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Modeling, Filtering and Optimization for AFM Arrays

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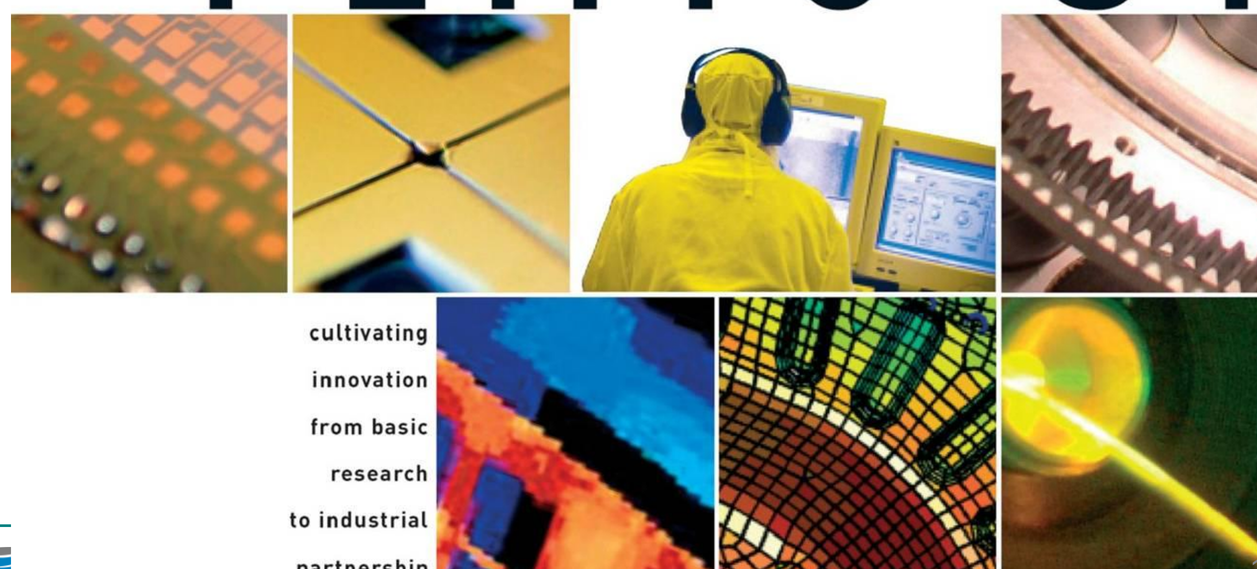

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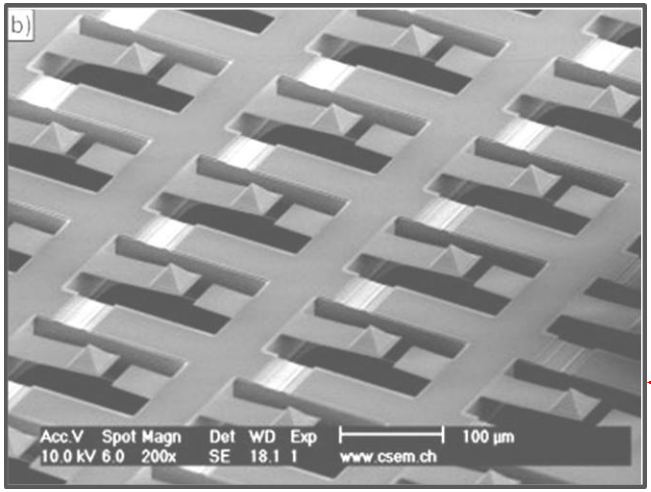
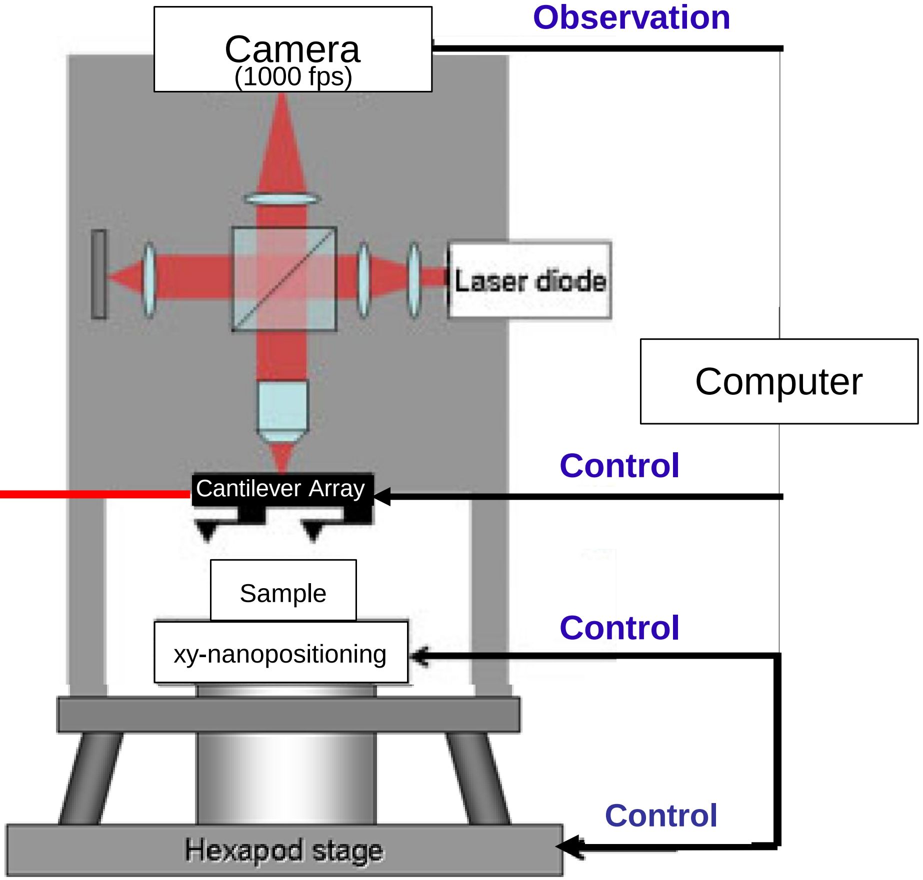
Objectives



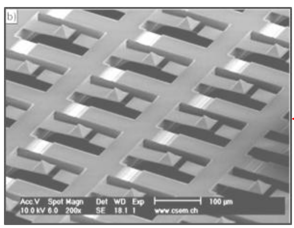


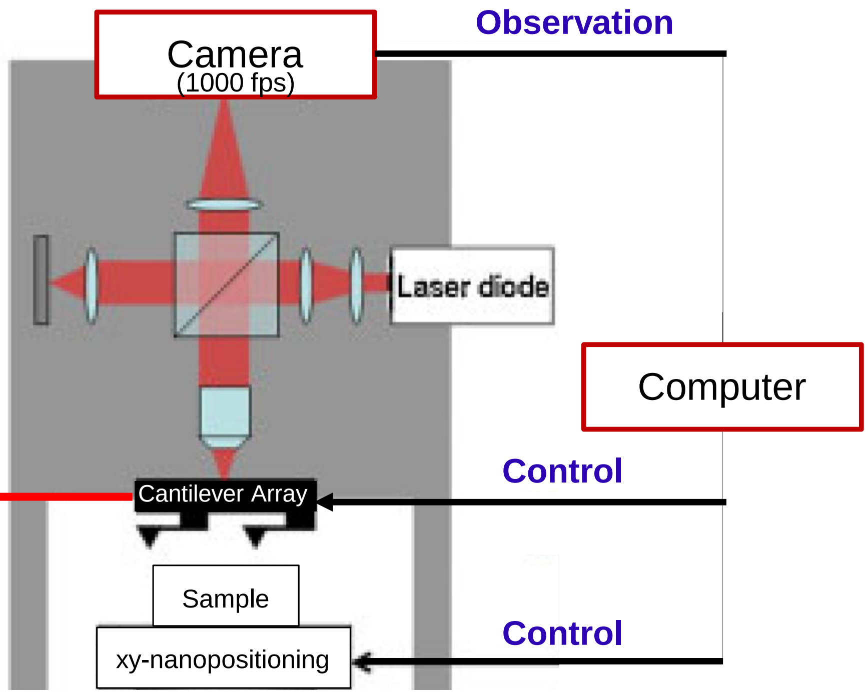
- **Design of the Array**
- **Functioning modes: Contact mode, Dynamic mode**

Requirements in Maths and Software

Real Time Interferometry Image Processing

Optimal Design Based on a Model





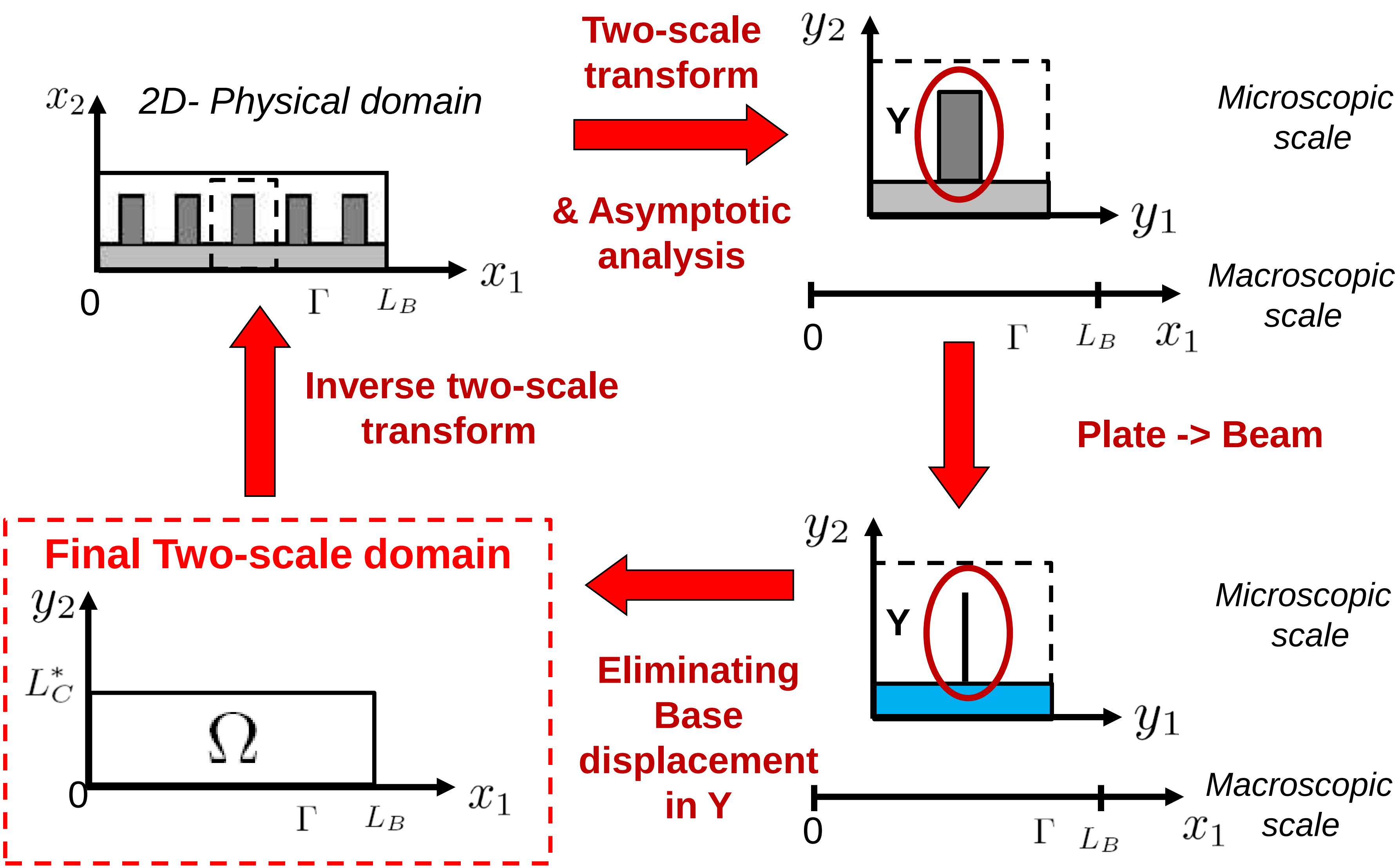
Real Time Processing of Distributed Control Law Based on a Model

- **Modeling: Two-scale Modelling**
- **Design: Robust Optimization Based on the Two-Scale Model**
- **Image Processing for Interferometry: Topographic scan**
- **Distributed Control: Semi-Decentralized**

Content

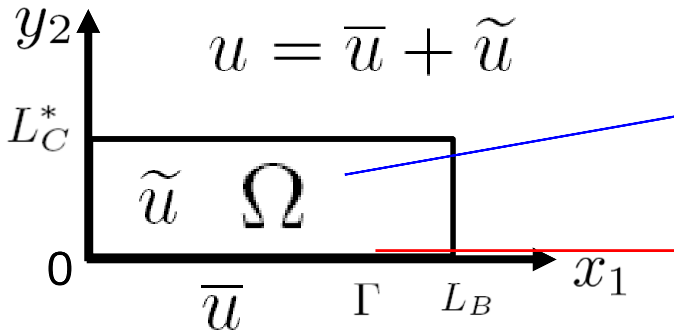
- Two-scale Modelling (AFMALab Software)
- Robust Parameter Optimization (SIMBAD Software)
- Image Processing for Interferometry
- Semi-Decentralized Distributed Control

Two-Scale Model Principle



Two-Scale Model Discretization

Final Two-scale domain

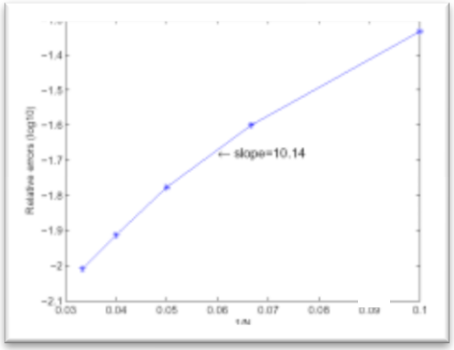


Motion equation in cantilevers at the microscale

$$m^C \partial_{tt}^2 \widetilde{u} + m^C \partial_{tt}^2 \bar{u} + r^C \partial_{y_2 \dots y_2}^4 \widetilde{u}$$

Motion equation in base at the macroscale

$$\rho^B \partial_{tt}^2 \bar{u} + R^B \partial_{x_1 \cdots x_1}^4 \bar{u} + \ell_C r^C (\partial_{y_2 y_2 y_2}^3 \widetilde{u})$$



L^2-norm error in a logarithmic scale with respect to cell size

FEM discretization of $\overline{u}$ in Base

Modal Decomposition of $\widetilde{u}$ in the Cantilevers

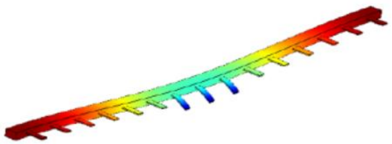
$$\widetilde{u}(t, x_1, y_2) \approx \sum_{k=1}^{N} \widetilde{u}_k(t, x_1) \phi_k(y_2) \text{ and } F^C(t, x_1, y_2) \approx \sum_{k=1}^{N} f_k^C(t, x_1) \phi_k(y_2)$$

where $\{\phi_k(y_2)\}_{k=1..N}$ are the cantilever modes

Verification & AFMALab Software

The One-dimensional Model

Solutions are approximated in the Multi-scale domain with respect to small thickness or large number of cells. This yields a two-scale model for a cantilever array. It is governing deflections for an infinite number of cantilevers.

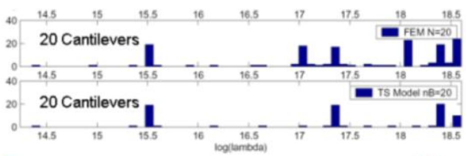


In base: $m^B \partial_{tt} u^0 + r^B \partial^4_{x_1 \ldots x_1} u^0 = -d^B \partial^3_{y_2 \ldots y_2} u^0 + f^B$

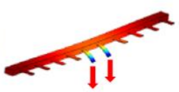
In cantilevers: $m^C \partial_{tt} u^0 + r^C \partial^4_{y_2 \ldots y_2} u^0 = f^C$

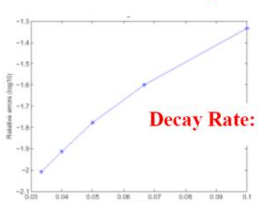
Interface: $u^0_{|cantilever} = u^0_{|base}$ and $(\partial_{y_2} u^0)_{|cantilever} = 0$

- Eigenvalues:



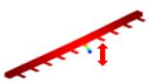
- Statics:

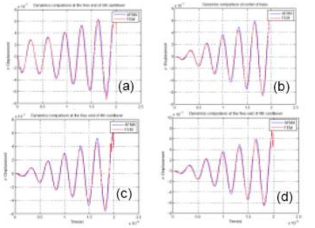




Logarithm of the L^2-norm error with respect to cell size

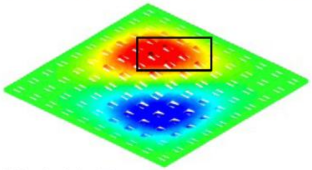
- Dynamics:





Exitation: First Eigenfrequency; Displacement (a) Fifth lever end, (b) Base middle, (c) Fourth lever end, (d) Sixth lever end

The Two-Dimensional Model

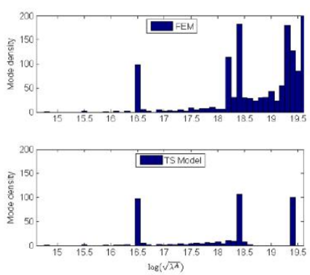


Global and local view of a Cantilever Array

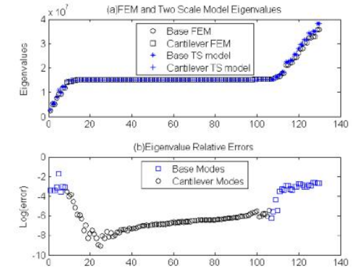
Model in Base:

$$\rho^B \partial_{tt} u^A + div_{\tilde{x}}(div_{\tilde{x}}(R^B : \nabla_{\tilde{x}} \nabla^T_{\tilde{x}} u^A)) + \ell^0_C r^C (\partial^3_{y_2 y_2 y_2} u^A)_{|junction} = f^B$$

MS Model / FEM Eigenfrequency comparison:

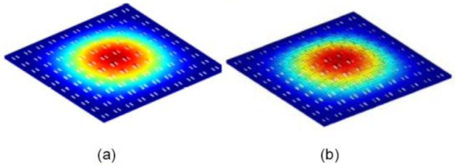


Comparison of Eigenvalue density distributions for (a) the FEM and for (b) the MSM

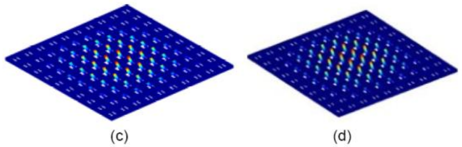


(a) Superimposition of eigenvalue distributions for the FEM and for the MSM (b) Relative errors

MS Model / FEM Eigenshapes comparison:



First base eigenmode shape
(a) Multi-scale Model
(b) FEM



First cantilever eigenmode shape
(c) Multi-scale Model
(d) FEM

Content

- Two-scale Modelling (AFMALab Software)
- Robust Parameter Optimization (SIMBAD Software)
- Image Processing for Interferometry
- Semi-Decentralized Distributed Control

Robust Parameter Optimization Toolbox

Sensitivity Analysis: Selection of design variables with the greatest impact

Deterministic Design Optimization: Optimization of design variables with constraints

Uncertainty quantification: Impact of manufacturing uncertainties

AFM Array Design Variables

Description	Initial value	Bound
Lever length	$200\mu m$	$[60\ 400]\mu m$
Lever width	$40\mu m$	$[40\ 80]\mu m$
Lever thickness	$0.5\mu m$	$[0.25\ 0.70]\mu m$
Young modulus of lever	$335Gpa$	fixed
Mass density of lever	$3100kg/m^3$	fixed
Tip presence	1 (or 0)	fixed
Base thickness	$30\mu m$	$[30\ 60]\mu m$
Base width	$40\mu m$	$[10\ 200]\mu m$
Young modulus of Base	$169Gpa$	fixed
Mass density of Base	$2330kg/m^3$	fixed
Array size in x-direction	$1000\mu m$	fixed
No. levers in x-direction	10	[2 20]
No. levers in y-direction	1	fixed
Tip base width	$x_2 - 10\mu m$	dependent
position of tip apex	$x_2/2$	dependent
Array size	$1 \times 1mm^2$	fixed
Array dimension	1 (or 2)	fixed
x-direction pitch	$floor(\frac{x11}{x12\times 50}) \times 50\mu m$	proportional to $50\mu m$
y-direction pitch	$floor(\frac{x11}{x12\times 50}+1) \times 50\mu m$	proportional to $50\mu m$
Longitudinal base width	$40\mu m$	$[10\ 200]\mu m$

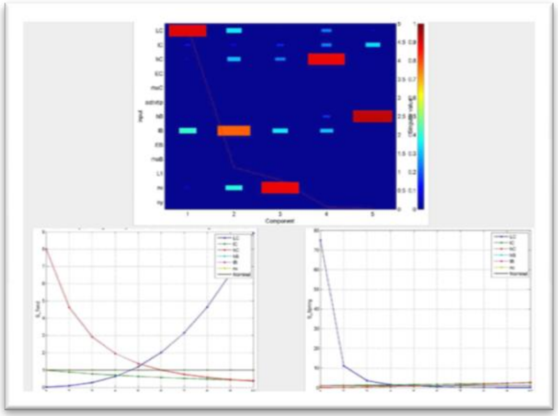
AFM Array Design objectives and constraints

Description	Equation
Maximum deflection angle at tip end less than 3/180*pi	$\frac{3S_2}{2x_1} - \frac{3\pi}{180} \leq 0$
Maximum displacement at base must smaller than $50nm$ for small array and $80nm$ for large array	$s_3 - 50nm \leq 0$ or $s_3 - 80nm \leq 0$
Minimum gap between two levers larger than lC/2	$\frac{lC}{2} - s_4 \leq 0$
Minimum gap percentage of each cell greater than 40%	$0.4 - s_5 \leq 0$
Number of levers in x-direction more than 2	$2 - s_6 \leq 0$
Number of levers in y-direction more than 2	$2 - s_7 \leq 0$
Maximum footprint size of small and large array	$s_9 - 1 \times 1mm^2 \leq 0$ or $s_9 - 2 \times 2mm^2 \leq 0$

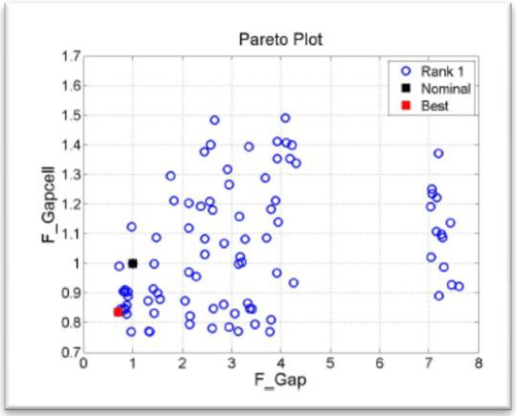
Robust Param. Optimization Toolbox

Design Optimization

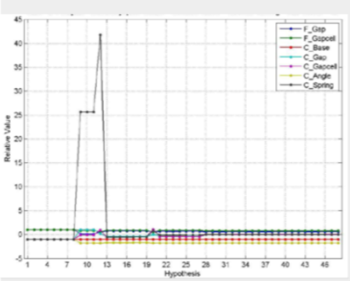
Sensitivity Analysis



Multi-Objective Design Optimization



Mono-objective Design Optimization



Design Objective	Objective Status	Initial Value	Optimal Value
F_Gap	active	6E-005	3.89E-005
F_Gapcell	active	0.52	0.454

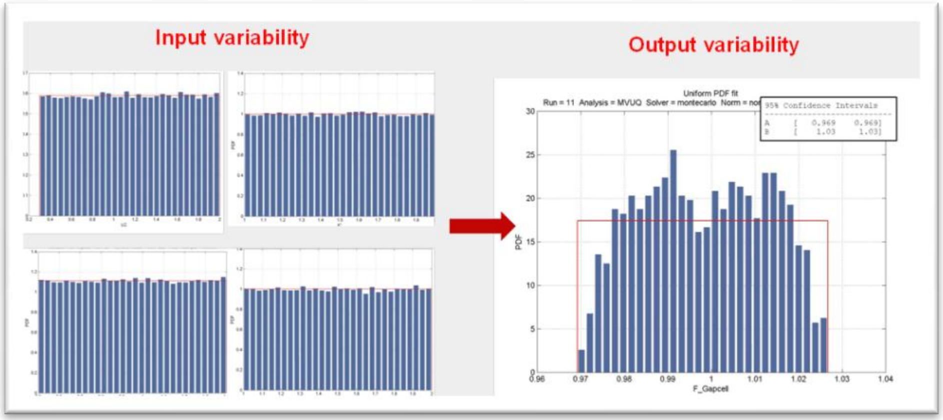
Design Constraint	Constraint Status	Initial Value	Optimal Value
C_Base	active	-4.97E-008	-4.97E-008
C_Gap	active	-4E-005	-8.33E-006
C_Gapcell	active	-0.12	-0.0542
C_Angle	active	-0.0291	-0.0513
C_Spring	active	-0.0294	-0.0042

→ 35% improvement of F_Gap

→ 12.6% improvement of F_Gapcell

→ All design constraints satisfied

Uncertainty Quantification



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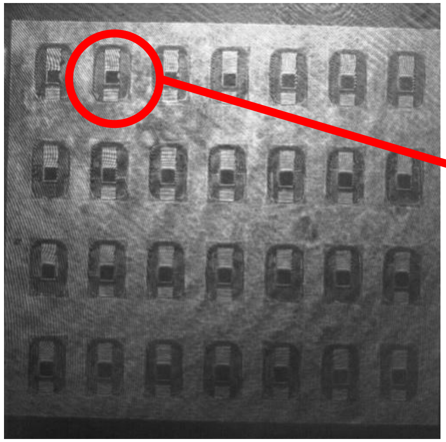
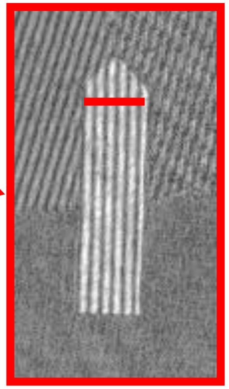
Displacement Measurement by Interferometry

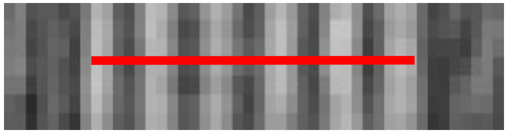
Image 1024 pixels x 1024 pixels

Less than 25 pixels in the width

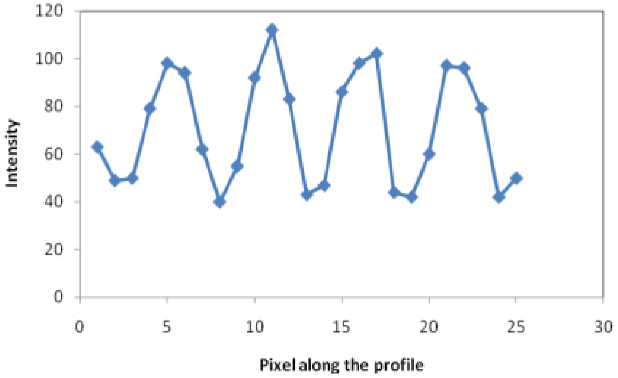




Intensity Profile



Intensity measurement



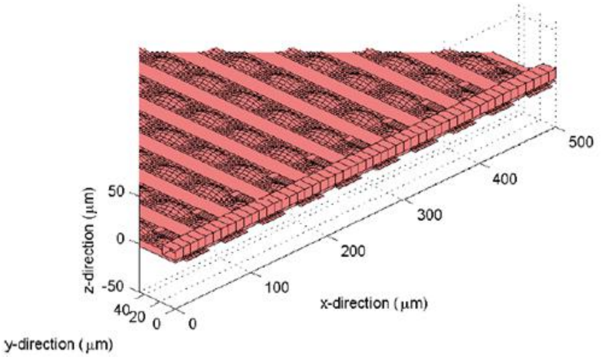
Processing (patent pending)

Standard deviation for displacement in air in static regime:

Free lever: 0.27 nm

In contact: 1.4 nm

Simulation of topographic scan



Sample topography $z_{obj}(i)$

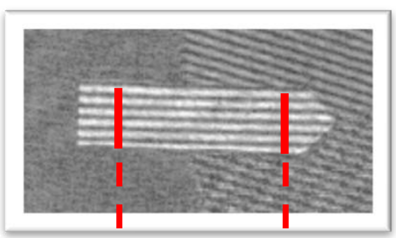
Static Regime $u^{tip}(i)$

carrier wave — phase

Intensity: $I(t, x_1, y_2) = A\cos\left(2\pi f x_1 + \theta(t, y_2)\right)$

with $\theta = \frac{2\pi}{\bar{\lambda}}(b - 2u)$

2 lines of measurements

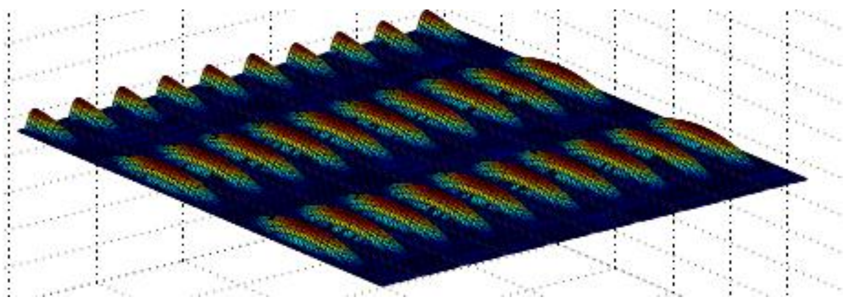


$y_2^{0,1}$ $y_2^{0,2}$

Least square algorithm θ^*

All tip forces:

$$f_3 = -\frac{\bar{\lambda}\delta\theta}{2\pi\left(K(y_2^{0,2}) - K(y_2^{0,1})\right)}$$



$z_{obj}^{est}(i)$

All displacements

Model

Content

- Two-scale Modelling (AFMALab Software)
- Robust Parameter Optimization (SIMBAD Software)
- Image Processing for Interferometry
- **Semi-Decentralized Distributed Control**

H-infinity filtering problem

Standard State formulation for the H-infini filter of the distributed pb:

State variable: $u = (\overline{u},\ (\widetilde{u}_k)_{k=1,..,N},\ \partial_t \overline{u},\ (\partial_t \widetilde{u}_k)_{k=1,..,N})^T$

State equation: $\partial_t u(t) = Au(t) + Bw_1(t), \quad u(0) = u_0,$

Observation: $y(t) = Cu(t) + Dw_2(t),$

Estimated state: $z(t) = Lu(t).$

Observation

$$Cu(t) = \frac{1}{|\beta - \alpha|} \int_\alpha^\beta u(x_1, y_2)\, dy_2$$

H-infinity problem:

$$\min_{\gamma} \max_{w_1, w_2} \frac{||z - \widehat{z}||^2}{||w_1||^2 + ||w_2||^2}$$

H-infinity filtering problem

Standard State formulation for the H-infini filter of the distributed pb:

State variable: $u = (\overline{u},\ (\widetilde{u}_k)_{k=1,..,N},\ \partial_t \overline{u},\ (\partial_t \widetilde{u}_k)_{k=1,..,N})^T$

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Estimated state: $z(t) = Lu(t).$

Observation

$$Cu(t) = \frac{1}{|\beta - \alpha|} \int_\alpha^\beta u(x_1, y_2)\, dy_2$$

Estimator equations

$$\partial_t \widehat{u}(t) = A\widehat{u}(t) + PC^*(y(t) - C\widehat{u}(t)),$$
$$\widehat{z}(t) = L\widehat{u}(t).$$

with operatorial Riccati equation

$$AP + PA^* - PC^*CP + \frac{1}{\gamma^2} PL^*LP + BB^*$$

To be computed in real time in FPGA

Difficult because:

P = Global Operator

Approximation using a Functional Calculus

Cauchy Integral for:

- **Holomorphic function f in C**

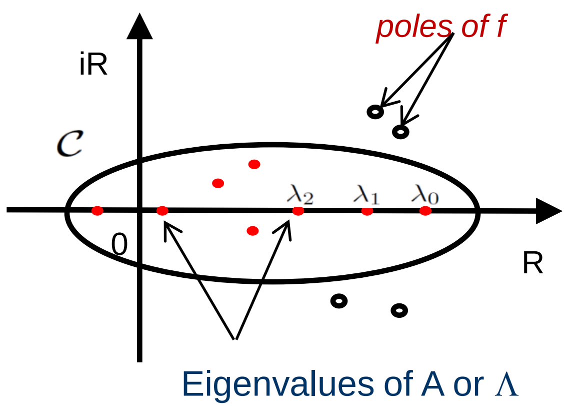
$$f(\lambda_0) = \frac{1}{2i\pi}\int_{\mathcal{C}} \frac{f(\lambda)}{\lambda-\lambda_0}\, d\lambda$$

- **Function of a matrix A:**

$$f(A) = \frac{1}{2i\pi}\int_{\mathcal{C}} f(\lambda)(\lambda I - A)^{-1}\, d\lambda$$

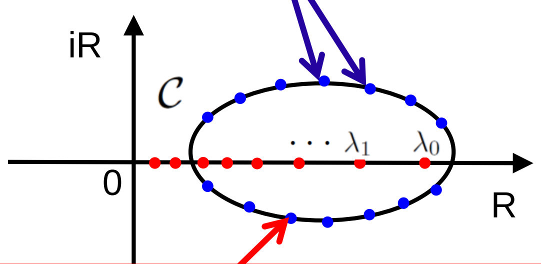
- **Function of an operator Λ:**

$$f(\Lambda) = \frac{1}{2i\pi}\int_{\mathcal{C}} f(\lambda)(\lambda I - \Lambda)^{-1}\, d\lambda$$



Application to PC^{N*} with $\Lambda^{-1} = \partial^4_{x\cdots x}$ after a preliminary preparation of the Riccati equation:

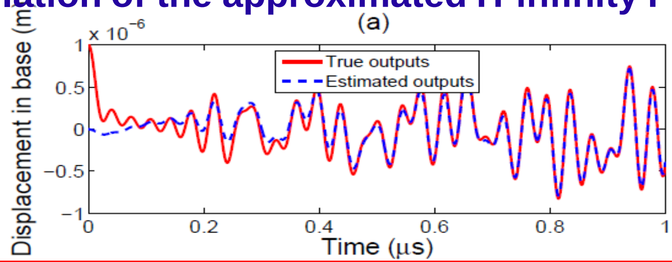
$$PC^{N*} \approx \frac{1}{2\pi}\sum_{l=1}^{M} v_1^{\zeta(\theta_l)}\omega_l.$$



For each ζ resolution of a 2d-system:

$$\begin{cases} \zeta_1 v_1^{\zeta} - \zeta_2 v_2^{\zeta} - \Lambda v_1^{\zeta} = Re\left(-i\zeta' k_R(\zeta)\right) z, \\ \zeta_2 v_1^{\zeta} + \zeta_1 v_2^{\zeta} - \Lambda v_2^{\zeta} = Im\left(-i\zeta' k_R(\zeta)\right) z. \end{cases}$$

Simulation of the approximated H-infinity Filter:



Conclusions and Perspectives

Conclusions

- Short review of our results for arrays of AFMs
- A two-scale model for the whole array sufficiently light to use in optimization procedures
- Software developed (best validated for 1D arrays)
- An optimization toolbox build and included
- Validated algorithm for interferometry with small number of pixels
- Topographic scan in static regime: recover displacements from 2 measurements and generate images for contact mode
- Dynamic regime: H-infinity filter approximation based on functional calculus

Perspectives (short term)

- Complete implementing the full setup and force spectroscopy in quasi-static regime
  - A new design based on our software is planed at CSEM
  - Verfiying the H-infinity filter approximation with the observation comes from interferometry measurements
  - Implementing the full measurement chain

End

Question & Remarks