	30	40	50	60	80	100
$\lambda = 1$	9.72	19.48	29.34	39.71	49.16	59.49	79.6	99.4
$\lambda = 2$	9.9	19.7	29.9	39.8	49.4	59.6	79.7	99.5
$\lambda = 3$	9.93	19.9	29.92	39.9	49.81	59.9	79.6	99.1
$\lambda = 5$	9.95	19.96	29.93	39.9	49.86	59.89	79.79	99.75

Table 4: Average number of reachable states, $\alpha = 3$ and $\beta = 5$.

	number of states \rightarrow	10	20	30	40	50	60	80	100
$\alpha = 2$ $\beta = 2$	$\lambda = 1$	293.1	-	-	-	-	-	-	-
	$\lambda = 1.5$	24.6	88.8	278.0	-	-	-	-	-
	$\lambda = 2$	6.9	20.3	14.9	13.2	17.9	25.2	65.9	95.8
	$\lambda = 3$	1.6	1.5	1.8	2.0	1.9	2.2	2.1	2.1
	$\lambda = 5$	1.1	1.2	1.1	1.2	1.1	1.2	1.2	1.2
$\alpha = 4$ $\beta = 2$	$\lambda = 1$	3.8	5.5	9.4	15.7	38.4	39.3	76.9	76.9
	$\lambda = 1.5$	1.7	2.0	1.7	1.8	1.9	2.0	2.0	1.9
	$\lambda = 2$	1.3	1.3	1.31	1.3	1.2	1.2	1.1	1.2
	$\lambda = 3$	1.1	1.1	1.1	1.1	1.0	1.1	1.1	1.1
	$\lambda = 5$	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
$\alpha = 2$ $\beta = 4$	$\lambda = 1$	2.3	5.5	16.9	23.8	20.1	44.1	80.4	214.2
	$\lambda = 1.5$	1.7	1.8	1.2	2.6	3.0	2.0	1.6	1.9
	$\lambda = 2$	1.4	1.2	1.2	1.1	1.3	1.4	1.3	1.1
	$\lambda = 3$	1.0	1.2	1.2	1.0	1.1	1.1	1.0	1.2
	$\lambda = 5$	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Table 5: Average number of random generations to obtain a reachable RDPDA.

out the stack. Proposition 5 confirms this outcome since averagely the number of pop transitions is close to $\frac{\alpha\beta n}{\lambda+1}$.

4.4 A Rejection Algorithm for Reachable Complete RDPDA

A natural question is to consider how to generate a complete accessible RDPDA with exactly n reachable states. An easy way would be to use a rejection approach by generating a complete accessible RDPDA with n states until obtaining a reachable RDPDA. Results presented in Tables 3 and 4 seems to prove that this approach might be fruitful for the parameters of Table 4 but more difficult for the parameters of Table 3. Several experiments have been performed to evaluate the average number of rejects and results are reported in Table 5: the average number random generations of RDPDA used to produce 10 reachable RDPDA was reported. When a “-” is reported in the table, it means that after 300 rejects, no such automata was obtained. These results seem to show that the rejection approach is tractable if $\lambda \geq 2$ and if the alphabets are not too small. With $\alpha = \beta = 2$, it works for $\lambda \geq 3$ and for smaller λ 's when the number of states is small.

5 Conclusion

In this paper a general framework for generating accessible deterministic pushdown automata is proposed. We also experimentally showed that with some accepting conditions, it is possible to generate pushdown automata where most states are reachable. The results on the random generation are synthesized in Table 6. In a future work we plan to investigate how to randomly generate real-time deterministic automata, with an empty-stack accepting condition and again, which most states are reachable. We also plan to remove the *real-time* assumption, but it requires a deeper work on the underlying automata.

Accepting Condition	
Empty Stack	<ul style="list-style-type: none"> • General frameworks does not work: the number of states reachable with an empty-stack is too small. • Fixing a minimal number of pop transitions (see Section 4.2) works for $\lambda < 1$.
Final States	<ul style="list-style-type: none"> • General framework works: a significant number of states are reachable. • A rejection approach is tractable for generating reachable RDPDA, when both $\lambda \geq 1.5$ and the alphabets are large enough.

Table 6: Random Generation of RDPDA.

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