Photonic nonlinear delay dynamics for advanced information processing: from secure chaos communications to brain-inspired computing

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Abstract—Photonics is a priviledged playground for the investigation of the dynamical complexity exhibited by delay equations. In this particular field of Physics, these infinite dimensional dynamics become tractable, controllable, and can be experimentally explored under a wide set of parameter settings. Beyond the fundamental and theoretical interest aimed at the understanding of their numerous dynamical mechanisms, photonic even provides opportunities to exploit this dynamical complexity for advanced information processing principles. In this contribution, we will review two such applications implemented in photonic, optical chaos communication, and Reservoir Computing.

1. Introduction

High speed photonic devices and systems have been strongly pushed by the development of optical telecommunications. Broadband electro-optic modulators and detectors have been successfully developed, as well as advanced modulation format involving phase or combined phase and amplitude modulations. Instead of being used in a point-to-point optoelectronic link where the information is forwarded or transmitted in a single way, one can imagine a feedback architecture of such an optoelectronic link, where the output of a photodiode receiver is amplified and used to modulate again the light at the emitter side. Such a simple architecture is actually the basic concept of a delayed feedback loop, in which the delay time is time of flight of the light from the emitter to the receiver and back to the emitter. Nonlinear transformation of the signal is also involved in such a delayed feedback loop oscillator, simply through the interference phenomena typically used to convert the electro-optic phase modulation into an intensity modulation; provided the amplitude of the optical phase modulation is large enough compared to π, the strength of the nonlinear effect can be considered as significant.

Following this concept of an electro-optic and optoelectronic nonlinear delayed feedback loop, high-dimensional nonlinear dynamical system becomes at disposal, moreover with high stability, high controlability, high structural flexibility, and of course high speed capability limited by electronic telecom devices (up to a few tens of GHz) only. Such a nonlinear delayed feedback dynamics can be modeled in
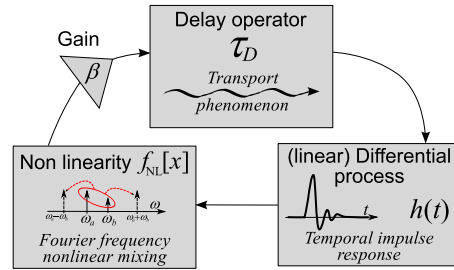

Figure 1: Bloc diagram of a nonlinear delayed dynamics.

different ways. In the Fourier domain, the definition of the Fourier filtering imposed by the electronic feedback reads as an input to output linear filter:

$$X(\omega) = H(\omega) \cdot Z(\omega) \quad (1)$$

where $Z(\omega) = \text{FT}\{f_{\text{NL}}[x(t - \tau_D)]\} = e^{-i\omega\tau_D} \cdot \text{FT}\{f_{\text{NL}}[x(t)]\}$ is the Fourier transform (FT) of the nonlinear delayed feedback signal, and $H(\omega)$ is the linear Fourier filtering transfer function of the electronic feedback. Considering a bandpass type electronic filtering delimiting the actual feedback bandwidth of the oscillator, the simplest model for is $H(\omega) = i\omega\theta/[(1 + i\omega\theta)(1 + i\omega\tau)]$, where τ and θ are the characteristic time scales determining the high and low cutoff frequencies $f_h = (2\pi\tau)^{-1}$ and $f_l = (2\pi\theta)^{-1}$ respectively. Under this simplifying asumption, an integro-differential delay equation can be deduced as one converts the Fourier domain description into the time domain one, leading to:

$$\frac{1}{\theta}\int_{t_0}^{t} x(\xi)\,\text{d}\xi + x(t) + \tau\frac{\text{d}x}{\text{d}t}(t) = z(t) = f_{\text{NL}}[x(t - \tau_D)], \quad (2)$$

where $\text{FT}[x(t)] = X(\omega)$ and $\text{FT}[z(t)] = Z(\omega)$. A third possible formulation of the dynamics, rarely used in the literature, can however bring useful insights for the understanding or illustration of the space-time analogies occurring when temporal information is processed by a delayed feedback dynamics:

$$x(t) = \int_{-\infty}^{t} h(t - \xi) \cdot f_{\text{NL}}[x(\xi - \tau_D)]\,\text{d}\xi, \quad (3)$$

where $h(t)$ is the impulse response of the linear filter, defined as the inverse Fourier transform of $H(\omega)$.

In the following, we will use the latter formulation of a nonlinear delay dynamics to describe theoretically two particular applications, namely optical chaos communication [1], and photonic Reservoir Computing [2].

2. Optical chaos communications

The emitter-receiver architecture for a chaotic optical secure link is based on the so-called open loop receiver scheme, also referred sometimes as to chaos modulation. The principle of operation consists of a closed loop emitter ruled by an equation of any of the forms (2) to (3) when operating in an autonomous configuration, and which parameters are set so that it generates a chaotic waveform. When used in an emitter-receiver chaos communication scheme, the chaotic oscillation is actually permanently perturbed by the information to be encoded, usually through a simple in-loop addition (see Fig.2). Under practical situation, the amplitude of the information can be as high as that of the free running chaotic oscillation, at the point where the signals are added. If the latter point is before the nonlinear transformation, a simple convolution product description of the encdoded signal generated at the emitter simply reads as follows:

$$e(t) = m(t) + \int_{-\infty}^{t} h(t - \xi) \cdot f_{NL}[e(\xi - \tau_D)] \, d\xi. \quad (4)$$

As suggested by the spelling of the encoded signal of concern here, $e(t)$, it has significantly different origines compared to the autonomous solution $x(t)$ described in the introduction. Rigorously speaking, $e(t)$ can not be considered as a chaotic signal, it is even partly non-deterministic depending on the properties of the message signal $m(t)$. If $m(t)$ is a real information signal, it is not predictable, and its role in the emitter dynamics is a strong perturbation of the original chaotic solution $x(t)$, thus necessarily destroying the phase space attractor observed in the autonomous operation. A more realistic view of this transmitted signal would thus be a complex recurrent nonlinear delayed transformation of itself. The nonlinear self delayed feedback used to generate a chaotic waveform, is transformed, for chaos communication, into a complex function (i.e. difficult to reversely transform without the knowledge of the nonlinear delayed feedback rule itself) of the signal to be transmitted and encoded.

At the receiver side, the authorized decoder explicitly makes use of the knowledge of this complex function, mathematically expressed by the convolution product:

$$r(t) = \int_{-\infty}^{t} h^R(t - \xi) \cdot f_{NL}^R[e(\xi - \tau_D^R)] \, d\xi, \quad (5)$$

where the superscript R underlines function and parameters that need to be precisely known by the authorized receiver

in order to achieve a real time recovery of the original information $m(t)$. This knowledge forms the key of the chaos encryption procedure. It is indeed obvious from Eqs. (4) and (5) that if we assume $(h^R, f_{NL}^R, \tau_D^R) \equiv (h, f_{NL}, \tau_D)$, one has then $m(t) = e(t) - r(t)$: the decoded information simply corresponds to the received signal $e(t)$, from which one would have to properly subtract $r(t)$, i.e. the locally *replicated* filtering and nonlinear delayed transformation of the same received signal $e(t)$. The receiver appears thus as a feedforward processing without any closed loop, which fact motivates its denomination as a replication receiver (open loop) better than a synchronization receiver (closed loop, or at least partially closed loop).
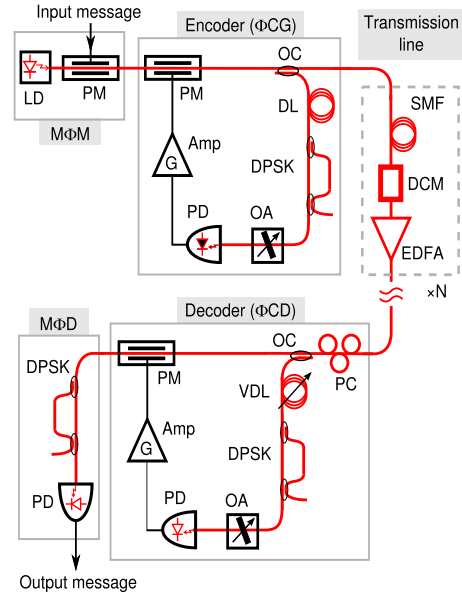

Figure 2: Experimental setup demonstrating optical phase chaos communication up to 10 Gb/s.

As long as the receiver parameters and functions can be accurately matched to the emitter ones, and controlled again possible drifts, continuous and unconditionally stable decoding can be achieved at a bit bit which is limited by the bandwidth of the filter, e.g. $f_h - f_l \simeq f_h$. This approach is indeed the one which led to successful demonstrations on field experiments, moreover with state of the art performances. Figure 2 shows the schematic of the electro-optic phase setup used to achieve the latest best performances in optical chaos communications [3] at 10 Gb/s over more than 100 km of an installed commercial fiber optic link.

The previous example exploits the dynamical complexity provided by a nonlinear delayed optoelectronic feedback loop, to perform a recurrent nonlinear processing on an information signal. The latter signal is thus somehow expanded in a continuously transient fashion into the infinite dimensional phase space of the delay dynamics. A similar expansion was also used more recently to develop another application based on a recurrent delayed feedback

loop, Reservoir Computing (RC), also known originally in the literature as Echo State Network [4] and Liquid State Machine [5].

3. Photonic Reservoir Computing

The concept of RC is derived from recurrent neural network (RNN) computing approaches, however simplifying extremely the learning phase of the computational steps. The latter indeed represents traditionally a very critical issue in standard RNN, because the optimal set of coupling parameters is very difficult to determine by a learning procedure, particularly when they concern many sets of such connectivity strength, the ones of the input and output layer, and the ones of the internal connectivity defining the network structure itself. RC considers that the output connectivity, also called the read-out or output layer, needs to be learnt only, the two other sets of connectiviy coefficients not needing optimization, and being chosen simply at random for example. Such a simplification transforms the learning phase into a very simple, very efficient, very fast, and always converging solution. Beyond this surprising simplification, RC has moreover shown surprising computation accuracy, with comparable results, and sometimes even better ones, compared to traditional neural network computing [7].

More recently, RC has reached another important step forward through its successful hardware demonstration [8], moreover with an initially unexpected structural solution for the so-called Reservoir: the usual network of interconnected nodes was physically realized through the internal complexity of a delay dynamical system. Delay dynamics are indeed known to present qualitative similarities with spatio-temporal dynamics such as a network of dynamical nodes, which fact has indeed motivated its choice of delay dynamics as a way to emulate a neural network. In the present section, we will again take the opportunity of the unusual modeling of delay dynamics through Eq.(3), in order to analytically derive a rigorous correspondence between a delay dynamics seeded by a time division multiplexed input information, and a network of interconnected nodes excited by an input information through the usual input layer.

A key concept in the use of a delay system to emulate a neural network, is to consider the dynamical nodes of the network as being temporal positions within the time interval corresponding to the delay. One needs then to re-define the time variable t, so that it can reflect the emulation of a virtual spatial position $\sigma \in [0, \tau_D]$, which is updated in time each round trip of the signal in the delayed feedbak loop, i.e. each time delay τ_D. Such an approach indeed reveals the intrinsic multiple time scale feature of a delay dynamics, the fast time scale τ related to the high cut-off frequency f_h, and the slow time related to the delay τ_D: $t = \sigma + n \cdot \tau_D$ (the role of the even slower integral time scale θ is omitted here, partly because it is not essential to

RC processing since the first delay-based RC demos were involving a high cut-off only, without low cut-off).

If one then assumes that the virtual nodes correspond to sampled positions $\sigma_k = k\,\delta\tau$, the number of virtual nodes in the delay dynamics amounts to $K = \tau_D/\delta\tau$. Addressing each of these nodes with an input vector $\mathbf{u}(n) \in \mathbb{R}^Q$ is achieved, as already stated, through a standard time division multiplexing technique. Distributing "randomly" each vector component of $\mathbf{u}(n)$ onto each of the K virtual nodes of the delay dynamics, is an operation typically performed according to a so-called input connectivity matrix $\mathbf{W}^I = [w_{kq}^I] \in \mathbb{R}^K \times \mathbb{R}^Q$. The resulting scalar signal obtained from this time division multiplexed distribution of the input vector reads as:

$$u_\sigma^I(n) = \sum_{k=1}^{K} \left[\sum_{q=1}^{Q} w_{kq}^I u_q(n) \right] p_{\delta\tau}(\sigma - \sigma_k), \qquad (6)$$

where $p_{\delta\tau}(t)$ is a rectangular temporal window having a unit amplitude over $[0; \delta\tau]$, and being zero everywhere else.

From Eq.(3), one can arrange the integration interval for the convolution product so that the node amplitude $x_k \equiv x_{\sigma_k}$ at time n can be expressed as an update of the amplitude of the same node, but at time $(n-1)$, i.e. $x_k(n-1)$. Taking also into account that the dynamics is seeded by the information to be processed according to the connectivity rule expressed in (6), one obtains:

$$x_k(n) = x_k(n-1) + \int_{\sigma_k - \tau_D}^{\sigma_k} h(\sigma - \sigma_k) \times$$
$$f_{NL}\left[x_\sigma(n-1) + \rho \cdot u_\sigma^I(n-1) \right] d\sigma. \qquad (7)$$

The latter expression reveals in a rigorous way the analogy of delay-based RC with the original Echo State Network approach as proposed in [4, 7].

The output layer consists also in a matrix multiplication, corresponding physically to a circular convolution operated on the response signal $x_\sigma(n)$ and involving the Read-Out matrix $\mathbf{W}^R = [w_{mk}^R] \in \mathbb{R}^M \times \mathbb{R}^K$. The computed output is a vector $\mathbf{y}(n) \in \mathbb{R}^M$, which is the expected calculation result obtained from the input information $\mathbf{u}(n)$:

$$y_m(n) = \sum_{k=1}^{K} w_{mk}^R x_k(n). \qquad (8)$$

The Read-Out matrix $\mathbf{W}^R$ is practically the solution of a ridge regression problem minimizing the error of the output vector considering a set of known pairs of answer / response $(\mathbf{u}(n); \tilde{\mathbf{y}}(n))$. This ridge regression step precisely corresponds to the learning phase of such a delay-based RC.

This delay-based RC concept was practically implemented recently by different authors, with very successful computational performances. A classical speech recognition problem was for example performed experimentally [2, 8], with record word error rate (WER) down to 0% for a
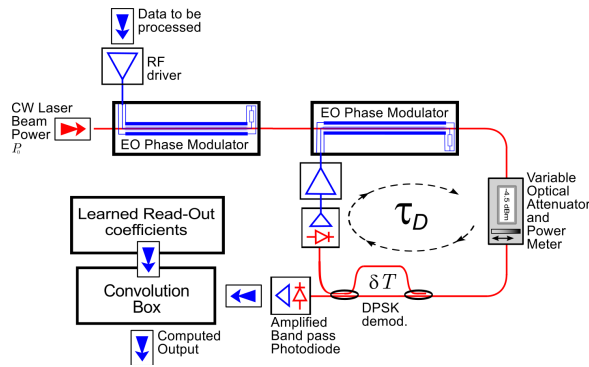
Figure 3: Electro-optic phase delayed dynamics for ultra-fast photonic Reservoir Computing demonstration.

clean spoken digit database, thus achieving state of the art performances.

With a very similar setup (see Fig.3) compared to the one reported previously for optical chaos communication, a record speech recognition speed up to 1 million words per second was demonstrated, with a very small WER degradation only [6].

4. Conclusion

We reported a signal theory approach for the theoretical description of two unusual applications involving complex nonlinear dynamics, namely optical chaos communications and photonic neuromorphic computing. In both cases, a convolution product description of the dynamics allowed on the first example to explain how chaos replication and decoding can be successfully operated at the receiver, and on the second example we could analytically derive a close correspondance of delay based RC processing with the original idea of Echo State Network. In the latter example, space-time analogy of purely temporal delay dynamics could have been also highlighted, thus justifying the emulation capability of such a particular class of infinite dimensional dynamics to emulate a virtual newtrok of coupled dynamical nodes. It is also remarkable to notice that a similar analogy recently revealed the existence of chimera states, a recently discovered self-organizationin networks of identical coupled oscillators, in nonlinear delay dynamics [9]. The same convolution product description allowed in the latter case to also reveal a clear analogy between delay-based chimera states, and the traditional network of coupled Kuramoto oscillators [10].

Acknowledgments

References

[1] A. Argyris *et al.*, "Chaos–based communications at high bit rates using commercial fiber–optic links", *Nature*, Vol. 438, pp. 343–346, 2005.

[2] L. Larger *et al.*, "Photonic information processing beyond Turing: an optoelectronic implementation of reservoir computing," *Optics Express*, Vol. 20, p. 3241, 2012.

[3] R. Lavrov, M. Jacquot, L. Larger, "Nonlocal nonlinear electro–optic phase dynamics demonstrating 10 Gb/s chaos communications," *IEEE J. Quant. Electron.* Vol.46, No.10, pp. 1430–1435, 2010.

[4] H. Jaeger, "The "echo state" approach to analysing and training recurrent neural networks,", *GMD Report* Vol. 148, 2001.

[5] W. Maass, T. Natschläger and H. Markram, "Real–Time Computing Without Stable States: A New Framework for Neural Computation Based on Perturbations," *Neural Computation* Vol. 14, pp. 2531–2560, 2002.

[6] L. Larger *et al.*, "Delayed electro-optic phase dynamics for ultra-fast Nonlinear Transient Computing," *Proceedings of NOLTA'14 symposium*, Luzern, Switzerland, p.553, Sept. 14-18, 2014.

[7] H. Jaeger and H. Haas, "Harnessing nonlinearity: predicting chaotic systems and saving energy in wireless communication,", *Science*, Vol.304, pp. 78–80, 2004.

[8] L. Appeltant *et al.*, "Information processing using a single dynamical node as complex system," *Nature Communications* Vol.2, No. 468, pp. 1-6, 2011.

[9] L. Larger, B. Penkovsky and Y. Maistrenko, "Virtual Chimera States for Delayed-Feedback Systems," *Physical Review Letters* Vol.111, 054103, 2013.

[10] L. Larger, B. Penkovsky and Y. Maistrenko, "Laser chimeras as a paradigm for multi-stable patterns in complex systems," *Nature Communications* accepted, June 2015.