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Dynamic and Acoustic Modeling of Capacitive Micromachined Ultrasonic Transducers

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***Abstract*—The Capacitive Micromachined Ultrasonic Transducers (CMUTs) are a promising alternative to piezoelectric ultrasound transducers. They are constituted by a very large number of silicon membranes electrostatically actuated. We propose an original method to calculate the acoustic pressure emitted by CMUTs by taking into account explicitly the dynamics of each membrane of the network. The electrostatic forces are linearized, the displacements of the membranes are projected on a mechanical modal base and the acoustic pressures are computed via the Rayleigh integral. The method has been validated by comparison with ANSYS for a single cell and a small hexagonal network of 7 cells. Application on a 2D CMUT network shows the interest of such a calculation method.**

I. Introduction

The numerical simulation of the radiated pressure of a CMUT Network is conducted by approximate methods, due to the huge number of membranes that have to be taken into account. Simple CMUT design tools, used piston like analytic solutions and simplified coupling hypothesis, as described in [5]. Commercial Finite Element Software are also currently used to study the radiation of a single membrane. Some authors [2], [3] have used ANSYS to study cross talk effects on models limited to a few number of membranes. The use of periodic conditions as in [1] is exact only for infinite networks. Other models take acoustic coupling into account with various simplification hypothesis [7], [6].

We present here a method where all the membranes are taken into account explicitly in the simulation. Computation time is reasonable, because acoustic couplings are calculated by the Rayleigh integral and membrane displacements are projected on mechanical mode shapes. We present first the model, then its validation by comparison with ANSYS simulations and finally the application of the method to a 2D CMUT network.

II. Model

A CMUT is composed of many cells organized as a network. Each cell comprises a small membrane over a sealed vacuum cavity. Electrodes are deposited at the bottom of the cavity and on the membrane to permit electrostatic actuation. Electrical connections allows to apply the same electrical tension to a group of cells called element. We suppose that the membranes are coupled by the semi infinite acoustic medium but not by the mechanical substrate. The objective of our method is to predict first the displacement of all the membranes of the network. Then radiated pressures, directivity diagrams and acoustic power can be deduced from the displacement of all the network membranes.

A. Cell electro-mechanical model

Each membrane, is statically deflected by bias voltage U_{dc}. We suppose that the alternating voltage U_{ac} is small compared to U_{dc}. In this case the alternating electrostatic forces F_{dyn} can be linearized around the static deflection of the membrane w_{dc}. We thus can write :

$$F_{dyn} = \varepsilon_0 U_{dc}^2 \int_S \frac{w}{(h_{gap} - w_{dc})^3} dS \quad (1)$$

$$+ \varepsilon_0 U_{dc} U_{ac} \int_S \frac{1}{(h_{gap} - w_{dc})^2} dS \quad (2)$$

where S is the surface of the electrode, h_{gap} the initial electric effective gap and w the dynamic membrane deflection.

B. Acoustic model

The CMUT if flat and each membrane can be considered as baffled. Thus the pressure $P(\mathbf{r}, \omega)$, in the frequency domain, radiated at the point M of coordinates $\mathbf{r}$, by the membrane of surface S_m, can be expressed by the Rayleigh integral :

$$P(\mathbf{r}, \omega) = -\frac{\omega^2 \rho_0}{2\pi} \int_{s_m} \frac{W(\mathbf{r}', \omega) e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} dS' \quad (3)$$

where $W(\mathbf{r}', \omega)$ is the harmonic deflection of the membrane current point with $w(\mathbf{r}',t) = Re[W(\mathbf{r}',\omega)e^{j\omega t}]$, ρ_0 is the mass per unit volume of the fluid and $k = \frac{\omega}{c_0}$ is the wave number with c_0 the speed of sound in the fluid.

The Rayleigh integral will be use in a first step to compute the acoustic direct and mutual impedances on the membranes. Then in a second step, it will be used to compute the pressure radiated by the network.

C. Cell mechanical model

The dynamic membrane deflection w can be projected on the first M modes shapes φ_k of the membrane in vacuum and clamped on its edge [4].

$$w = \sum_{k=1}^{M} \varphi_k q_k$$

where q_k is the generalized coordinate associated with mode number k. We will further consider an harmonic domain approach where

$$q_k(t) = Re[Q_k(\omega)e^{j\omega t}]$$

In addition, for each mode k, we will introduce a mechanical modal viscous damping.

D. Network model

If we use M mode shapes to represent the displacements of the membrane number i, for $i = 1, N$, the unknown vector for the membrane i $is \{Q^i\}^T = \{Q_1^i, ..., Q_M^i\}$ and the unknown vector for the entire network is $\{Q\}^T = \left\{\{Q^1\}^T, ..., \{Q^N\}^T\right\}$. The momentum equation for the unknown $\{Q\}$ is obtained after projection of the forces on the modal vectors φ_k^i, for $i = 1, N$ and $k = 1, M$. It leads to the following linear system of size $N \times M$ to be solved in the frequency domain :

$$(-\omega^2[M]+j\omega[C]+[K]-[K_{elec}]+\frac{\omega^2\rho_0}{2\pi}[A(\omega)])\{Q\} = \{F_{elec}\}$$

Where :

- $[M], [C], [K]$ are the diagonal structural mass, damping and stiffness matrices
- $[K_{elec}]$is the linearized electrostatic softening matrix evaluated from (1)
- $\{F_{elec}\}$is the linearized electrostatic forces vector evaluated from (2)
- $[A(\omega)]$ is the acoustic coupling matrix evaluated from (3)

Most of the physics is thus included in the model for small amplitude responses. Only two parameters have to be chosen; the number of modes and the radius out of which acoustic interactions between membranes are neglected. The adequate choice depends on the physics of the problem. A trade off has to be made between accuracy and computational cost. The method has been implemented in an in house software called OptiMUT.

III. VALIDATION

We consider a generic circular membrane that we will use through all the simulations described in this paper. It is a poly silicon membrane of 1,5 μm thickness and 25 μm radius. The electrode radius is 20 μm and the air gap height is $0,3\ \mu m$. Collapse voltage has been calculated to 106 V and the bias voltage U_{dc} is equal to 95 V (90% of the collapse voltage). Alternating voltage U_{ac} is equal to 1 V. Water is the acoustic radiation medium. A modal viscous damping ratio of 0.5% has been taken for all the mechanical modes. The first ten eigen frequencies of this membrane in vacuum are given on table I

Table I
FIRST TEN EIGEN FREQUENCIES OF THE MEMBRANE IN VACUUM

Frequency MHz	Multiplicity	Nb. of nodal diameters	Nb. of nodal circles
9.9	1	0	0
20.7	2	1	0
33.9	2	2	0
38.7	1	0	1
49.6	2	3	0
59.2	2	1	1
67.8	2	4	0
82.3	2	2	1
86.7	1	0	2
88.3	2	5	0

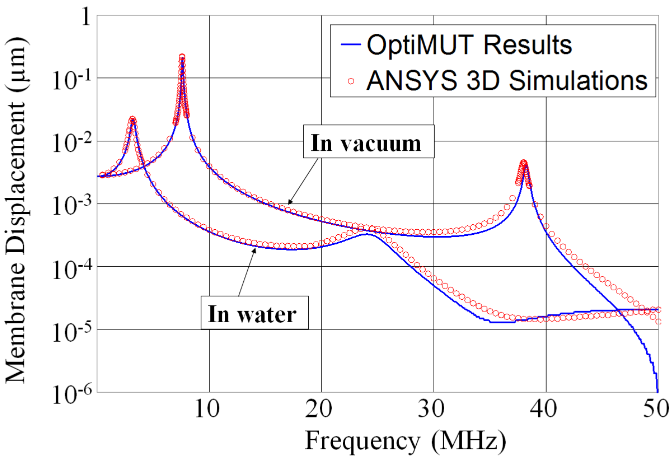

Figure 1. Single membrane center displacement

A. Single cell model

The radiation of a single membrane in vacuum and in water has been simulated with ANSYS and OptiMUT. The first 6 modes have been considered in the OptiMUT model. Displacement vs frequency at the center of the membrane is represented on figure 1. We can see that the solutions with ANSYS and OptiMUT are very close. A slight difference observed on the second peak in vacuum is due to the mechanical damping that is defined differently with ANSYS. In vacuum we can see the effect of the electrostatic softening on the first two modes, with zero nodal diameters, that are the only modes excited here. The first mode shifts from 9.9 MHz to 7.6 MHz and the second shifts from 38.7 MHz to 38 MHz. In water, we can notice that the frequency shift and the strong damping effect are identically computed with both models.

B. Hexagonal cell model

This test inspired from [3] is done to validate cross talk effects. A central membrane is surrounded by 6 identical membranes disposed according to an hexagonal pattern. The minimum distance between two adjacent membranes is $5\ \mu m$. Only the central membrane is electrically excited by an alternating voltage of 1V. The outer membranes are not electrically excited. Seventeen modes have been considered for each membrane in the OptiMUT simulation. The ANSYS meshing,

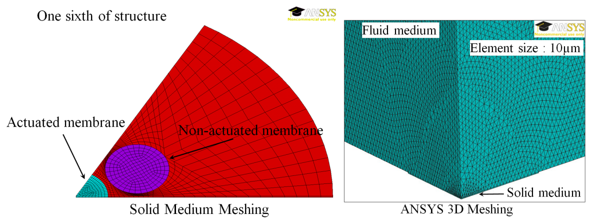

Figure 2. ANSYS meshes

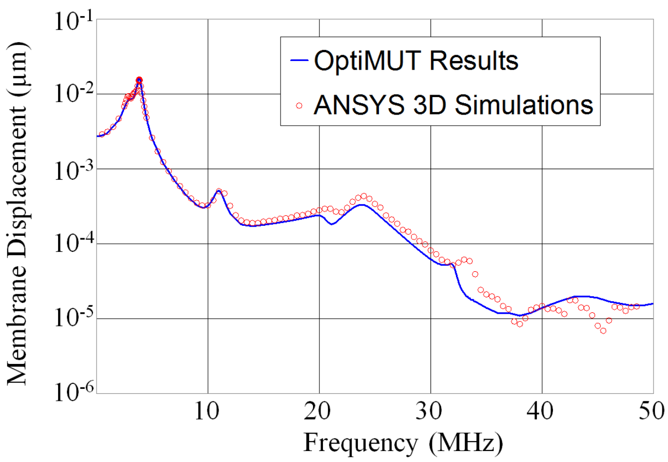

Figure 3. Displacement at the center of the central membrane

of one sixth of the structure, and of the fluid is represented Fig. 2.

The displacement at the center of the central cell is plotted on Fig.3. A good agreement is found between ANSYS and OptiMUT from 0 to 30 MHz. Between 30 MHz and 50 MHz, the ANSYS meshing is too coarse relative to the wavelength. Indeed, beyond 30 MHz each wavelength includes less than 5 fluid elements. On Fig. 3 we can distinguish clearly 5 different frequency peaks at 3.86 MHz, 11 MHz, 19.9 MHz, 23.9 MHz and 32.8 MHz. These frequencies correspond to strong coupling between the different modes of the central and the surrounding cells. This is illustrated by the instantaneous displacements represented at these frequencies on Fig.4. The plot of the pressure at 1mm in front of the central cell in Fig.5 shows again a good agreement between ANSYS and OptiMUT up to 30 MHz.

IV. Application

This section is devoted to show the potential of the method to study larger networks. We choose an academic 2D Network composed of 64 elements of 16 cells each. Six mode shapes are considered for each membrane. The interaction radius for acoustic coupling is 500 μm. This means roughly that each cell is coupled with its eight close neighbors in each direction. The alternating voltage is applied with a phase of 10 degrees between adjacent elements in the x and y directions. The displacement at the center of a cell in the middle of the CMUT is represented on Fig.6. We can see a series of peaks and in particular a very sharp one at 11.84 MHz. This

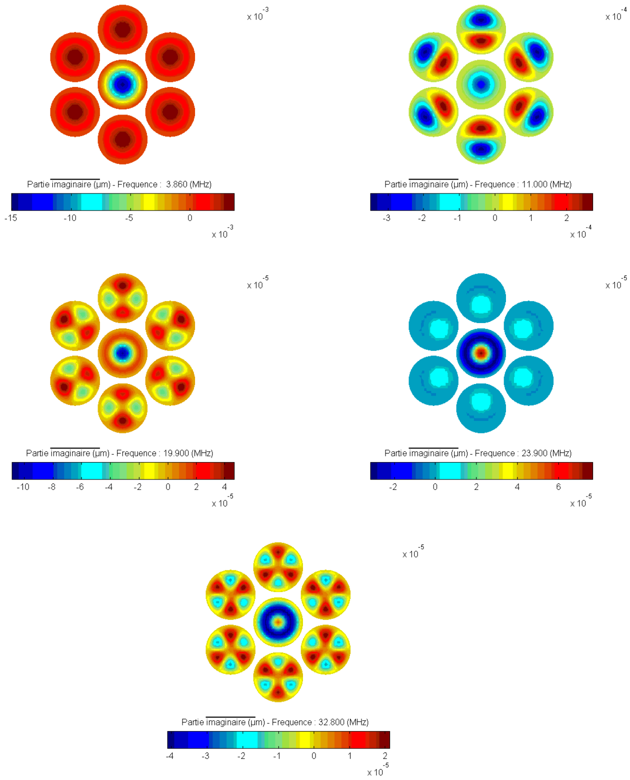

Figure 4. Instantaneous displacement of the membranes at coupling frequencies

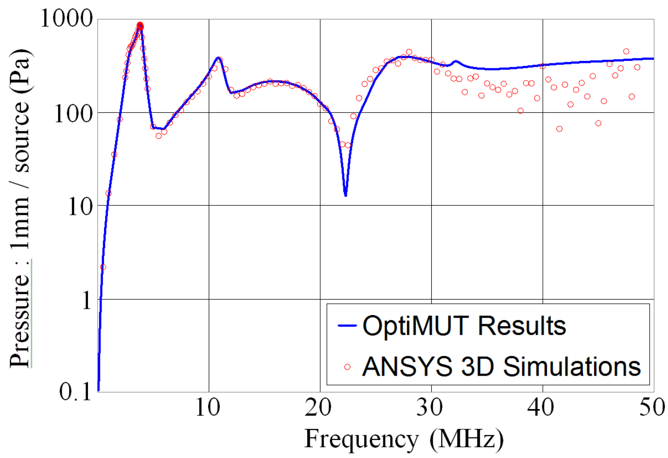

Figure 5. Pressure at 1mm in front of the central membrane

peak correspond to a global displacement involving mostly the second mode of the cells. An instantaneous displacement field at the center of each cell, for 3.18 MHz, is represented on Fig.7. We can see large differences on the amplitudes of different cells belonging to the same element. Also at that frequency, we can notice that the elements are not responding according to the phase imposed on the excitation. Finally the far field pressure in front of the CMUT is shown on Fig. 8. Those results shows the importance of cross talk in such

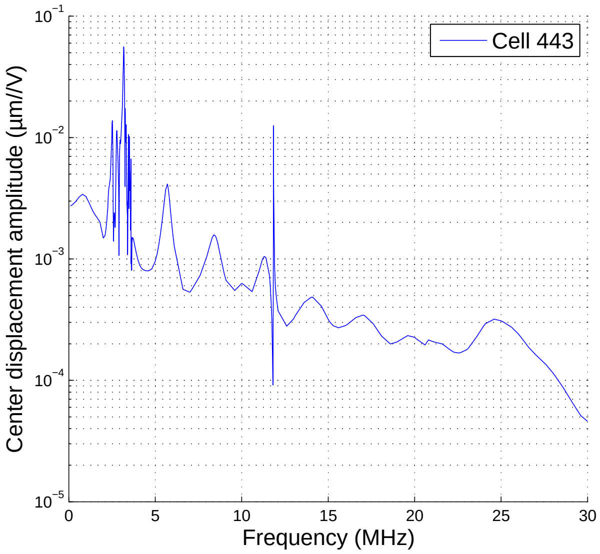

Figure 6. Displacement at the center of a cell in the middle of the CMUT

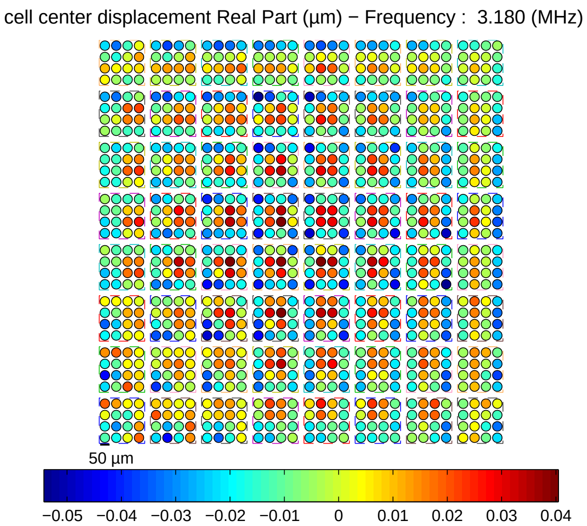

Figure 7. Instantaneous displacement at the center of each cell for 3.18 MHz

a network. Validation of those results with Finite Element commercial software is not possible due to the size of the model. Experimental validation needs to be done in the next future.

V. Conclusion

We propose a method that can predict the behavior of CMUT operating in linear regime, taking into account cross talk and edge effects. The model represent explicitly all the membranes. Most of the physics is taken into account. The only approximations are the linearisation of the electrostatic forces, the modal truncation and a radius above which acoustic

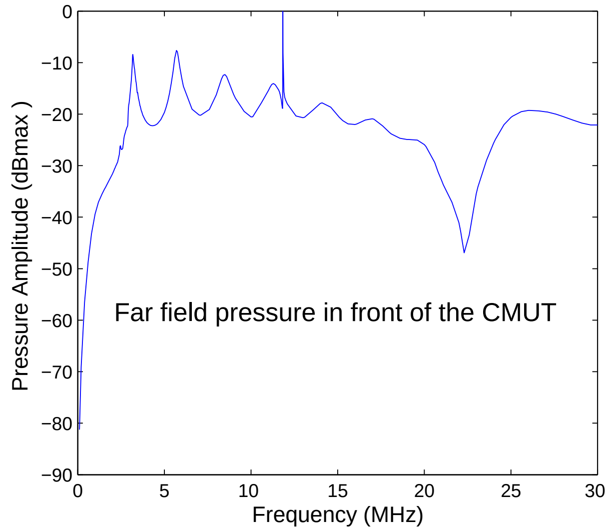

Figure 8. Far field pressure in front of the CMUT

coupling is neglected. The model has been validated by comparison with ANSYS for small networks. Application to a larger 2D network shows very strong coupling effects. Further investigations will be necessary to understand all the phenomena in order to optimize the performance of future networks. Experimental validation of the results is necessary and will be done in a near future.

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