Network Characterization of Lattice-Based Modular Robots with Neighbor-to-Neighbor Communications

André Naz*, Benoît Piranda*, Thadeu Tucci*,
Seth Copen Goldstein** and Julien Bourgeois*

*FEMTO-ST Institute, UMR CNRS 6174, Univ. Bourgogne Franche-Comte (UBFC)
{andre.naz, benoit.piranda, thadeu.tucci, julien.bourgeois}@femto-st.fr

**School of Computer Science, Carnegie Mellon University
seth@cs.cmu.edu

Abstract

Modular robots form autonomous distributed systems in which modules use communications to coordinate their activities in order to achieve common goals. The complexity of distributed algorithms is generally expressed as a function of network properties, e.g., the number of nodes, the number of links and the radius/diameter of the system. In this paper, we characterize the networks of some lattice-based modular robots which use only neighbor-to-neighbor communications. We demonstrate that they form sparse and large-diameter networks. Additionally, we provide tight bounds for the radius and the diameter of these networks. We also show that, because of the huge diameter and the huge average distance of massive-scale lattice-based networks, complex distributed algorithms for programmable matter pose a significant design challenge. Indeed, communications over a large number of hops cause, for instance, latency and reliability issues.

1 Introduction

Modular robots form autonomous distributed systems in which modules communicate with each other to coordinate their activities in order to achieve common goals. In this paper, we focus our attention on lattice-based modular robots composed of identical modules that communicate together using only neighbor-to-neighbor communications. In lattice-based modular robots, modules are arranged in some regular 2-dimensional or 3-dimensional lattice structure. Modules are connected to their immediate neighbors in the lattice. We consider different kinds of lattices, namely the square, the hexagonal, the simple cubic and the face-centered cubic lattices (see Figure 1). In the neighbor-to-neighbor communication model, modules communicate only with adjacent modules. This communication model is fundamentally different than the global communication model where all modules can directly communicate together through a global bus. This approach works well in small networks but it is not scalable. Indeed, the number of hosts a bus can support is limited and packet collisions may frequently occur. Some hybrid approaches have been proposed but they are not common in modular robotics.

The considered class of modular robots captures a wide variety of existing systems, e.g., the Telecubes [1], the Miche [2] and the Distributed Flight Array [3] modular robots, some of the self-assembling systems used in [4] and the modular robotic systems developed in the Smart Blocks [5] and the Claytronics [6, 7] projects. As this work is part of the Smart Blocks and the Claytronics projects, we illustrate it using the modular robots designed in these projects, namely the Smart Blocks, the millimeter-scale

2D Catoms [8], the Blinky Blocks [9] and the 3D Catoms [10] (see Figure 1). These modular robots are respectively arranged in the square, the hexagonal, the simple cubic, and the face-centered cubic lattice.

The Smart Blocks and the Claytronics projects propose some interesting applications based on large-scale modular robotic systems. The Smart Blocks project aims to build a large distributed modular system to convey small and fragile objects, by attaching many modules together, each one equipped with a conveyance surface. The goal of the Claytronics project is to use up to millions of modules to build programmable matter, i.e., matter that can change its physical properties in response to external and programmed events.

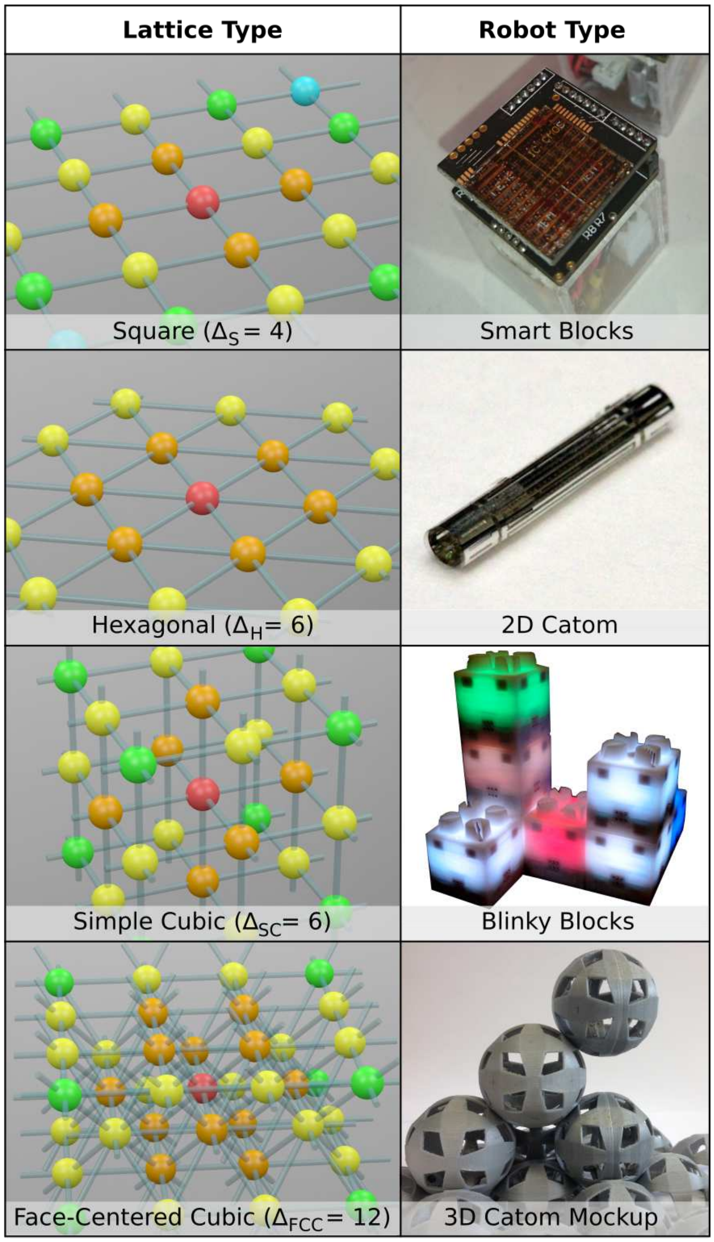

Figure 1: The different arrangement lattices considered in this paper associated with the modular robots used to illustrate our work. For a lattice L, Δ_L denotes its coordination number, i.e, the maximum number of modules to which a module can be connected.

Communication is central to module coordination. Message and time complexities of distributed algorithms are generally expressed as a function of network properties (e.g., number of nodes, number of links, node degree, radius/diameter of the system). Many algorithms target a specific class of networks. For instance, some algorithms are more efficient in sparse networks than in dense networks (e.g, the virtual coordinate-based routing protocol in [11]). Moreover, the diameter indicates the number of hops that is required to broadcast information through the whole system. Many distributed algorithms have a worst-case time complexity that is linear to the diameter (e.g., leader election algorithms [12, 13]) or a precision that decreases with the number of hops that messages have to travel (e.g., time synchronization protocol [14]). Thus, it is crucial to take into account the network properties in order to design and choose appropriate algorithms, especially in large-scale systems.

The contribution of this paper is to characterize the network of our class of modular robots based on their lattice type and the number of modules in the system. We demonstrate that these modular robots form sparse and large-diameter networks. Moreover, we provide tight bounds on the radius and the diameter of these networks. Note that we assume perfect alignment of the modules in the lattice. However, defects in the lattice which may cause unreliable and intermittent connections, will only make the network sparser and increase both its radius and its diameter. We also discuss our results and show that efficient and effective distributed algorithms for programmable matter and more generally for massive-scale lattice-based networks may be challenging to design. Indeed, communications over a large number of hops cause, for instance, latency and reliability issues. To the best of our knowledge, this paper is the first work to characterize the networks of our class of modular robots.

The rest of this paper is organized as follows. Section 2 presents the related work. Then, section 3 defines the system model and some terms. Afterwards, Section 4 characterizes the network density for our class of modular robots. Section 5 provides tight bounds of the radius and the diameter of the networks for our class of modular robots. Then, section 6 discusses our results. Finally, section 7 concludes this paper and section 8 suggests future research directions.

2 Related Work

To the best of our knowledge, network characterization has attracted little attention in the modular robotic community. In [15], the authors compare the efficiency of neighbor-to-neighbor communication and global communication. Based on experimentally validated models, the authors compare the information transmission time in different scenarios for systems composed of 10 to 1000 modules. As mentioned in section 1, global communication through a shared medium is less scalable with system size. Since we envision systems composed of millions of units, global communication is not an option. In this paper, we focus our attention on lattice-based modular robots in which modules communicate with each other using neighbor-to-neighbor communications. These modular robots form lattice-based networks.

As characterizing network properties is crucial for choosing appropriate algorithms and designing efficient new ones, graphs and networks have been extensively studied. Studies have been conducted on various graphs and networks, e.g., the Internet [16, 17, 18], the World Wide Web [19], sensor networks [20], small-world networks [21, 22], unit disk graphs [23], and lattice-based networks [22, 24, 25]. These studies are network specific. They are either measurement based (e.g., [16, 17, 19]), or purely theoretical using the intrinsic characteristics of the network (e.g., [20, 23, 24, 25]).

Due to the regular tiling of the space in lattices, lattice-based networks obey certain geometric rules that can be used to analyze these networks. In [22, 24], the authors study some lattice-based networks, but they only consider networks embedded in the square lattice and restrict their analyze to specific network topologies, e.g., the square, the ring, etc. Their results are not generalizable to other lattices and arbitrary network topologies. In [25], the author states that the average distance between nodes in lattice networks is on the order of $n^{\frac{1}{D_L}}$, where n is the number of nodes and D_L is the dimension of the considered lattice.

In this paper, we consider lattice-based networks embedded in any of the square, hexagonal, simple-cubic and face-centered lattices. We show that these networks are sparse and large diameter. Moreover, we provide tight lower and upper bounds for the radius and the diameter of these networks.

3 System Model and Definitions

In this paper, we consider lattice-based modular robots with neighbor-to-neighbor communications. In lattice-based modular robots, modules are arranged in some regular 2-dimensional or 3-dimensional lattice L. Here, we consider the Square (S), the Hexagonal (H), the Simple Cubic (SC) and the Face-Centered Cubic (FCC) lattices. Modules can only occupy a set of discrete positions defined by L. Note that modular robots may contain holes, i.e., some positions of L may be unoccupied. Because we assume neighbor-to-neighbor communications, L also defines the module connectivity: Modules can directly communicate only with their immediate neighbors in L. D_L denotes the dimension of L and Δ_L its coordination number, i.e, the maximum number of modules to which a module can be connected.

Arbitrarily arranged modular robotic systems form lattice-based networks that can be modeled by connected, undirected, unweighted and lattice-based graphs $G = (V, E)$, where V is the set of vertices (representing the modules), E the set of edges (representing the connections), $|V| = n$, the number of vertices and $|E| = m$, the number of edges. $\delta(v_i)$ denotes v_i's degree, i.e., the number of vertices to which v_i is connected. $d(v_i, v_j)$ refers to the distance between vertices v_i and v_j, i.e., the number of edges on a shortest path between v_i and v_j. The radius, r, and the diameter, d, of G are respectively defined as $r = \min_{v_i \in V} \max_{v_j \in V} d(v_i, v_j)$ and $d = \max_{v_i \in V} \max_{v_j \in V} d(v_i, v_j)$.

Notice that we assume perfect alignment of the modules in the lattice. However, defects in the lattice which may cause unreliable and intermittent connections, will only make the network sparser and increase both its radius and its diameter.

We now define some specific graphs used in this paper. Let V_L be the infinite set of vertices representing the infinite set of positions in L. $L\text{-}Sphere(v_c, r)$ is a sphere embedded in L, where vertex v_c is the center of the sphere and $r \in \mathbb{N}$ its radius. It contains the set of vertices in V_L whose distance from v_c is equal to r:

$$L\text{-}Sphere(v_c, r) = \{v_i \in V_L \mid d(v_i, v_c) = r\} \tag{1}$$

$L\text{-}Ball(v_c, r)$ is a ball embedded in L, where v_c the center of the ball and $r \in \mathbb{N}$ its radius. It contains the set of vertices in V_L whose distance from v_c is less than or equal to r:

$$L\text{-}Ball(v_c, r) = \{v_i \in V_L \mid d(v_i, v_c) \leq r\} \tag{2}$$

$$= \bigcup_{i=0}^{r} L\text{-}Sphere(v_c, i) \tag{3}$$

By abuse of notation, L-*Sphere* and L-*Ball* can respectively refer to sphere and ball graphs embedded in L where the connectivity between vertices is induced by the lattice structure of L. $L\text{-}Sphere(r)$ and $L\text{-}Ball(r)$ respectively refer to a sphere and a ball of radius r in the lattice L. In all the illustrations of this paper, $L\text{-}Sphere(r)$ are gradually colored from red to blue according to the value of r.

4 Network Density

In this section, we show that the networks formed by our class of modular robots are all sparse.

Corollary 1. Let $G = (V, E)$ be the network graph of an arbitrarily arranged modular robotic system that fits the model described in section 3. The vertex degree, $\delta(v_i)$, of any vertex $v_i \in V$ is bounded by:

$$0 \leq \delta(v_i) \leq \Delta_L \tag{4}$$

Lemma 1. Let $G = (V, E)$ be the network graph of an arbitrarily arranged modular robotic system that fits the model described in section 3. The number of edges of G, m, is bounded as follows:

$$n - 1 \leq m \leq n\Delta_L \tag{5}$$

Proof. **Lower Bound.** A connected graph must have at least n-1 edges [22].
Upper Bound. Because of Corollary 1, every module cannot be connected to more than Δ_L others. Thus, the number of edges of G is upper-bounded by $n\Delta_L$. Note that a tighter upper bound can be established by considering the lattice structure of L.

Theorem 1. Let $G = (V, E)$ be the network graph of an arbitrarily arranged modular robotic system that fits the model described in section 3. If $|V| = n$ is large, then G is a sparse graph, i.e., $m \ll n^2$.

Proof. If n is large, then $\Delta_L \ll n$. Thus, we have $n\Delta_L \ll n^2$. Then, because of Lemma 1, we obtain $m \ll n^2$.

5 Network Radius and Diameter

In this section, we establish tight lower and upper bounds of the radius and the diameter of the networks of our class of modular robots.

5.1 Preliminary Materials

This section presents some preliminary results used in the computations and the demonstrations of the radius and the diameter bounds of modular robot networks. We recall that V_L is the infinite set of vertices representing the set of positions in the lattice L.

Corollary 2. $\forall v_c \in V_L,\ \forall r \in \mathbb{N},\ L\text{-}Ball(v_c, r)$ is centrally symmetric: The reflection v_j of every vertex v_i at distance $d(v_i, v_c) = k$ through v_c is also at distance k from v_c and $d(v_i, v_j) = 2k$.

Proof. Let $L\text{-}Ball(v_c, 1)$ be the ball of radius 1 and v_c its center. All the vertices except v_c are at distance 1 from v_c. Along every axis of the lattice L, two vertices, v_1 and v_2, are connected to v_c, one in each direction. These two vertices are symmetric through v_c, at distance 1 from v_c and at distance 2 from each other.

Let $L\text{-}Ball(v_c, r)$ be the ball of radius r and v_c its center. We assume that $L\text{-}Ball(v_c, r)$ is centrally symmetric. Let $L\text{-}Ball(v_c, r + 1)$ be the ball of radius $r + 1$ with v_c its center. By construction, $L\text{-}Ball(v_c, r + 1)$ is obtained from $L\text{-}Ball(v_c, r)$ by adding all the vertices at distance $r + 1$ from v_c. Let us consider v_3 and v_4 in $L\text{-}Ball(v_c, r)$ such that v_3 and v_4 are symmetric through v_c and $d(v_3, v_4) = 2r$. In order to construct $L\text{-}Ball(v_c, r + 1)$, we add to v_3 and v_4 two vertices v_5 and v_6 on the same axis but in the opposite direction such that $d(v_5, v_c) = d(v_6, v_c) = r + 1$. v_5 and v_6 are symmetric through v_c. Moreover, there is no shortcut between v_5 and v_6, thus, $d(v_5, v_6) = 1 + d(v_3, v_4) + 1 = 2 + 2r = 2(r + 1)$. Thus, $L\text{-}Ball(v_c, r + 1)$ is centrally symmetric.

By induction, $\forall v_c \in V_L,\ \forall r \in \mathbb{N},\ L\text{-}Ball(v_c, r)$ is centrally symmetric.

Lemma 2. $\forall v_c \in V_L,\ \forall r \in \mathbb{N}$, the diameter, d, of $L\text{-}Ball(v_c, r)$ is equal to $2r$.

Proof. As stated in Corollary 2, $L\text{-}Ball(v_c, r)$ is centrally symmetric. Thus, $\forall v_i \in L\text{-}Ball(v_c, r)$ such that $d(v_i, v_c) = r$, $\exists v_j \in L\text{-}Ball(v_c, r)$ with $d(v_i, v_j) = 2r$. By construction, $\nexists v_i \in L\text{-}Ball(v_c, r),\ d(v_i, v_c) > r$. As a consequence, the diameter of $L\text{-}Ball(v_c, r)$, i.e., the largest distance between any two vertices is equal to $d = 2r$.

Corollary 3. $\forall v_c \in V_L,\ \forall r \in \mathbb{N},\ L\text{-}Ball(v_c, r)$ is the minimum-radius and minimum-diameter existing graph composed of $n_{L\text{-}Ball}(v_c, r) = |\ L\text{-}Ball(v_c, r)|$ vertices in L.

Proof. By construction, in $L\text{-}Ball(v_c, r)$ all the positions of the lattice L at distance less than or equal to r from v_c are occupied. Thus, if we remove a vertex v_1 and add it to an empty place adjacent to a full one (the system should remain connected) occupied by the vertex v_2, the new location of v_1 must be at distance $r + 1$ from v_c. Moreover, every vertex would be at distance $r + 1$ or more from at least one other vertex. Thus, the radius of the graph would be equal to $r + 1$. Moreover, because $L\text{-}Ball(v_c, r)$ is centrally symmetric (See Corollary 2), $\exists v_3 \in L\text{-}Ball(v_c, r),\ d(v_2, v_3) = 2r$. Because of Lemma 2, $d(v_2, v_3)$ is the diameter of $L\text{-}Ball(v_c, r)$. Since there is no shortcut between v_1 and v_3 in its new location, $d(v_1, v_3) = d(v_2, v_3) + 1 = 2r + 1$. Thus, the diameter of the graph would be equal to $2r + 1$.

5.2 Radius and Diameter Bounds

Theorem 2. Let $G = (V, E)$ be the network graph of an arbitrarily arranged modular robotic system that fits the model described in section 3. Let $L\text{-}Ball(r_b)$ and $L\text{-}Ball(r_b+1)$ be two ball graphs embedded in L such that the number of vertices of G, n, is between the number of vertices of these two balls, i.e., $n_{L\text{-}Ball}(r_b) \leq n < n_{L\text{-}Ball}(r_b+1)$. The radius, r, and the diameter, d, of G are tightly bounded as follows:

$$r_b \leq r \leq \lfloor \frac{n-1}{2} \rfloor \tag{6}$$

$$2r_b \leq d \leq n-1 \tag{7}$$

Proof. **Upper Bound.** In a connected graph, any two vertices are at most separated by all the others. In such a graph, the n vertices form a line of $n-1$ edges. Thus, the largest distance between any two vertices, i.e., the diameter of G, is at most equal to $n-1$ edges. The radius of G is at most equal to the half of that line, i.e, $r \leq \lfloor \frac{n-1}{2} \rfloor$.
Lower Bound. Because of Corollary 3, $L\text{-}Ball(r_b)$ is the minimum-radius and minimum-diameter graph composed of $n_{L\text{-}Ball}(r_b)$ vertices. Thus, with n vertices, G has a radius at least equal to r_b and a diameter at least equal to the diameter of $L\text{-}Ball(r_b)$, which is, because of Lemma 2, equal to $2r_b$.

In the rest of this section, we establish the formula to compute the exact radius of an *L-Ball* according to its number of vertices in the different lattices we consider.

5.2.1 Systems in Two Dimensions: The Square and Hexagonal Lattices

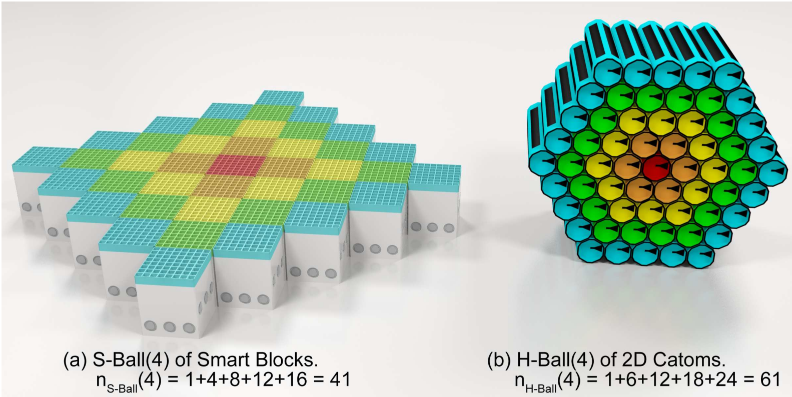

Figure 2: An *S-Ball*(4) and an *H-Ball*(4) with color gradient from the center of the ball.

In this section, we compute the exact radius of an *L-Ball* given the number of vertices it has for the case of two-dimensional systems embedded in the Square (S) and Hexagonal (H) lattices. Figure 2 depicts an *S-Ball* and an *H-Ball* of radius 4, respectively composed of Smart Blocks and 2D Catoms.

Lemma 3. In the square and the hexagonal lattices, the number of vertices in a sphere of radius $r \geq 1$, $n_{L\text{-}Sphere}(r, \Delta_L)$, can be computed by:

$$n_{L\text{-}Sphere}(r, \Delta_L) = r\Delta_L \tag{8}$$

Proof. As illustrated in Figure 2, in the square and the hexagonal lattices, a sphere of radius $r \geq 1$ is composed of Δ_L segments of length r modules. Consequently, the number of vertices is equal to $r\Delta_L$.

Theorem 3. In the square and the hexagonal lattices, the radius of a ball composed of $n \geq 1$ vertices, $r_{L\text{-}Ball}(n, \Delta_L)$, can be computed by:

$$r_{L\text{-}Ball}(n, \Delta_L) = \frac{1}{2}\left(\sqrt{1 + \frac{8(n-1)}{\Delta_L}} - 1\right) \tag{9}$$

Proof. By definition, $L\text{-}Ball(r)$ is the union of all the $L\text{-}Sphere(i)$ for i ranging from 0 to r. Thus, in the square and the hexagonal lattices, for $r \geq 1$, the number of vertices in an $L\text{-}Ball(r)$, $n_{L\text{-}Ball}(r, \Delta_L)$, can be computed as follows:

$$n_{L\text{-}Ball}(r, \Delta_L) = \sum_{i=0}^{r} n_{L\text{-}Sphere}(i, \Delta_L) \tag{10}$$

$$= 1 + \sum_{i=1}^{r} i\Delta_L \tag{11}$$

$$= \frac{1}{2}r^2\Delta_L + \frac{1}{2}r\Delta_L + 1 \tag{12}$$

To obtain Equation 9, we solve Equation 12 for r and keep only the positive root.

5.2.2 Systems in Three Dimensions: The Simple Cubic and Face-Centered Cubic Lattices

In this section, we compute the exact radius of an *L-Ball* given the number of vertices it contains for the case of three-dimensional systems embedded in the Simple Cubic (SC) and Face-Centered Cubic (FCC) lattices. Figures 3 and 4 depict the *SC-Ball* and the *FCC-Ball* of radius 2, respectively composed of Blinky Blocks and 3D Catoms. Both systems can be decomposed into horizontal layers.

The Simple Cubic Lattice

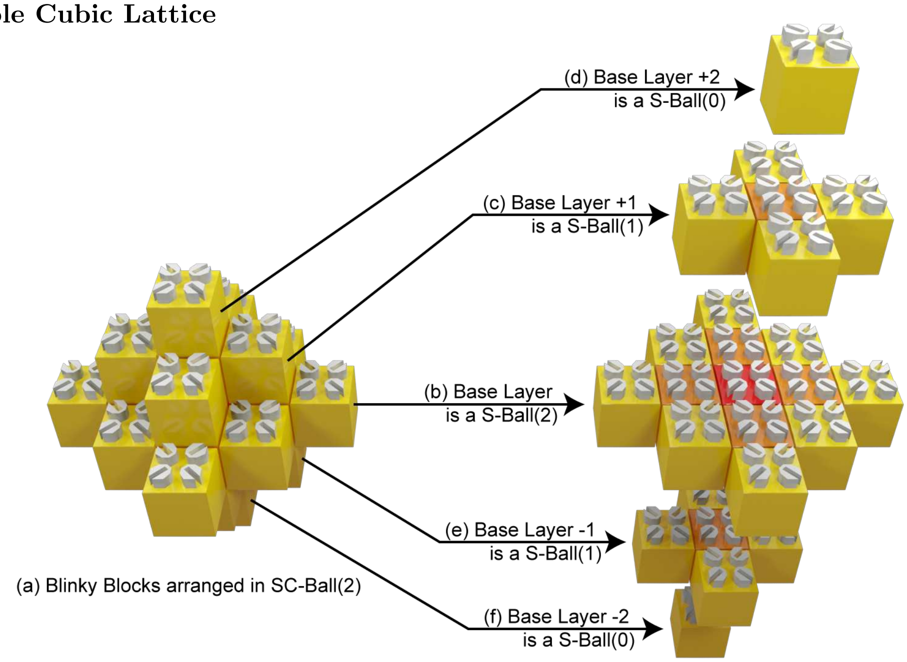

Figure 3: An $SC\text{-}Ball(2)$ of Blinky Blocks and its decomposition in horizontal layers with color gradient from the center of the ball.

Lemma 4. In the simple cubic lattice, the number of vertices in a sphere of radius $r \geq 1$, $n_{SC\text{-}Sphere}(r)$, can be computed by:

$$n_{SC\text{-}Sphere}(r) = n_{S\text{-}Sphere}(r) + 2\sum_{i=0}^{r-1} n_{S\text{-}Sphere}(i) \tag{13}$$

$$= 2(2r^2 + 1) \tag{14}$$

Proof. As illustrated in Figure 3, a sphere of radius r in the simple cubic lattice can be decomposed into $2r+1$ horizontal *S-Spheres* of different radii. Equation 13 is obtained by summing up all the size of the *S-Spheres*.

Theorem 4. In the simple-cubic lattice, the radius of a ball composed of $n \geq 1$ vertices, $r_{SC\text{-}Ball}(n)$, can be computed by:

$$r_{SC\text{-}Ball}(n) = \frac{1}{2}\left(\frac{(\sqrt{3}\sqrt{243n^2+125}+27n)^{\frac{1}{3}}}{3^{\frac{2}{3}}} - \frac{5}{3^{\frac{1}{3}}(\sqrt{3}\sqrt{243n^2+125}+27n)^{\frac{1}{3}}} - 1\right) \tag{15}$$

Proof. By definition, *L-Ball*(r) is the union of all the *L-Sphere*(i) for i ranging from 0 to r. Thus, for $r \geq 1$, the number of vertices in an *SC-Ball*(r), $n_{SC\text{-}Ball}(r)$, can be computed as follows:

$$n_{SC\text{-}Ball}(r) = \sum_{i=0}^{r} n_{SC\text{-}Sphere}(i) \tag{16}$$

$$= 1 + \sum_{i=1}^{r} 2(2i^2+1) \tag{17}$$

$$= \frac{4}{3}r^3 + 2r^2 + \frac{8}{3}r + 1 \tag{18}$$

To obtain Equation 15, we solve Equation 18 for r and keep only the real root.

The Face-Centered Cubic Lattice

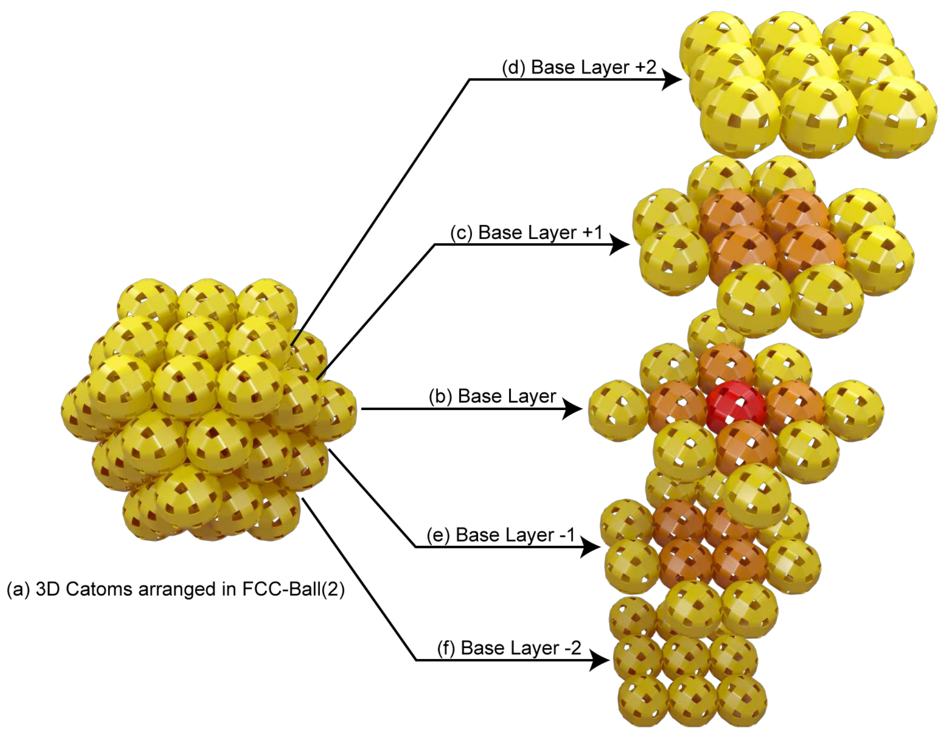

Figure 4: An *FCC-Ball*(2) of 3D Catoms and its decomposition in horizontal layers with color gradient from the center of the ball.

Lemma 5. In the face-centered cubic lattice, the number of vertices in a sphere of radius $r \geq 1$, $n_{FCC\text{-}Sphere}(r)$, can be computed by:

$$\begin{aligned} n_{FCC\text{-}Sphere}(r) &= 4r + 2(r+1)^2 + 2(r-1)4r & (19) \\ &= 2(5r^2 + 1) & (20) \end{aligned}$$

Proof. As shown in Figure 4, a sphere of radius r in the face-centered cubic lattice can be decomposed into $2r + 1$ horizontal layers. The base layer is an *S-Sphere*(r) and contains $4r$ vertices. The bottom and the top layers both contain $(r + 1)^2$ vertices. The $2(r - 1)$ other layers contain $4r$ vertices each. Equation 19 is obtained by summing up the number or vertices of each layer.

Theorem 5. In the face-centered cubic lattice, the radius of a ball of $n \geq 1$ vertices, $r_{FCC\text{-}Ball}(n)$, can be computed by:

$$r_{FCC\text{-}Ball}(n) = \frac{1}{2}\left(\frac{(\sqrt{15}\sqrt{4860n^2 + 343} + 270n)^{\frac{1}{3}}}{15^{\frac{2}{3}}} - \frac{7}{15^{\frac{1}{3}}(\sqrt{15}\sqrt{4860n^2 + 343} + 270n)^{\frac{1}{3}}} - 1\right) \quad (21)$$

Proof. By definition, *L-Ball*(r) is the union of all the *L-Sphere*(i) for i ranging from 0 to r. Thus, for $r \geq 1$, the number of vertices in an *FCC-Ball*(r), $n_{FCC\text{-}Ball}(r)$, can be computed as follows:

$$\begin{aligned} n_{FCC\text{-}Ball}(r) &= \sum_{i=0}^{r} n_{FCC\text{-}Sphere}(i) & (22) \\ &= 1 + \sum_{i=1}^{r} 2(5i^2 + 1) & (23) \\ &= \frac{10}{3}r^3 + 5r^2 + \frac{11}{3}r + 1 & (24) \end{aligned}$$

To obtain Equation 21, we solve Equation 24 for r and keep only the real root.

6 Discussion

In this paper, we demonstrated some properties of networks of lattice-based modular robots with neighbor-to-neighbor communications. As shown in [26], this class of modular robots is particularly suitable to design programmable matter, i.e., matter that can change its physical properties in response to some events. In our vision, programmable matter will be composed of up to millions of modules [6, 7]. This section discusses our theoretical results and the impact on the efficiency of distributed algorithms for programmable matter and more generally for massive-scale lattice-based networks.

More precisely, we compare lattice-based networks to small-world networks [21] (e.g., the Internet network [18]) and to wireless ad-hoc networks (e.g., wireless sensor networks, multi-robot networks, etc.). Since many large real-world networks are small-world networks, it is legitimate to consider them for comparison. Wireless ad-hoc networks are highly spatially dependent, like our class of networks. Indeed, in wireless ad-hoc networks, nodes can only communicate with some neighboring nodes within some limited range. Note that wireless ad-hoc networks can fall in the class of lattice-based networks if they are deployed in a lattice structure.

We demonstrated that lattice-based networks are sparse networks (i.e., $m \ll n^2$). Because of Lemma 1 and because Δ_L is bounded by a constant for all the lattices we consider, the number of edges is $\Theta(n)$. Thus, lattice-based networks are sparser than small-world networks that have $\Omega(nlog(n))$ edges [21]. Wireless ad-hoc networks can be sparse or dense depending on the deployment environment (area/volume, obstacles, etc.), the deployment density and the node communication range.

In regular lattice networks, the typical distance between two nodes is $\sim n^{\frac{1}{D_L}}$ [25]. Thus, in lattice-based networks, i.e., lattice networks with potential holes, this distance is lower bounded by $\Omega(n^{\frac{1}{D_L}})$, while in small-world networks, this distance is $\sim log(n)$ [25]. Small-world networks have typically short distances between arbitrary pairs of nodes due to the presence of few long-range edges. As a consequence, small-world networks tend to have a small diameter. In lattice-based and sparse wireless ad-hoc networks, such long-range edges do not exist. Thus, these networks tend to have a larger average distance and a larger diameter. These phenomena are accentuated as the number of nodes in the network increases. Because programmable matter is formed of millions of modules [6, 7], the networks we consider are much larger and thus have a larger diameter than usual wireless ad-hoc networks that are typically composed of dozens of nodes to tens of thousands of nodes. We demonstrated that the radius and the diameter of lattice-based networks are lower bounded by $\Omega(\sqrt[3]{n})$ (Equations 9, 15 and 21 are all $\Omega(\sqrt[3]{n})$).

Studies indicate that the diameter of the Internet is around 30 hops [16, 17]. This is corroborated by the suggested values for Time-To-Live (TTL) for Internet Protocol (IP) packets. The TTL should be twice the diameter of the Internet [27] and the actual value recommended is 64 [28, 29]. As shown in Figure 5, systems with a million 3D Catoms have a diameter of at least 132 hops, while systems with 100 million 3D Catoms have a diameter of at least 620 hops. Blinky Blocks systems have similarly large diameter, e.g., a 40,000 Blinky Blocks system has a diameter greater than 30 hops. Thus, a 40,000 Blinky Blocks system which fits in a $1.4m^3$ cube, would have a diameter larger than the entire Internet that spans the whole world.

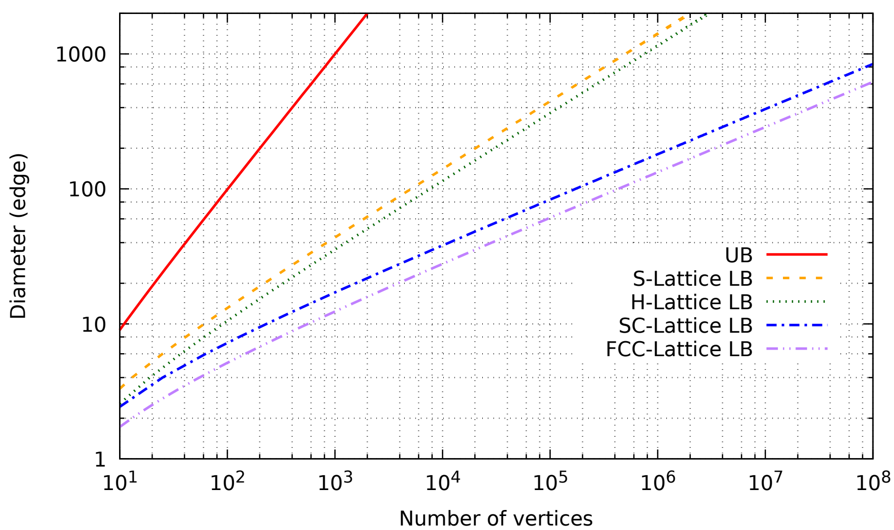

Figure 5: Diameter bounds versus the number of vertices in the network graph for the different considered lattices. The terms "LB" and "UB" respectively stand for "lower bound" and "upper bound".

It is crucial to take into account the large diameter and large average distance to design efficient and effective distributed algorithms for large-scale modular robotic systems. For example, communication over a large number of hops causes latency and reliability issues. Assuming link faults are independent and identically distributed, the probability that a multi-hop communication fails increases exponentially with the number of hops [30]. Let us consider time synchronization and data sharing algorithms. These algorithms are required for real-time responsive programmable matter and to distribute, store and access geometry data for self-reconfiguration. However, these algorithms are challenging to design for such huge diameter and huge average distance systems. Unpredictable delays (due, for example, to queueing or retransmissions) accumulate every hop, which tends to disturb the time synchronization process and decrease the achievable synchronization precision. Moreover, in data sharing algorithms, lookup latency may be extremely long if it involves messages that have to travel a large number of hops.

7 Conclusions

In this paper, we characterize the networks of some lattice-based modular robots which use only neighbor-to-neighbor communications. We demonstrate that they form sparse and large-diameter networks. Moreover, we provide tight bounds of the radius and the diameter of these networks. Our results are generalizable to other networks embedded in the considered lattices. We also show that, it may be challenging to design efficient distributed algorithms for massive-scale lattice-based networks because of their huge diameter and their huge average distance.

8 Future Work

In future work, we will take into account the properties of huge diameter and huge average distance of massive-scale lattice-based networks in order to design efficient and effective distributed algorithms for programmable matter.

In addition, we will experimentally evaluate the practical impact of the diameter and the average distance values on the performance of some distributed algorithms executed in our class of modular robotic systems.

As previously mentioned, different communication models exist in modular robotic systems. In large-scale systems, the global communication model where all modules can directly communicate together through a global bus is not an option because the number of hosts a bus can support is limited by packet collisions. As shown in this paper, using the neighbor-to-neighbor communication model in large-scale systems implies a large diameter and a large average distance. In future work, we plan to study the network properties of modular robotic systems that use hybrid communication models in which modules communicate together through small buses, each one with a few participating modules, as proposed in [31].

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