Modal parameter identification of perforated microplates from output data only

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ABSTRACT

In several applications the design of MEMS includes perforated microplates supported by an elastic suspension which can oscillate. To control such oscillations it is necessary to know the modal parameters of the microplate such as eigenfrequencies, damping and stiffness. The purpose of this communication is to identify the vibratory parameters of a perforated microplate from the measure of the microplate displacement only. Dynamic measurements of a perforated microplate are conducted in time domain and a comparison between experimental and analytical results is presented.

Key-Words
MEMS, microplate, oscillating system, time domain, modal parameters, identification, subspace method

1. INTRODUCTION

The design of MEMS includes oscillating elements and components which are often perforated microplates supported by an elastic suspension. The main purpose of perforations is to reduce the damping and spring forces acting in the MEMS due to the fluid flow inside and around the micro structure. The study of the damping caused by surrounding fluid and by the dissipations in the material is very important to predict the dynamic response of the microsystem and to estimate some important parameters such as the quality factor. G. De Pasquale and T. Veijola [1] used numerical strategies for the estimation of the damping force acting on a perforate movable MEMS, using FEM methods with ANSYS. It was shown that ANSYS results contained a systematic error at small perforations and were not usable for large perforations. Our purpose is to identify the modal parameters of a perforated microplate : the eigenfrequency, the damping ratio and the stiffness from the displacement response only of the microstructure.

2. THE SUBSPACE IDENTIFICATION METHOD

The subspace identification method assumes that the dynamic behaviour of a vibrating system can be described by a discrete time state space model [2,3]

$$\boldsymbol{z}_{k+1} = \boldsymbol{A}\, \boldsymbol{z}_k + \boldsymbol{w}_k \qquad \text{state equation} \tag{1}$$

$$\boldsymbol{y}_k = \boldsymbol{C}\, \boldsymbol{z}_k + \boldsymbol{v}_k \qquad \text{observation equation} \tag{2}$$

where $\boldsymbol{z}_k$ is the unobserved state vector of dimension n; $\boldsymbol{y}_k$ is the (mx1) vector of observations or measured output vector at discrete time instant k; $\boldsymbol{w}_k$ contains the external non measured force or the excitation which can be a random force, an impulse force, a step force...and $\boldsymbol{v}_k$ is a measurement noise. $\boldsymbol{A}$ is the (nxn) transition matrix describing the dynamics of the system and $\boldsymbol{C}$ is the (mxn) output or observation matrix. The subspace identification problem deals with the determination of the two state space matrices $\boldsymbol{A}$ and $\boldsymbol{C}$ using output-only data $\boldsymbol{y}_k$. The modal parameters of a vibrating system are obtained by applying the eigenvalue decomposition of the transition matrix $\boldsymbol{A}$ [2,3]

$$\boldsymbol{A} = \boldsymbol{\Psi}\ \boldsymbol{\Lambda}\ \boldsymbol{\Psi}^{-1} \tag{3}$$

where $\Lambda = diag(\lambda_i)$, i=1,2,…,n, is the diagonal matrix containing the complex eigenvalues and $\boldsymbol{\Psi}$ contains the eigenvectors of $\boldsymbol{A}$ as columns. The eigenfrequencies F_i and damping factors ξ_i are obtained from the eigenvalues which are complex conjugate pair

$$F_i = \frac{1}{2\pi\,\Delta t}\sqrt{\frac{[\ln(\lambda_i\,\lambda_i^*)]^2}{4} + [\cos^{-1}(\frac{\lambda_i + \lambda_i^*}{2\sqrt{\lambda_i\,\lambda_i^*}})]^2} \tag{4}$$

$$\xi_i = \sqrt{\frac{[\ln(\lambda_i\,\lambda_i^*)]^2}{[\ln(\lambda_i\,\lambda_i^*)]^2 + 4[\cos^{-1}(\frac{\lambda_i + \lambda_i^*}{2\sqrt{\lambda_i\,\lambda_i^*}})]^2}} \tag{5}$$

with Δt the sampling period of analyzed signals.
Define the ($mpx1$) future data vector as $\boldsymbol{y}^+_k = [\boldsymbol{y}^T_k, \boldsymbol{y}^T_{k+1}, \ldots, \boldsymbol{y}^T_{k+p-1}]^T$ and the ($mpx1$) past data vector as $\boldsymbol{y}^-_{k-1} = [\boldsymbol{y}^T_{k-1}, \ldots, \boldsymbol{y}^T_{k-p}]^T$, where the superscript T denotes the transpose operation. The ($mpxmp$) covariance matrix between the future and the past is

$$\boldsymbol{H} = E[\boldsymbol{y}^+_k\, \boldsymbol{y}^{-T}_k] = \begin{bmatrix} \boldsymbol{R_1} & \boldsymbol{R_2} & . & \boldsymbol{R_p} \\ \boldsymbol{R_2} & \boldsymbol{R_3} & . & \boldsymbol{R_{p+1}} \\ . & . & . & . \\ \boldsymbol{R_p} & \boldsymbol{R_{p+1}} & . & \boldsymbol{R_{2p-1}} \end{bmatrix} \tag{6}$$

where E denotes the expectation operator. $\boldsymbol{H}$ is the block Hankel matrix formed with the (mxm) individual theoretical auto-covariance matrices $\boldsymbol{R}_i = E[\boldsymbol{y}_{k+i}\, \boldsymbol{y}^T_k] = \boldsymbol{C}\boldsymbol{A}^{i-1}\boldsymbol{G}$, with $\boldsymbol{G}=E[\boldsymbol{x}_{k+1}\, \boldsymbol{y}^T_k]$. In practice, the auto-covariance matrices are estimated from N data points [2]. The block Hankel matrix $\boldsymbol{H}$ can be written as

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{CA} \\ . \\ \boldsymbol{CA}^{p-1} \end{bmatrix} [\boldsymbol{G}\ \ \boldsymbol{AG} \ldots \boldsymbol{A}^{p-1}\boldsymbol{G}] = \boldsymbol{O}\,\boldsymbol{K} \tag{7}$$

By identification we obtain the block observability matrix $\boldsymbol{O}$ and the block controllability matrix $\boldsymbol{K}$

$$\boldsymbol{O} = \begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{CA} \\ \cdot \\ \boldsymbol{CA}^{p-1} \end{bmatrix} \quad \text{and} \quad \boldsymbol{K} = [\boldsymbol{G}\ \ \boldsymbol{AG} \ldots \boldsymbol{A}^{p-1}\boldsymbol{G}\ \ ] \quad (8)$$

This last matrix can be written as $\boldsymbol{K} = [\ \boldsymbol{K_1}\ \ \boldsymbol{A}^{p-1}\boldsymbol{G}] = [\boldsymbol{G}\ \ \boldsymbol{K_2}]$ where the block matrices $\boldsymbol{K_1}$ and $\boldsymbol{K_2}$ are

$$\boldsymbol{K_1} = [\boldsymbol{G}\ \ \boldsymbol{AG} \ldots \boldsymbol{A}^{p-2}\boldsymbol{G}] \text{ and } \boldsymbol{K_2} = [\boldsymbol{AG} \ldots \boldsymbol{A}^{p-1}\boldsymbol{G}\ ] \quad (9)$$

$\boldsymbol{K_1}$ and $\boldsymbol{K_2}$ are $nxm(p-1)$ matrices obtained by deleting respectively the last and the first block column of the block controllability matrix $\boldsymbol{K}$. It is easy to show that $\boldsymbol{K_2} = \boldsymbol{A}\ \boldsymbol{K_1}$ and the transition matrix obtained by the deleted block column of the controllability matrix method can then be calculated via pseudo-inverse $\boldsymbol{A_K} = \boldsymbol{K_2}\ \boldsymbol{K_1}^+$ .The eigenvalues of the transition matrix $\boldsymbol{A_K}$ can be used to identify the modal parameters and one gets $\lambda(\boldsymbol{A_K}) = \lambda(\boldsymbol{K_2}\ \boldsymbol{K_1}^+)$.

3. MODAL PARAMETER IDENTIFICATION OF A PERFORATED MICROPLATE

The perforated microplate supported by an elastic suspension as shown in Figure 1, and the model used to study the microplate behavior is constituted by the following parameters : the plate mass m concentrated in the center of the plate, the damping coefficient c and the stiffness coefficient k.

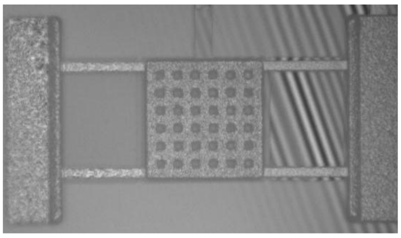

Figure 1. Optical image of the perforated microplate

The dynamic measurements are conducted in time domain by means of a laser vibrometer and Figure 2 shows the time response of the center of the microplate. Only this time response is used in our subspace identification procedure where the sampling frequency is 2 MHz and 3000 time samples are used.

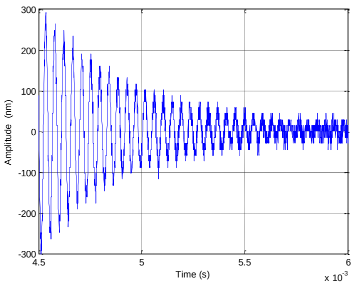

Figure 2. Displacement response of the microplate

The perforated microplate area is A = 3.127x10^{-8} m^2 and its mass is m = 3.814x10^{-9} kg. The microplate stiffness is given by $k = m\,(2\,\pi F)^2$ and the damping coefficient is $c = 4\,\pi\, m\, F\, \xi$ where F is the resonance frequency of the perforated microplate and ξ the damping factor. These modal parameters are obtained by an average over the orders of the stabilization diagram [2] on frequency and damping ratio obtained by the subspace approach and presented in Figure 3.

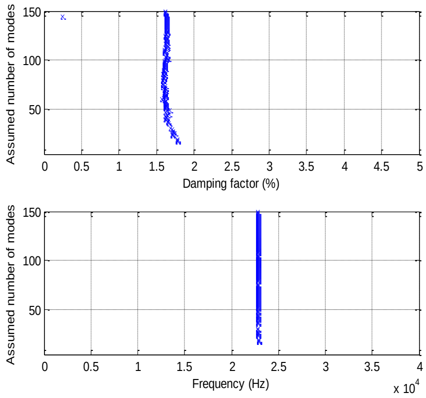

Figure 3.Stabilization diagram on frequency and damping ratio

Table 1 shows the identified microplate parameters and Figure 4 shows a comparison between the measured time response of the perforated microplate and the reconstructed response obtained from the identified modal parameters.

Resonance frequency F	Damping factor ξ	Stiffness coefficient k	Damping coefficient c
22810 Hz	1.62 %	78.33 N.m^{-1}	17.71x10^{-6} N.s.m^{-1}

Table 1. Parameters identification of the perforated microplate

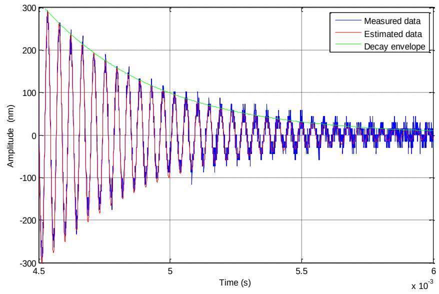

Figure 4. Comparison between the measured (in blue) and the reconstituted (in red) displacement response

4. CONCLUSION

The effectiveness of the subspace identification procedure developed in the paper has been applied to modal parameter identification of a perforated microplate. Analytical and experimental results are presented showing the accuracy of the method.

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