Post-Prognostics Decision for Optimizing the Commitment of Fuel Cell Systems *†

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Abstract

In a post-prognostics decision context, this paper addresses the problem of maximizing the useful life of a platform composed of several parallel machines under service constraint. Application on multi-stack fuel cell systems is considered. In order to propose a solution to the insufficient durability of fuel cells, the purpose is to define a commitment strategy by determining at each time the contribution of each fuel cell stack to the global output so as to reach the demand as long as possible. Two algorithms making use of convex optimization are proposed to cope with the assignment problem. First one is based on the Mirror-prox for Saddle Points method and second one uses the Lasso (Least Absolute Shrinkage and Selection Operator) principle. Results based on computational experiments assess the efficiency of these two approaches in comparison with an intuitive resolution performing successive basic convex projections onto the sets of constraints associated to the optimization problem.

Keywords: Decision making, Post-prognostics decision, PHM, Fuel cell, Convex optimization

1 Introduction and related work

In the context of the decline of fossil fuel resources, the use of fuel cells appears to be of growing interest as a potential alternative to conventional power systems Jouin et al. (2013). Fuel cells can be used in many applications, such as stationary ones for domestic use, but also in transportation and portable power applications Borup et al. (2007). Fuel cells suffer however from insufficient durability. In fact, their lifetime reaches between 1500 and 3000 hours, whereas 5000 hours are required for transportation applications and up to 100000 hours for stationary ones. Improvement of their performance, reliability and lifetime is then an important challenge Borup et al. (2007), for which techniques of

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Prognostics and Health Management (*PHM*) can help. It has been pointed out by Jouin et al. (2013) that researches in PHM dealing with fuel cells have been mainly focused on data acquisition and data processing. Less attention has been paid to condition assessment and diagnostics and very few works address prognostics and decision making. Papers taking into account the decision part propose furthermore only corrective actions (see Bosco and Fronk (2000) and Wells and Parr (2004)), for which physical parameters (such as inlet and outlet gas flows, pressures and temperatures, single cell and stacks voltages or current) are controlled to master each fuel cell operating conditions as accurately as possible. These corrective actions correspond to real-time control (from nanoseconds to seconds), necessary to compensate the natural fluctuation of fuel cells parameters and to avoid too early irreversible degradations. At each time it allows also to set the operating current to meet the needs in power for each fuel cell. Decision making addressed in this paper differs from the studies proposed so far. Larger scale of time (hours to weeks) is indeed considered and decision comes within the scope of Prognostic Decision Making (PDM), which aims at choosing an appropriate system configuration Balaban and Alonso (2012). The system considered here is composed of several fuel cells, used in parallel to provide a global power output. The problem is to provide the power output value for each fuel cell as a function of time, on the basis of a global power demand. Target application considered here is based on stationary power generation for domestic usage, also known as micro combined heat and power (micro-CHP).

In order to deliver suitable power outputs, fuel cells are used in the form of stacks, composed of many individual connected cells. Each stack is supposed to be independent, but the multi-stack fuel cell system has to deliver a given global power output based on a need of energy. At each time, the total provided power output is the sum of each output of the stacks that are currently running. Each fuel cell stack is able to deliver an output that can vary continuously and take any value within a given interval. The optimization problem consists in determining the appropriate output for each fuel cell stack during the whole production horizon. All the stacks are not supposed to be running at each time if the target output can be reached by using only a subset of them. All the stacks may moreover not be always available if their end of life has been reached. Considering a global needed power output, the multi-stack system useful life depends not only on each stack useful life, but also on both the schedule and the operating condition settings that define the contribution of each stack over time. The same statement applies to batteries in a health management context. Saha et al. Saha et al. (2012) have for instance addressed the maximization of the battery charge used while constraining the probability of a battery shut off in flight for electric unmanned aerial vehicles. Predictions on remaining battery life are used to optimize mission plans without exceeding the available battery charge. In a same way, we propose to use prognostics results in the form of *RUL* to maximize the global useful life of a multi-stack fuel cell system under service constraint.

A similar problem has been addressed in Herr et al. (2014a) and Herr et al. (2014b), where the purpose was to define a schedule of machines that maximizes the production horizon, based on the knowledge of each machine remaining useful life (*RUL*) in a *PHM* framework. In these studies, machine throughputs have been considered to be in a discrete domain. It has been shown in Herr et al. (2014b) that optimal solutions can be found in limited time only for small size instances considering a very limited number of machines, very few throughput values and short production horizons. An other study considering this time machines whose performances can vary continuously between two bounds has been proposed in Herr et al. (2015). The considered model has been built to fit the fuel cells behavior, but the proposed resolution approach gives suboptimal solutions and is limited to systems of reasonable size. In order to overcome these two limitations, we propose in this paper to change totally the paradigm of the resolution and to build the solutions globally on the whole production horizon. Contribution of each machine during its lifetime is considered as a whole and optimized on the whole horizon. Each machine contribution is determined through convex optimization, whose interest is to allow the solving of big optimization problems in limited time.

The considered scheduling problem is proposed to be addressed via optimizing a composite function subject to several constraints due to fuel cell intrinsic characteristics. Two different algorithms are developed and used to define the contribution of each fuel cell stack to the global output over the whole production horizon. First one is based on the Mirror Prox method proposed by Nemirovski (2004) as a variant of the Mirror Descent developed by Nemirovsky and Yudin (1983) to minimize a smooth convex function subject to convex constraints. Estimation of the variable is efficient in that it depends very little on its dimension. This is why these methods can be used to solve big optimization problems Beck and Teboulle (2003). Second resolution method is based on a penalization through an ℓ_1 norm, which has been extensively studied in many domains such as artificial intelligence for machine learning, statistics, image processing or data analysis Donoho and Elad (2003); Candès et al. (2008). We propose to use a variant of the Lasso (Least Absolute Shrinkage Operator) algorithm, proposed by Tibshirani (1996) as a method for sparse model selection in statistics.

The organization of the paper is as follows: the tackled problem is first described in Section 2, with a brief presentation of the application framework and the optimization problem. After a mathematical formulation of the problem, resolution methods are then described in Section 3. Efficiency of these methods is assessed through simulation results in Section 4. Conclusion and future work are finally given in Section 5.

2 Problem statement

The application addressed in this paper is based on a multi-stack fuel cell system which is supposed to meet energy requirements for domestic usage in a stationary power generation framework. This system is supposed to be composed of m fuel cell stacks M_j $(1 \leqslant j \leqslant m)$. All the stacks are supposed to be always supplied with raw material required for the energy conversion. They can be used simultaneously and independently from each other.

This corresponds to a parallel machines system, in which each machine is supposed to be able to deliver power outputs P_j that can vary continuously within a given power output range $[P\text{min}_j; P\text{max}_j]$. For each machine M_j $(1 \leqslant j \leqslant m)$, the minimal power output $P\text{min}_j$ is supposed to be strictly greater than zero and constant over time. The maximal power output $P\text{max}_j$ decreases with time when the machine M_j is used. The range of available power outputs depends then on the time t: for each machine M_j, $0 < P\min_j < P_j(t) < P\max_j(t)$. Useful life of each power output P_j, $RUL_j(P_j)$, is moreover limited by the decrease of $P\max_j$ of equation $P\max_j(t) = a_j \cdot t + P\max_j(0)$, with $a_j < 0$.

At each time, the global outcome $P\mathrm{tot}$ is the sum of each stack power contribution. During the whole production horizon, denoted H, this global outcome has to reach a given load demand $\sigma(t)$. In the stationary power generation framework considered here, this demand is supposed to be constant over time. Storage being not considered in this study, overproduction is lost. Stop-and-start of fuel cell stacks have moreover to be avoided as far as possible. Stopping and restarting a fuel cell can indeed induce considerable damage and lead to premature aging Borup et al. (2007). Change of power output during the use of fuel cell stacks is however still authorized.

Considering these assumptions, the point is to manage the system by defining the commitment of fuel cell stacks so as to reach the demand as long as possible. During the whole production horizon, the purpose is then to define at each time each stack contribution to the global power output.

3 Resolution

A mathematical formulation of the problem making use of convex elements is first defined. Two different convex resolution methods are then proposed to cope with the assignment problem. First one is based on the Mirror Prox method and second one makes use of the Lasso technique to optimize the commitment of machines.

3.1 Mathematical formulation

Let $f_j(t)$ ($1 \leqslant j \leqslant m$, $0 \leqslant t \leqslant T$) be the vector defining the evolution over time of the power output delivered by the machine M_j, with T the length of the decision horizon. Link between this decision horizon and the solution production horizon is clarified in Section 4.3. Contributions of all the machines are gathered together in a vector $F \in \mathbb{R}^{m(T+1)}$ such that:

$$F = [f_1(0), \dots, f_1(T), \dots, f_j(t), \dots f_m(0), \dots, f_m(T)] .$$

The general idea is to minimize a convex function subject to a set of constraints. The objective function aims at ensuring that the power demand is reached. At each time t ($0 \leqslant t \leqslant T$), it is about minimizing the difference between the global power output delivered by the set of machines and the demand $\sigma(t)$. This is expressed by Equation (1).

$$\min \quad \sigma(t) - \sum_{j=1}^{m} f_j(t) \qquad \forall\ 0 \leqslant t \leqslant T \tag{1}$$

Constraints on each function f_j relate to the definition domain of each contribution and to the limited availability of machines. At each time t, each machine contribution is either equal to zero or constrained between two bounds (see

Equation (2)), in accordance with the hypotheses detailed in the application framework. $f_j(t) = 0$ means that the machine M_j is not used at time t.

$$f_j(t) = 0 \quad \text{or} \quad fmin_j(t) \leqslant f_j(t) \leqslant fmax_j(t) \tag{2}$$

$$\forall\, 1 \leqslant\; j \leqslant\; m,\; \forall\, 0 \leqslant\; t \leqslant\; T$$

Each contribution f_j is constrained by the maximal power output decrease of the associated machine M_j, which expresses its limited availability. Evolution over time of this maximal power output, $fmax_j(t)$, is a function of the use of machine M_j, $f_j(t)$. Indeed, $fmax_j(t)$ evolves only if M_j is used, that is, if $f_j(t) > 0$. A first formulation is proposed in Equation (3), with a_j the speed associated to the maximal power output decrease.

$$fmax_j(t) = \begin{cases} fmax_j(t-1) + a_j & \text{if } f_j(t) > 0\,; \\ fmax_j(t-1) & \text{if } f_j(t) = 0 \end{cases} \tag{3}$$

$$\forall\, 1 \leqslant\; j \leqslant\; m,\; \forall\, 1 \leqslant\; t \leqslant\; T$$

Equations (2) and (3) being not convex, they can not be used as is within the proposed convex programming paradigm. A second formulation of the constraints is proposed in set of equations (4), which details the mathematical program associated to the optimization problem. This program does not respect the real evolution over time of the maximal power output that can be delivered by machines (see Equations (2) and (3)), but presents the advantages of being convex and thus consistent with the convex resolution methods proposed in next section. In the following, the machines behavior follows the simplified model depicted in Figure 1.

$$\begin{cases} \min \quad \sigma(t) - \displaystyle\sum_{j=1}^{m} f_j(t) \qquad \forall\, 0 \leqslant t \leqslant T & \text{(4a)} \\ \text{s.t.} \quad fmax_j(t) \leqslant fmax_j(t-1) + \mu \cdot a_j \cdot (f_j(t-1))^{\upsilon} & \text{(4b)} \\ \qquad\quad (\text{with } \mu \text{ and } \upsilon \in \mathbb{R}^{*+}) & \\ \qquad\quad \forall\, 1 \leqslant j \leqslant m,\; \forall 1 \leqslant t \leqslant T & \\ \text{with} \quad 0 \leqslant f_j(t) \leqslant fmax_j(t) & \text{(4c)} \\ \qquad\quad \forall 1 \leqslant j \leqslant m,\; \forall 0 \leqslant t \leqslant T & \end{cases}$$

3.2 A Mirror-Prox-based algorithm

The first resolution method is based on the Mirror Prox algorithm proposed by Nemirovski (2004) as a variant of the Mirror Descent. Both are based on the resolution of a primal-dual saddle point problem, which allows to take constraints into account. The purpose is to minimize a smooth convex function under constraints. The Mirror Descent algorithm makes use of a gradient descent to find the minimum of the considered function. A mirror function allows to transition from the primal space, where all the constraints of the problem are defined, to the dual space. The Mirror Prox method applies at each iteration two consecutive steps of Mirror Descent. A very instructive description of the Mirror Prox algorithm has been proposed by Bubeck (2014).

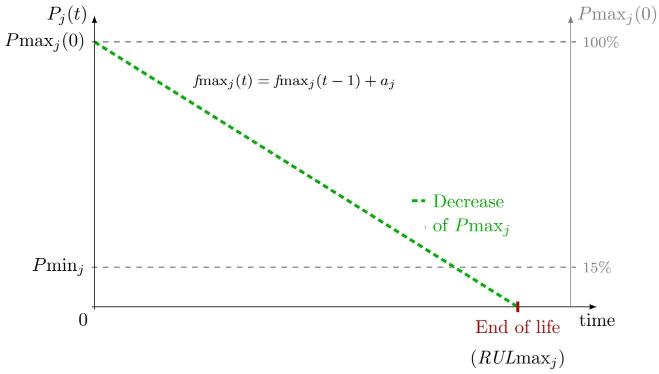

Figure 1: Simplified evolution of fuel cells characteristics

The objective function taken into account is detailed in equation (5). This proposed formulation includes directly the different constraints of the application. First part of the function, detailed in Equation (6), is aimed at satisfying the demand $\sigma(t)$ at each time t. Second one is used to control the evolution of each $f\!\max_j$ as a function of each associated contribution f_j (see equation (7)). The mirror function considered in the Mirror Prox formula is defined on $\mathbb{R}^{m(T+1)}$ by Equation (8).

$$\phi(F, f\!\max, \sigma) = \lambda_{dem} h_{dem}(F, \sigma) + \lambda_{slope} h_{slope}(F, f\!\max) \quad \text{with } \lambda_{dem}, \lambda_{slope} > 0 \tag{5}$$

$$h_{dem}(F, \sigma) = \sum_{t=0}^{T} \frac{1}{T+1} \exp\left(-\gamma \left(\sum_{j=1}^{m} f_j(t) - \sigma(t) \right) \right) \quad \text{with } \gamma > 0 \tag{6}$$

$$h_{slope}(F, f\!\max) = \sum_{t=1}^{T} \sum_{j=1}^{m} \exp\left(\delta \Big(f\!\max_j(t) - f\!\max_j(t-1) - \mu' a_j (f_j(t-1))^{\upsilon'} \Big) \right) \quad \text{with } \delta > 0,\ \mu' > 0,\ \upsilon' > 1 \tag{7}$$

$$\theta(F) = \sum_{t=0}^{T} \sum_{j=1}^{m} F \ln(F) \tag{8}$$

3.3 An algorithm based on the Lasso

The second resolution method makes use of a ℓ_1 penalization. The idea is the following : in order to maximize the production horizon, it would be better to use at each time only the machines strictly necessary to reach the demand. This means minimizing both the overproduction and the number of machines used at the same time, which allows to keep some potential for the end of the schedule and eventually to reach the demand for a longer time. This can be done by controlling the sparsity of the solution vector F. This vector has then the same structure as if the number of its nonzero component were constrained. The Lasso (Least Absolute Shrinkage and Selection Operator) algorithm proposed by Tibshirani (1996) allows to tackle such problems. It is a regularization technique performing simultaneously an estimation and a variable selection Tibshirani (1996). We propose to use the Adaptive Lasso, which associates the Lasso regularization technique with an adaptive weighted ℓ_1 penalty. This weighted penalization is used to adapt the penalization to the initial value of the vector F to optimize, allowing to accelerate the convergence of the method. This variant of the Lasso has been shown by Zou (2006) to give good results in comparison with other sparse modeling techniques.

For the considered optimization problem and following the notations introduced previously, the Lasso estimate is defined in Equation (9), with the weight w defined in Equation (10). Optimization of the solution F is done following this objective function under constraints. These constraints set the range of available power outputs and its evolution over time (see Equations (4b) and (4c)).

$$\widehat{F} = \text{argmin}_F \left(\frac{1}{2} \sum_{t=0}^{T} \left(\sigma(t) - \sum_{j=1}^{m} f_j(t) \right)^2 + \lambda \sum_{t=0}^{T} \sum_{j=1}^{m} w_j(t) f_j(t) \right) \quad \text{with } \lambda \in \mathbb{R}^+ \tag{9}$$

$$w_j(t) \leftarrow \begin{cases} \min\left(\frac{1}{f_j^{(l)}(t)}, 10 \right) & \text{if } f_j(t) > 0; \\ 1 & \text{if } f_j(t) = 0 \end{cases} \tag{10}$$

$$\forall\ 1 \leqslant j \leqslant m,\ \forall\ 0 \leqslant t \leqslant T$$

4 Simulation results

4.1 Problem generation

Random problem configurations have been generated using a simulator and configured with many parameters including the number of stacks in the considered multi-stack fuel cell system, m, and intrinsic fuel cell characteristics. The latter have been defined on the basis of fuel cell manufacturer specifications and considering a maximal lifetime $RUL\text{max}_j = 1500$ hours $\pm 20\%$ for each machine M_j ($1 \leqslant j \leqslant m$). Each $RUL\text{max}_j$ value is drawn following a uniform distribution between 1200 and 1800 hours. Power output values are determined in the same way, with $P\text{max}_j(0) = 500\,W \pm 5\%$ and $P\text{min}_j = 0.15 \cdot P\text{max}_j(0)$ for each machine M_j.

For the results presented hereafter, the power demand has been assumed to be constant during the whole scheduling horizon: $\sigma(t) = \sigma$. Without any loss of generality, only one demand value has then been associated to each problem configuration, but many demands corresponding to different configurations have been tested. Many loads α have been defined such that $\sigma = \alpha \cdot P\text{nom}_{tot}$, with $P\text{nom}_{tot}$ the nominal total power output reachable with the considered multi-stack system and $30\% \leqslant \alpha \leqslant 90\%$. $P\text{nom}_{tot} = \sum_{j=1}^{m} P\text{nom}_j$, with $P\text{nom}_j = 0.75 \cdot P\text{max}_j(0)$ the power output recommended by fuel cell manufacturers for a nominal use of fuel cells. In the following figures, results are represented as a function of the load α.

4.2 Resolution methods configuration

Initial values of each solution vector F has first been set to zero: $f_j(t) = 0$ $\forall\ 1 \leqslant j \leqslant m,\ \forall\ 0 \leqslant t \leqslant T$. Quality of solutions from the point of view of the reached production horizon globally increases with the number of iterations and stabilizes starting from a certain value. For each resolution method proposed earlier, several iterations of the associated process are then performed to optimize this solution. The global process is stopped when the variation of the solution vector is not significant anymore.

Tuning of the different parameters involved in the two resolution methods allows to comply with the constraints and to adapt the shape of each $f\text{max}_j$. Values for the parameters used in the Mirror-Prox algorithm are the following: $\lambda_{dem} = \lambda_{slope} = 100$, $\gamma = 100$, $\delta = 100$, $\mu' = 1$, $\upsilon' = 1$. The parameter λ involved in the Adaptive Lasso has been defined as a function of the number of machines: $\lambda = 10 \cdot m$. This allows to adapt the control of the sparsity to the size of the optimization problem. For the evolution of the maximal power output $f\text{max}$, values have been defined as follows: $\mu = 0.2$, $\upsilon = 0.3$.

4.3 Post-processing

The main constraint of the optimization problem is the reaching of the power output demand $\sigma(t)$. This constraint being tackled through the minimization of an objective function, solutions may contain time periods during which this demand is not reached. But, with the two resolution methods proposed in previous section, solutions are built so that the time periods for which the power demand $\sigma(t)$ is reached are gathered at the beginning of the schedules. This is consistent with the objective to maximize the production horizon of the set of machines. This behavior is linked with the shape of the functions $f\text{max}_j(t)$ representing for each machine M_j the evolution over time of the maximal power output reachable, which limits the contribution of each machine. These functions being strictly decreasing with the use of machines, it is in fact more likely to reach the demand at the beginning of the scheduling process than after some time. As already mentioned, resolution algorithms detailed previously can then be applied on overestimated horizons T, named decision horizons. The production horizon of each solution, H, is simply the maximal time during which all the constraints are strictly satisfied. In practice, the production horizon corresponds to the time during which the demand $\sigma(t)$ is reached.

4.4 Results

Efficiency of the proposed commitment strategies defined in Section 3 is assessed through a comparison with a basic strategy which performs successive convex projections onto each set of constraints defined previously. The idea is to generate a sequence of points that is supposed to converge to a solution of the optimization problem Bauschke and Borwein (1996).

Results do not vary significantly with the number of stacks considered. For the results proposed hereafter, the parameter m has been set to 25, which is consistent with stationary applications. For readability reasons, points associated to the different strategies have been scattered around the corresponding load value on the abscissa in Figure 2, which shows the distance of production horizons obtained with the considered strategies to a theoretical upper bound. Considering a constant demand σ and a set of fuel cell stacks, this upper bound, denoted UB and defined in Equation (11), corresponds to the theoretical maximal time during which the demand can be reached. This upper bound being never reachable, results are actually better than showed in the following figure.

$$\text{UB} = \left\lfloor \frac{\sum_{j=1}^{m} 0.6 \cdot P\text{max}_j(0) \cdot RUL(P\text{min}_j)}{\sigma} \right\rfloor \tag{11}$$

One can see in Figure 2 that the strategy performing successive convex projections allows to reach a mean relative horizon of around 39%. Resolution methods based on the Mirror Prox and on the Adaptive Lasso give better results. They allow to reach respectively 64.3% and 65.7% of the upper bound UB. These two latter algorithms allow to reach similar performances when considering the production horizon reached. However, they differentiate themselves by the shape of the solutions and by their computation time. Figures 3 and 4 show schedules of three machines obtained respectively with the algorithm based on the Mirror Prox and with the Adaptive Lasso. Evolution of each machine contribution and of the associated maximal power output reachable, $f\text{max}_j$, is also shown in these figures. One can see in Figure 3 that the algorithm based on the Mirror Prox defines a smooth use of machines, which complies with a continuous use. In fact, once a machine has been started, it is used until its end of life. The postponed start-up of some machines allows to reach better production horizons than with the method based on successive projections.

The method using the Adaptive Lasso algorithm performs a better postponement of machines start-up by minimizing at each time the number of machines used in parallel to reach the demand (see Figure 4). This allows to reach better production horizons in most of the cases, but solutions does not always comply with a continuous use of machines once they have been started up. This drawback apart, this resolution method gives globally the better solutions in terms of production horizons reached and of computation time. It is indeed faster than the Mirror Prox and gives solutions in less than 40 seconds[1] for all the tested scenarios.

5 Conclusion

A management of fuel cell systems has been proposed in a PHM framework. Decision coming within the scope of Prognostic Decision Making has been ad-

[1]Simulations have been made using Matlab (Parameters: Processeur Intel® Core^{TM} i5-3550 CPU@3.30GHz×4, 15.6 Gio, 64 bits)

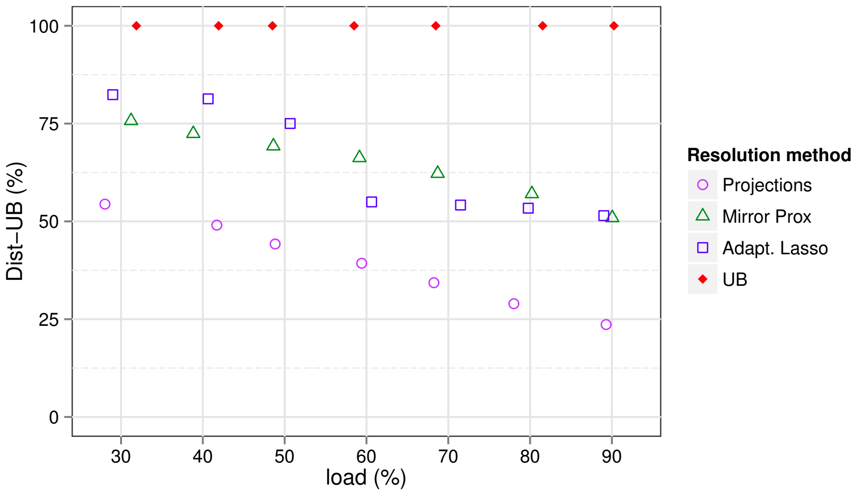

Figure 2: Distance to the upper bound UB of production horizons reached with each convex resolution method – m = 25 machines

dressed considering longer timeframes than those proposed so far in the literature on fuel cells. The use of convex programming has been proposed to cope with the scheduling problem of multi-stack fuel cell systems under service constraint. A mathematical formulation of the problem has been proposed as well as two different convex resolution methods performing a minimization of the objective function under constraints. First one is based on the Mirror Prox algorithm and second one on the Adaptive Lasso algorithm. All the fuel cell properties are not observed by the solutions obtained with the proposed approaches, but this first study is promising. It shows indeed that a global resolution on the scale of the whole production horizon can be used to define the commitment of machines over time with the horizon maximization as objective.

As future work, enhancement of the considered mathematical formulation will be addressed to suit the associated model to a realistic evolution over time of fuel cell characteristics. The respect of a continuous use of fuel cells will also be improved using a minimization of the ℓ_1 norm of the successive differences of the components of the solution vector F. Consideration of such a norm could allow to control the speed of the variation of each machine contribution and then to obtain smoother contribution profiles. This will require a new determination of parameters involved in the resolution methods depending on the importance attached to the solution characteristics. Respect of a strict continuous use of machines can indeed be contradictory with the objective of maximization of the production horizon.

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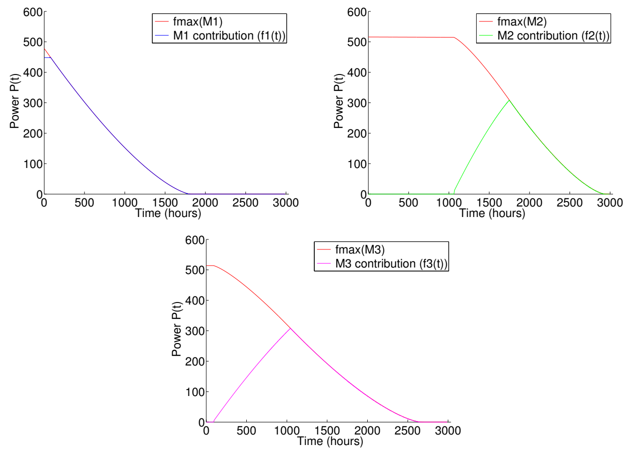
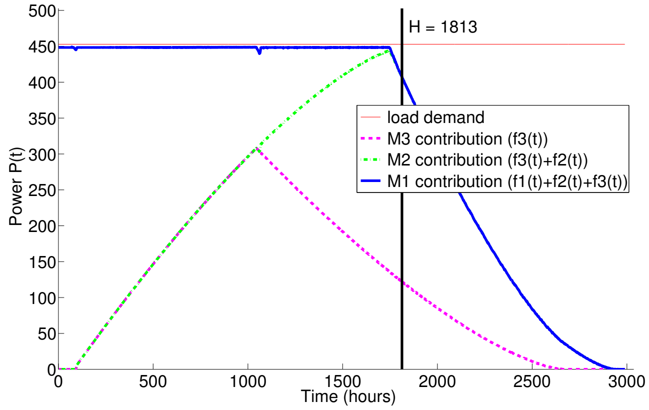


(a) Machines contributions



(b) Schedule

Figure 3: Solution obtained with the Mirror-Prox-based algorithm – $m = 3$ machines, $\sigma = 0.4 \cdot P\text{nom}_{tot}$

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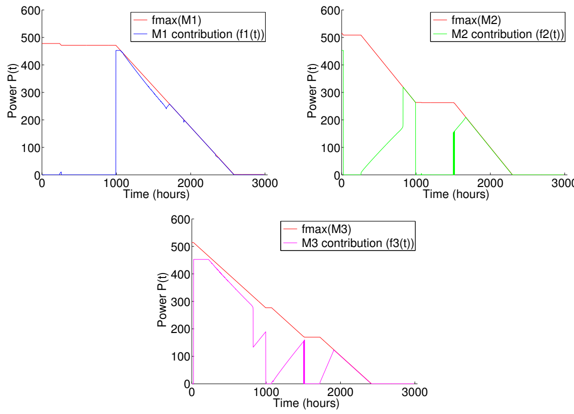
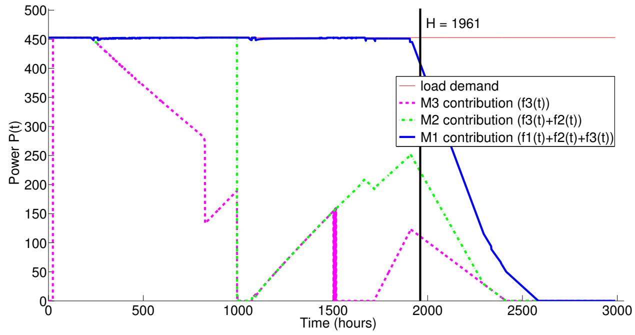


(a) Machines contributions



(b) Schedule

Figure 4: Solution obtained with the algorithm based on the Lasso – $m = 3$ machines, $\sigma = 0.4 \cdot P\text{nom}_{tot}$

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