Identification of non self-adjoint systems: toward the use of a MIMO strategy

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This paper deals with identification of non self-adjoint second order problems in vibroacoustics. During the identification procedure, a modal analysis can be performed to obtain the complex modes of the system, before estimating the matrices using an inverse procedure. This procedure is known to be very sensitive to experimental noise. In particular, damping terms are badly estimated. An elegant way to enhance the conditioning of the procedure is to correct the complex vectors such that they verify the properness condition. This optimal correction has been first developed for structural dynamics and recently extended to non-symmetric problems. Test-cases have shown the efficiency of the procedure for active control and rotordynamics, while for vibroacoustics, some improvements are still required to provide robust identification. Reciprocity measurements have been used to allow the use of either structural or acoustic sources during the procedure. In this paper we show that it is necessary to use a Multi-Input Multi-Output (MIMO) technique in order to take into account both types of excitations in the procedure. This allows excitation of the structure and the cavity, in order to capture the coupled behavior in an efficient way. The identified vectors can then be optimally corrected to obtain a confident vibroacoustic reduced model.

1 INTRODUCTION

Identification of reduced models from experimental modal analysis is a well-known problematic in structural dynamics. The main objective is to provide efficient tools to identify the damping matrix of a system directly from measurements, considering the fact that modeling dissipation is often costly and uncertain. Mass and stiffness terms can also be identified with a higher confidence level. The modern trends in this context mainly deal with problematic associated to spatial and modal incompleteness of data, ill-conditioning of matrices, non-uniqueness of solutions, computational time for large structural models and noise and errors in measurement. Two categories of approaches are popular today: those which are based on direct use of FRFs (which are basically very efficient if excitation on all dofs are possible) [1], and those which are based on the use of complex modes, identified from the FRFs. Among these methods, the one which are based on the use of a full modal basis (number of identified modes = number of dofs) are the most efficient for correct damping localization. The complex modes identified from measurements can be modified in an optimal way in order that they constitute a complete basis of a reduced model which has a topology in accordance with physical behaviors [2].
Extension of these techniques for vibroacoustic modal analysis [8, 10] is still a challenge, in particular because of the non-symmetry of the problem. This paper contributes to this topic, by introducing the use of reciprocity measurements [6, 7] to improve reduced system identification.

2 PROBLEM STATEMENT

The problem which is considered in this paper is related to internal vibroacoustics. The general formulation of this problem can be classically written using a matrix system which is not symmetric [3-5]:

$$-\omega^2 [M]\{q\} + i\omega [C]\{q\} + [K]\{q\} = \{f\} \tag{1}$$

where the matrices are constituted by block matrices related to the structural part (those indexed with subscript S), to the fluid part (subscript F) and to the vibroacoustical couplings (L matrix):

$$[K] = \begin{bmatrix} K_S & -L \\ 0 & K_F \end{bmatrix}, [C] = \begin{bmatrix} C_S & 0 \\ 0 & C_F \end{bmatrix}$$
$$[M] = \begin{bmatrix} M_S & 0 \\ L^T & M_F \end{bmatrix}, \ \{q\} = \begin{Bmatrix} U \\ P \end{Bmatrix}, \ \{f\} = \begin{Bmatrix} F_S \\ \dot{Q} \end{Bmatrix} \tag{2}$$

The degrees of freedom are the structural displacement *U* and the acoustic pressure *p*. The block matrices are supposed to be symmetric, which is not the case of the full system matrices. The damping terms are limited to equivalent viscous effects. This is reasonable while the losses are small and well distributed in the system, but could lead to erroneous results when large damping effects are considered.
In the force vector, the acoustic source is a volume acceleration.

2.1 Complex modes

Since the global system is not symmetric, right and left eigenshapes are not equal. The right eigenvectors are solution of

$$\left([M]\,\lambda_j^2 + [C]\,\lambda_j + [K]\right)\left\{\phi_{Rj}\right\} = 0, \tag{3}$$

while the right ones verify

$$\left([M]^T\,\lambda_j^2 + [C]^T\,\lambda_j + [K]^T\right)\left\{\phi_{Lj}\right\} = 0, \tag{4}$$

with obviously the same eigenvalues. The orthogonality conditions can be obtained using the state-space representation

$$[U]\left\{\dot{Q}(t)\right\} - [A]\left\{Q(t)\right\} = \left\{F(t)\right\} \tag{5}$$

with

$$[U] = \begin{bmatrix} C & M \\ M & 0 \end{bmatrix},\ [A] = \begin{bmatrix} -K & 0 \\ 0 & M \end{bmatrix},\ \{Q(t)\} = \begin{Bmatrix} q(t) \\ \dot{q}(t) \end{Bmatrix},\ \{F(t)\} = \begin{Bmatrix} f(t) \\ 0 \end{Bmatrix} \tag{6}$$

One has then

$$[\theta_L]^T\,[U]\,[\theta_R] = [\xi] \text{ or } [\theta_L]^T\,[A]\,[\theta_R] = [\xi]\,[\Lambda] \tag{7}$$

where

$$[\Lambda] = \left[{}^{\backslash}\lambda_{j\backslash}\right] \quad [\theta_R] = \begin{bmatrix} \phi_R \\ \phi_R\Lambda \end{bmatrix} \quad [\theta_L] = \begin{bmatrix} \phi_L \\ \phi_L\Lambda \end{bmatrix} \quad [\xi] = \left[{}^{\backslash}\xi_{j\backslash}\right] \tag{8}$$

It should be emphasized that for vibroacoustic modes, left eigenvectors can be directly derived from the right ones [8]:

$$[\phi_R] = \begin{bmatrix} U \\ P \end{bmatrix} \quad [\phi_L] = \begin{bmatrix} U \\ -P\Lambda^2 \end{bmatrix} \tag{9}$$

2.2 Inverse procedure for matrices estimation – properness condition

The matrices of the initial system can then be identified by inverting the modal matrices:

$$\begin{bmatrix} C & M \\ M & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & M^{-1} \\ M^{-1} & -M^{-1}CM^{-1} \end{bmatrix} = \begin{bmatrix} \phi_R\phi_L^T & \phi_R\Lambda\phi_L^T \\ \phi_R\Lambda\phi_L^T & \phi_R\Lambda^2\phi_L^T \end{bmatrix} \tag{10}$$

and

$$\begin{bmatrix} -K & 0 \\ 0 & M \end{bmatrix}^{-1} = \begin{bmatrix} -K^{-1} & 0 \\ 0 & M^{-1} \end{bmatrix} = \begin{bmatrix} \phi_R\Lambda^{-1}\phi_L^T & \phi_R\phi_L^T \\ \phi_R\phi_L^T & \phi_R\Lambda\phi_L^T \end{bmatrix} \tag{11}$$

yielding

$$M = \left[\phi_R\Lambda\phi_L^T\right]^{-1},\ \ K = -\left[\phi_R\Lambda^{-1}\phi_L^T\right]^{-1},\ \ C = -\left[M\phi_R\Lambda^2\phi_L^T M\right] \tag{12}$$

These relationships are valid only if the properness condition is verified:

$$\left[\phi_R\phi_L^T\right] = 0 \tag{13}$$

The complex modes of the system must verify this relation if they constitute the full basis of a physical system.

3. OPTIMAL CORRECTION OF COMPLEX MODES

When the complex modes are available from experimental identification, one can use inverse relationships in order to find the reduced model which is supposed to have the same behavior as the measured one. The fact is that in general, the modes do not verify the properness condition (13). In the particular case of vibroacoustics, one can try to follow the same methodology as the one used in structural dynamics. The following constrained optimization problem should then be solved:

$$\begin{array}{l} \text{Find } \tilde{U} \text{ and } \tilde{P} \text{ minimizing } \left\|U-\tilde{U}\right\| \text{ et } \left\|P-\tilde{P}\right\| \\ \text{while } \tilde{U}\tilde{U}^T = 0\ ;\ \tilde{U}\tilde{P}^T = 0\ ;\ \tilde{U}\Lambda^{-2}\tilde{P}^T = 0\ ;\ \tilde{P}\Lambda^{-2}\tilde{P}^T = 0 \end{array} \quad (14)$$

This problem can be re-written using four Lagrange multipliers matrices

$$\begin{cases} 0 = \begin{bmatrix} \tilde{U} \\ \tilde{P} \end{bmatrix} - \begin{bmatrix} U \\ P \end{bmatrix} + \frac{1}{2}\begin{bmatrix} \delta_1+\delta_1^T & \delta_2 \\ \delta_2^T & 0 \end{bmatrix}\begin{bmatrix} \overline{\tilde{U}} \\ \overline{\tilde{P}} \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & \delta_3 \\ \delta_3^T & \delta_4+\delta_4^T \end{bmatrix}\begin{bmatrix} \overline{\tilde{U}\Lambda^{-2}} \\ \overline{\tilde{P}\Lambda^{-2}} \end{bmatrix} \\ 0 = \tilde{U}\tilde{U}^T \\ 0 = \tilde{U}\tilde{P}^T \\ 0 = \tilde{U}\Lambda^{-2}\tilde{P}^T \\ 0 = \tilde{P}\Lambda^{-2}\tilde{P}^T \end{cases} \quad (15)$$

Solving this problem is clearly not easy because of the presence of the Λ matrices that makes impossible to find explicitly the expression of multipliers versus the unknown vectors. An iterative procedure could be investigated but this is not the best way to obtain quick results that can be used in real-time during modal analysis. Some simplified methods have been proposed [10], among which one is called over-properness: considering the fact that the method developed for structural dynamics [2] is valid for all matrix subjected to a properness condition $xx^T = 0$, one can use

$$x = \begin{bmatrix} U \\ P \\ -P\Lambda^2 \end{bmatrix} \quad (16)$$

It can be observed that the four required terms of equation (13) are included in the matrix xx^T while two of them are not theoretically required. Using this vector in the procedure detailed in ref. [2] leads to a so-called over-proper solution which includes more constraints than those required, but that includes the required ones.

4. EXPERIMENTAL SETUP

The experimental illustration of the methodology is performed on an acoustic cavity with 5 rigid surfaces, closed with a clamped elastic plate. The full set-up, available at LVA-INSA Lyon, is presented in ref. [11, 12]. Basically, it includes 3 force sensors on the structural plate, and 3 microphones in the cavity, one of them being in front on the loudspeaker used as acoustic source. The transducers positions have been optimized using a QR decomposition of the modal matrix including the modes of interest [9, 12]. A sample of mineral wool is used to damp the acoustic modes.

5. VIBROACOUSTIC RECIPROCITY

The easiest way to identify a vibroacoustic reduced model using the inverse procedure presented in section 3 is to use an impedance head at structural excitation point(s) in order to identify the input and cross FRFs. Only one excitation point is theoretically required to identify the full model. It is nevertheless clear that increasing the number of excitation points will yield to an improvement of the experimental model since new information will be added to increase the conditioning of the inverse problem.

In particular, it can be interesting to use acoustic sources. If these sources are monopoles, the vibroacoustic reciprocity relation holds [6, 7]:

$$\left.\frac{P_i}{F_j}\right|_{\dot{Q}_i=0} = -\left.\frac{\ddot{U}_j}{\dot{Q}_i}\right|_{F_j=0} \tag{17}$$

This relation must be verified by the structural-acoustic cross FRFs. In particular, it can be useful to identify the acoustic source strength which is not easy to measure directly. As indicated in ref. [8], the source can be calibrated in an anechoic chamber using transfer functions between input voltage and structural velocity of the loudspeaker used as acoustic source. Another strategy consists in using reciprocity relationship (17) associated with input voltage measurement in order to identify the unknown transfer function between acoustic volume acceleration and input voltage. The figure 2 illustrates the value of the identified transfer function for the experimental setup considered.

6. EXPERIMENTAL ILLUSTRATION

The efficiency of the methodology can be illustrated using FRFs. When using only structural excitation, the methodology provides efficient results. The figure 3 exhibits the following curves:

- The Reference FRF, calculated from the initial given matrices. It is supposed to represent the behavior of the system to be identified. For illustration purpose, only one of the many FRFs is presented here (corresponding to the collocated input and output on the first degree of freedom).
- The Modal FRF re-built after noise introduction on eigenvectors. This is associated to "experimentally" identified eigenvalues and eigenvectors. An important point is that this curve is visually coincident to the reference one. In an experimental procedure, this would indicate that the identification is correct.
- The Direct FRF reconstructed using the matrices obtained after solving the inverse problem from the eigenvalues and eigenvectors affected by noise. This procedure clearly fails in this case, because equations (12) are not verified strictly since the properness condition is not verified. Some small differences in the eigendata can induce large discrepancies on the identified system.
- The Proper FRF obtained using data with enforcement of the properness condition, to be discussed in the next paragraph. This curve is also visually coincident with the reference one.

Using only structural excitation leads to coherent results. The corresponding reduced model can be applied for any structural excitation in a very efficient way to estimate the vibroacoustic levels. When this model is used with acoustic excitation, the levels are unfortunately badly estimated.

In order to have better estimation in this case, the complex modes identified with acoustic excitation can be used to build the reduced model. The results are shown in figure 4: one can clearly see that the levels at resonances estimated with identified model are in agreement with measurements. Concerning the anti-resonances, one can observe that the measured FRFs have a non-classical frequency evolution, which explains the differences observed between the modal synthesis and the measured curve. This point is related to the acoustic source identification procedure, and needs more investigation for correct estimation of amplitude at anti-resonance levels. It should be emphasized that the problem comes from the modal identification coupled with the frequency evolution of acoustic source, and not from the original procedure proposed here, which cannot lead to better results than those of the modal synthesis, since starting point is the modal basis.

7. CONCLUSIONS

In this paper, a methodology has been provided for identification of vibroacoustic reduced models, starting from complex modes. These modes are modified in an optimal way to enhance the conditioning of the inverse procedure. The methodology is based either on structural or acoustic excitation. In both cases, levels at resonances are evaluated with a good degree of confidence. The future steps in the methodology will concern:

- the extension to MIMO (Multi-Input Multi-Output) identification procedure, in order to identify reduced models which are able to use both structural and acoustic sources;
- the extension of the methodology to frequency-dependant damped configurations, in order to identify reduced models of structures including porous materials for example;
- the development of an hybrid experimental/numerical modeling strategy to build efficient vibroacoustic reduced models.

8. ACKNOWLEDGMENTS

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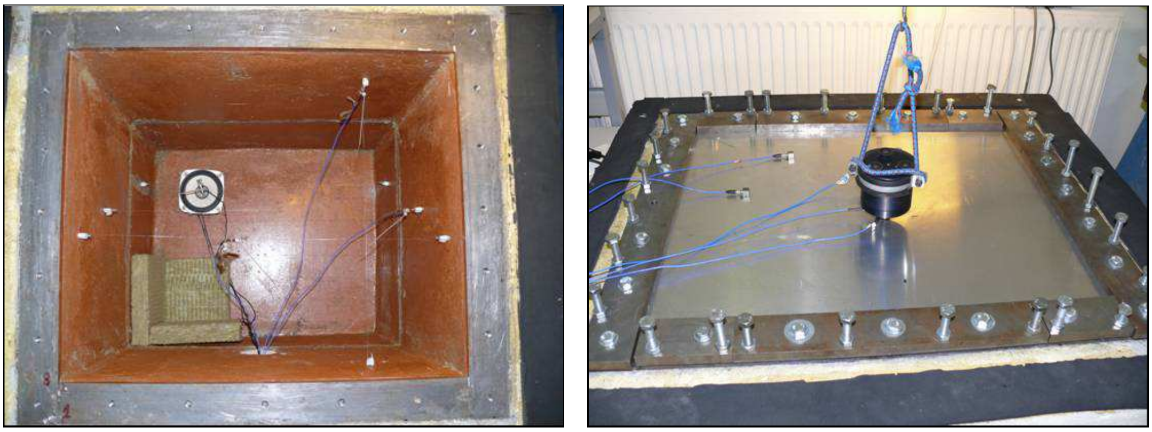

Figure 1. Views of the experimental setup (LVA-INSA Lyon)

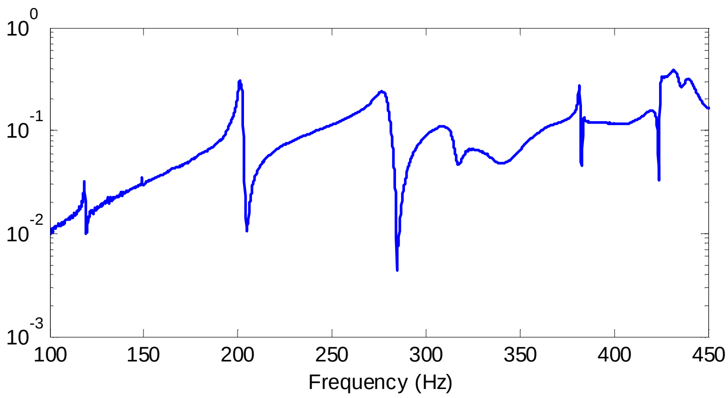

Figure 2. Identification of transfer function between acoustic volume acceleration and input voltage from reciprocity

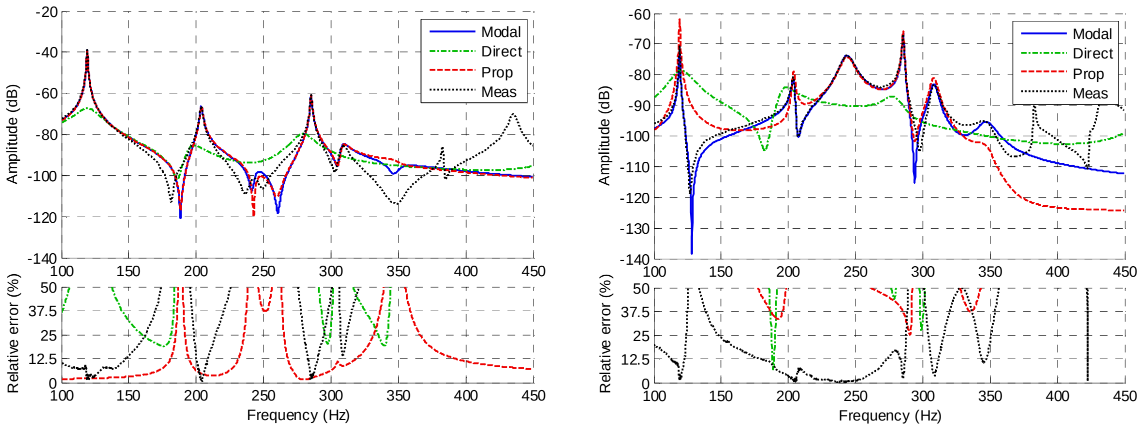

Figure 3. Impact of properness enforcement on reconstructed FRFs (left: FRFs1-s1, right: FRFs1-a6, s=structural, a=acoustic)

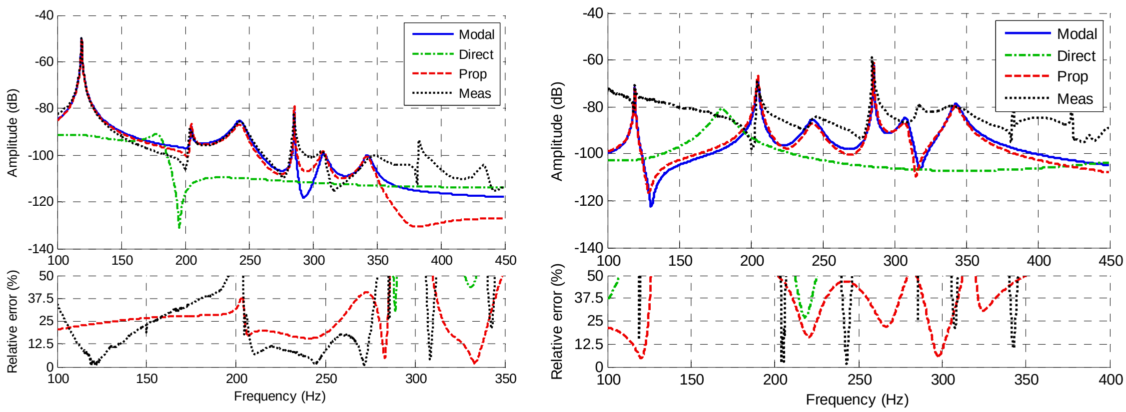

Figure 4. Impact of properness enforcement on reconstructed FRFs (left: FRFa6-s1, right: FRFa6-a6, s=structural, a=acoustic)