Scheduling independent parallel machines with convex programming*

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Abstract
In the field of production scheduling, this paper adresses the problem of maximizing the production horizon of fuel cell systems under service constraint. Convex optimization is used to define at each time the contribution of each fuel cell to the global output so as to satisfy a power demand as long as possible. An algorithm based on the Mirror-prox for Saddle Points method is proposed to cope with the assignment problem. Results based on computational experiments assess the efficiency of this approach in comparison with an intuitive resolution performing successive basic convex projections onto the sets of constraints associated to the optimization problem.
Keywords: Convex optimization ; Production scheduling ; Parallel machines ; Fuel cell systems

1 Introduction and related work

Due to the decline of fossil fuel resources, the use of fuel cells appears to be of growing interest as a potential alternative to conventional power systems. Fuel cells are expected to be used in stationary applications and in transportation and portable power applications. Their durability are however not consistent with such application. In fact, their lifetime reaches between 1500 and 3000 hours, whereas 5000 hours are required for transportation applications and up to 100000 hours for stationary ones. An important challenge highlighted by Borup et al. (2007) consists then in improving the performance, reliability and lifetime of fuel cells.

In the fuel cells systems considered in this paper, each fuel cell is supposed to be independent but the whole system has to globally deliver a given power output based on a need of energy. This global output is determined by the sum of each output of the fuel cells that are currently running. The scheduling of such systems is addressed, with the objective to satisfy the power demand as long as possible. All the machines are not supposed to be in use at any time because the target power output can be reached by using only a subset of the machines within the platform or because some machines are not available. Machines are indeed assumed to suffer from wear and tear. Their lifetime is then limited and maintenance is required. Many reasons justify to postpone maintenance operations as late as possible and to maintain all the machines at the same time. Maintenance can for instance be challenging and costly (Kovacs et al., 2011). Isolated or embedded equipment can also require to wait for the end of a global task before performing maintenance (Balaban et al., 2011), for example in the aerospace, the railway or the automobile domain. One challenging objective is then to maximize the production horizon of the set of machines between two maintenance periods. This production horizon corresponds to the lifetime of the whole set of machines. This global lifetime depends on each machine lifetime, but also on the schedule of the machines. A fuel cell lifetime is indeed variable and dependent on its use (Borup et al., 2007).

Basically, the problem is to provide the power output value for each fuel cell as a function of time, on the basis of a global power demand. This corresponds to the scheduling of heterogeneous parallel machines performing independent and identical tasks, with the maximization of the production horizon as objective. A similar problem has been addressed by Herr et al. (2014) considering machines able to provide a discrete number of power outputs. Considered approach can not be applied to fuel cell systems. Power output provided by fuel cells can indeed vary continuously and take any value within a given interval. Furthermore, due to the aging, each maximal power output is supposed to be decreasing with time. A scheduling process taking into account these fuel cells specific features has been proposed by Herr et al. (2015). The proposed resolution method makes use of many successive sub-problem resolutions based on a mixed integer linear program. Solutions provided by each linear program are optimal but global schedules obtained with the succession of many linear programs are not necessarily optimal. This approach is moreover limited to systems of reasonable size. In order to overcome these two limitations, we propose in this paper to change totally the paradigm of the resolution and to build the solutions globally on the whole production horizon. Contribution of each fuel cell during its lifetime is considered as a whole and determined through convex optimization, whose interest is to allow the solving of big optimization problems in limited time.

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2 Problem statement and mathematical model

The application addressed in this paper is based on a parallel fuel cells system, in which each fuel cell is supposed to be able to deliver power outputs P_j that can vary continuously within a given power output range $[P\text{min}_j; P\text{max}_j]$. For each machine M_j ($1 \leqslant j \leqslant m$), the minimal power output $P\text{min}_j$ is supposed to be strictly greater than zero and constant over time. The maximal power output $P\text{max}_j$ decreases with time when the machine M_j is used. The range of available power outputs depends then on the time t: for each machine.

Let $f_j(t)$ ($1 \leqslant j \leqslant m, 0 \leqslant t \leqslant T$) be the vector defining the evolution over time of the power output delivered by the machine M_j, with T the length of the decision horizon. Contributions of all the machines are gathered together in a vector $F \in \mathbb{R}^{m(T+1)}$ such that: $F = [f_1(0), \ldots, f_1(T), \ldots, f_j(t), \ldots f_m(0), \ldots, f_m(T)]$.

The general idea is to minimize a convex function subject to a set of constraints. The objective function aims at ensuring that the power demand is reached. At each time t ($0 \leqslant t \leqslant T$), it is about minimizing the difference between the global power output delivered by the set of machines and the demand $\sigma(t)$. This is expressed by Equation (1).

$$\min \quad \sigma(t) - \sum_{j=1}^{m} f_j(t) \qquad \forall\, 0 \leqslant t \leqslant T \tag{1}$$

Constraints on each function f_j relate to the definition domain of each contribution and to the limited availability of machines. At each time t, each machine contribution is either equal to zero or constrained between two bounds (see Equation (2)). $f_j(t) = 0$ means that the machine M_j is not used at time t.

$$f_j(t) = 0 \quad \text{or} \quad f\text{min}_j(t) \leqslant f_j(t) \leqslant f\text{max}_j(t) \qquad \forall\, 1 \leqslant j \leqslant m,\ \forall\, 0 \leqslant t \leqslant T \tag{2}$$

Each contribution f_j is constrained by the maximal power output decrease of the associated machine M_j, which expresses its limited availability. Evolution over time of this maximal power output, $f\text{max}_j(t)$, is a function of the use of machine M_j, $f_j(t)$. Indeed, $f\text{max}_j(t)$ evolves only if M_j is used, that is, if $f_j(t) > 0$. A first formulation is proposed in Equation (3), with a_j the speed associated to the maximal power output decrease. Evolution of the available power outputs range following these equations is depicted in Figure 1.

$$f\text{max}_j(t) = \begin{cases} f\text{max}_j(t-1) + a_j & \text{if } f_j(t) > 0\,; \\ f\text{max}_j(t-1) & \text{if } f_j(t) = 0 \end{cases} \qquad \forall\, 1 \leqslant j \leqslant m,\ \forall\, 1 \leqslant t \leqslant T \tag{3}$$

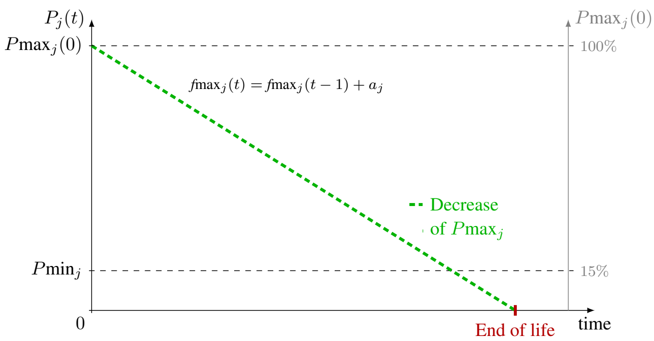

Figure 1: Simplified evolution of fuel cells characteristics

Equations (2) and (3) being not convex, they can not be used as is within the proposed convex programming paradigm. A second formulation of the constraints is proposed in set of equations (4), which details the mathematical program associated to the optimization problem. This program does not respect the real evolution over time of the maximal power output that can be delivered by machines (see Equations (2) and (3)), but presents the advantages of being convex and thus consistent with the convex resolution methods proposed in next section.

$$\left\{ \begin{array}{ll} \min \quad \sigma(t) - \sum_{j=1}^{m} f_j(t) \qquad \forall\, 0 \leqslant t \leqslant T & \text{(4a)} \\ \text{s.t.} \quad f\text{max}_j(t) \leqslant f\text{max}_j(t-1) + \mu \cdot a_j \cdot (f_j(t-1))^{\upsilon} \qquad \forall\, 1 \leqslant j \leqslant m,\ \forall 1 \leqslant t \leqslant T & \text{(4b)} \\ \qquad \text{(with } \mu \text{ and } \upsilon \in \mathbb{R}^{*+}) & \\ \text{with} \quad 0 \leqslant f_j(t) \leqslant f\text{max}_j(t) \qquad \forall 1 \leqslant j \leqslant m,\ \forall 0 \leqslant t \leqslant T & \text{(4c)} \end{array} \right.$$

3 Resolution: a Mirror-Prox-based algorithm

The convex resolution method is based on the Mirror Prox algorithm proposed by Nemirovski (2004) as a variant of the Mirror Descent. Both are based on the resolution of a primal-dual saddle point problem, which allows to take constraints into account. The purpose is to minimize a smooth convex function under constraints. The Mirror Descent algorithm makes use of a gradient descent to find the minimum of the considered function. A mirror function allows to transition from the primal space, where all the constraints of the problem are defined, to the dual space. The Mirror Prox method applies at each iteration two consecutive steps of Mirror Descent. A very instructive description of the Mirror Prox algorithm has been proposed by Bubeck (2014).

The objective function taken into account is detailed in equation (5). This proposed formulation includes directly the different constraints of the application. First part of the function, detailed in Equation (6), is aimed at satisfying the demand $\sigma(t)$ at each time t. Second one is used to control the evolution of each $f\max_j$ as a function of each associated contribution f_j (see equation (7)). The mirror function considered in the Mirror Prox formula is defined on $\mathbb{R}^{m(T+1)}$ by Equation (8).

$$\phi(F, f\max, \sigma) = \lambda_{dem} h_{dem}(F, \sigma) + \lambda_{slope} h_{slope}(F, f\max) \qquad \text{with } \lambda_{dem},\ \lambda_{slope} > 0 \tag{5}$$

$$h_{dem}(F, \sigma) = \sum_{t=0}^{T} \frac{1}{T+1} \exp\left(-\gamma \left(\sum_{j=1}^{m} f_j(t) - \sigma(t) \right) \right) \qquad \text{with } \gamma > 0 \tag{6}$$

$$h_{slope}(F, f\max) = \sum_{t=1}^{T} \sum_{j=1}^{m} \exp\left(\delta \left(f\max_j(t) - f\max_j(t-1) - \mu' a_j (f_j(t-1))^{\upsilon'} \right) \right) \tag{7}$$
$$\text{with } \delta > 0,\ \mu' > 0,\ \upsilon' > 1$$

$$\theta(F) = \sum_{t=0}^{T} \sum_{j=1}^{m} F \ln(F) \tag{8}$$

4 Simulation results

Random problem configurations have been generated using a simulator and configured with many parameters including the number of machines in the considered system, m, and intrinsic machines characteristics. The power demand has been assumed to be constant during the whole scheduling horizon: $\sigma(t) = \sigma$. Without any loss of generality, only one demand value has then been associated to each problem configuration, but many demands corresponding to different configurations have been tested. Many loads α have been defined such that $\sigma = \alpha \cdot P\text{nom}_{tot}$, with $P\text{nom}_{tot}$ the nominal total power output reachable with the considered fuel cells system and $30\% \leqslant \alpha \leqslant 90\%$. $P\text{nom}_{tot} = \sum_{j=1}^{m} P\text{nom}_j$, with $P\text{nom}_j = 0.75 \cdot P\max_j(0)$ the power output recommended by fuel cell manufacturers for a nominal use of fuel cells. In the following figure, results are represented as a function of the load α.

Initial values of each solution vector F has first been set to zero: $f_j(t) = 0\ \forall\ 1 \leqslant j \leqslant m,\ \forall\ 0 \leqslant t \leqslant T$. Quality of solutions from the point of view of the reached production horizon globally increases with the number of iterations and stabilizes starting from a certain value. Several iterations of the associated process are then performed to optimize each solution. The global process is stopped when the variation of the solution vector is not significant anymore. Tuning of the different parameters involved in the resolution method allows to comply with the considered model and to adapt the shape of each $f\max_j$. The resolution algorithm based on the Mirror Prox formulation detailed previously is applied on overestimated horizons T, named decision horizons. The production horizon of each solution, H, is simply the maximal time during which all the constraints are strictly satisfied. In practice, the production horizon corresponds to the time during which the demand $\sigma(t)$ is reached.

Efficiency of the proposed commitment strategy is assessed through a comparison with a very basic strategy which performs successive convex projections onto each set of constraints defined previously. The idea is to generate a sequence of points that is supposed to converge to a solution of the optimization problem (Bauschke and Borwein, 1996).

Results do not vary significantly with the number of fuel cells considered. For the results proposed hereafter, the parameter m has been set to 25, which is consistent with stationary applications. Figure 2 shows the distance of production horizons obtained with the considered strategies to a theoretical upper bound. Considering a constant demand σ and a set of fuel cells, this upper bound, denoted UB and defined in Equation (9), corresponds to the theoretical maximal time during which the demand can be reached. $RUL(P\min_j)$ corresponds to the remaining useful life of fuel cell M_j when it is used with its minimal power output $P\min_j$. This upper bound being never reachable, results are actually better than shown in the following figure. One can see in Figure 2 that the strategy performing successive convex projections allows to reach a mean relative horizon of around 39%. Resolution method based on the Mirror Prox gives better results. It allows to reach on average 64.3% of the upper bound UB.

$$\mathrm{UB} = \left\lfloor \frac{\sum_{j=1}^{m} 0.6 \cdot P\mathrm{max}_j(0) \cdot RUL(P\mathrm{min}_j)}{\sigma} \right\rfloor \tag{9}$$

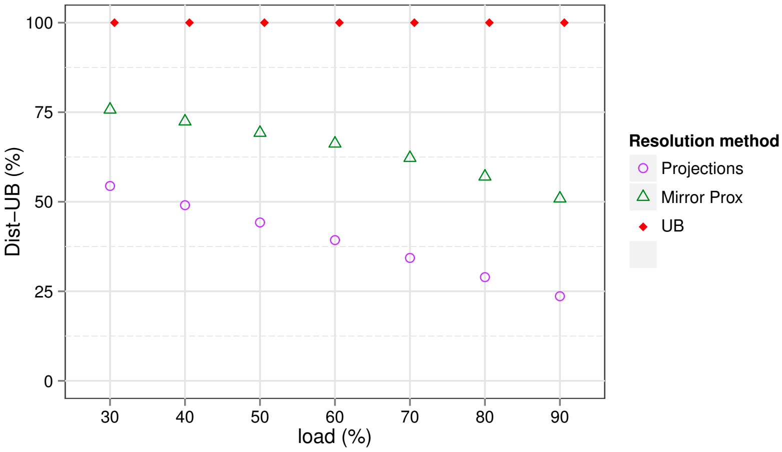

Figure 2: Distance to the upper bound UB of production horizons reached with each convex resolution method for different load demand values – m = 25 machines

5 Conclusion

A management of a set of fuel cells able to provide different power outputs has been proposed when the objective is to maximize the production horizon, that is, the time during which a global power demand is reached. The use of convex programming has been proposed to cope with the scheduling problem under service constraint. A mathematical formulation of the problem has been defined as well as a convex resolution method performing a minimization of the objective function under constraints, based on the Mirror Prox algorithm. All the fuel cell properties are not observed by the solutions obtained with the proposed approaches, but this first study is promising. It shows indeed that a global resolution on the scale of the whole production horizon can be used to define the commitment of machines over time with the horizon maximization as objective.

As future work, enhancement of the considered mathematical formulation will be addressed to suit the associated model to a realistic evolution over time of fuel cell characteristics.

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