Study on the origin of 1/f noise in bulk acoustic wave resonators

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Abstract

In this contribution, we first give some relevant information about the realization of the 5 MHz SC-cut resonators used in this study. Then, we report the resulting short term stability of the resonators as a function of the position of the resonators inside the crystal block. Concerning the second goal, a theoretical approach, based on the fluctuation-dissipation theorem, is used in order to put numerical constraints on a model of 1/f noise caused by an internal (or structural) dissipation proportional to the amplitude and not to the speed. The order of magnitude of the noise is then discussed using a candidate physical process. Finally, we conclude on the work that could be done to solve the remaining open problems.

Resonator realization and noise measurements

All the resonators were fabricated from a single quartz crystal block. This crystal block was obtained from a seed cut in a previous synthetic crystal which was grown using a natural seed.

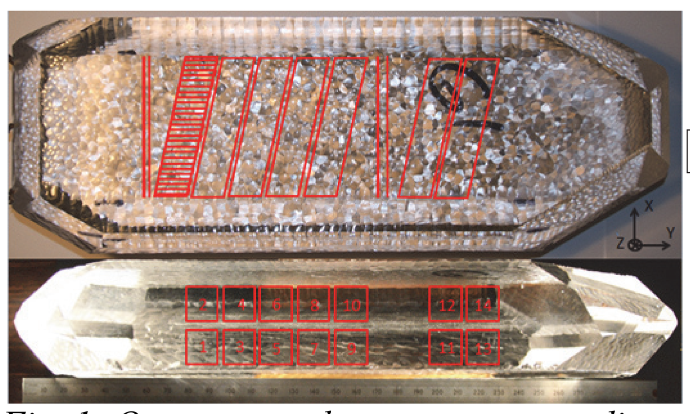


Fig. 1: Quartz crystal resonators according to their positions in the mother block. Crystal dimensions: 220 mm along Y-axis, 36 mm along Z-axis, 110 mm along X-axis

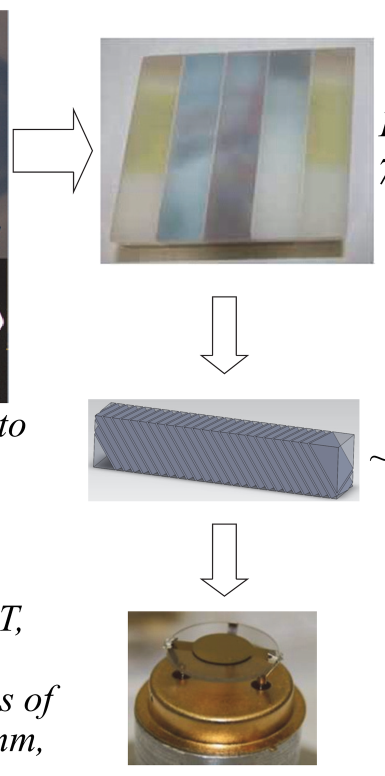


Fig. 2: Final resonator: 5 MHz SC-cut, 3rd OT, Plano-convex shape
diameter: 14 mm, Thickness: 1.09 mm, radius of curvature: 130 mm, Electrodes diameter: 8 mm, temperature turnover point: ~ 80°C

A hundred of resonators were used for noise measurements.

X-ray topography shows residual dislocations on thin plates ("blanks") cut perpendicularly to the Y-axis, which corresponds to the longest dimension of the crystal block (Fig. 3).

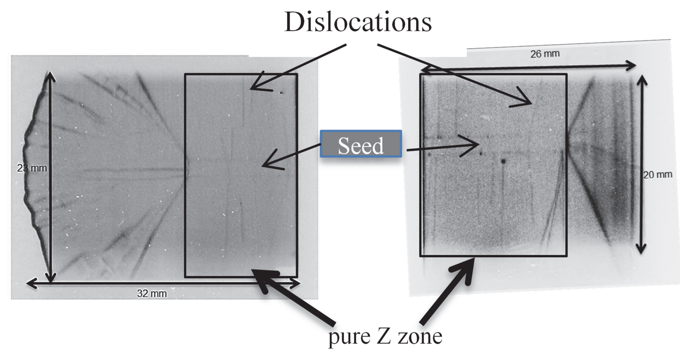


Fig. 3: X-ray topography of Y-cut plates. The duration of exposure of the photos is about 5h30 with an X-ray vertical beam given by a generator of 45 kV with 25 mA.

The number of dislocations, observed inside the black rectangular zone is about 1 to 3 per cm^2 which corresponds to a very high quality quartz crystal.

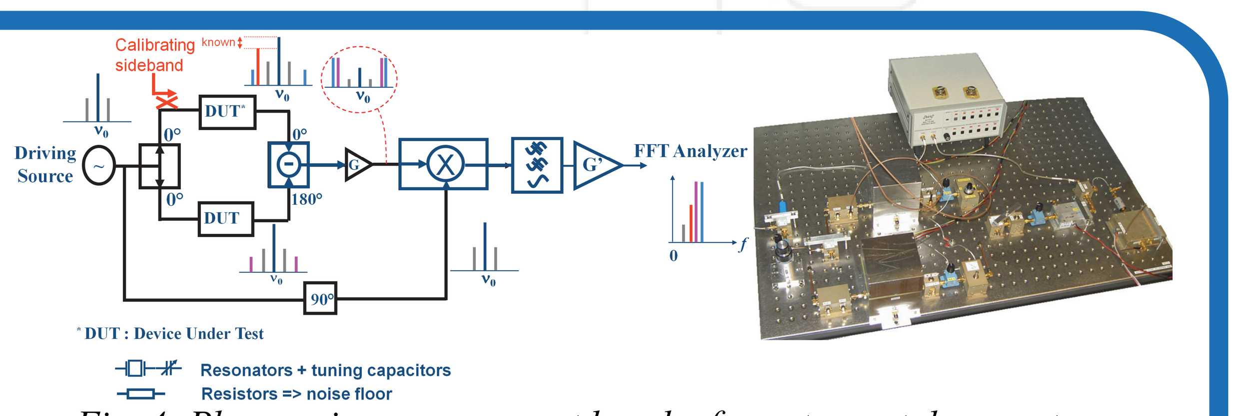
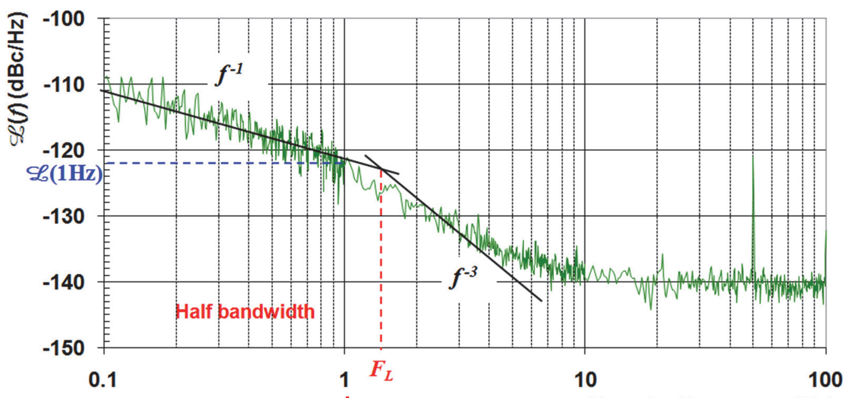


Fig. 4: Phase noise measurement bench of quartz crystal resonators.



- SC-cut
- $f_{res.}$ = 5 MHz
- P_{xtal} = 20 µW
- Q = 2.6·10^6
- F_L = 1.3 Hz

$$Q_L = \frac{f_{res.}}{2 \cdot F_L}$$

Allan Standard Deviation

$$\sigma_{y\_flicker} = \sqrt{\left[\frac{F_L^2 + 1}{f_{res.}^2}\right] 2\ln 2\, \mathcal{L}(1Hz)}$$

$$\sigma_{y\_flicker} = 1 \cdot 10^{-13}$$

Fig. 5: Typical resonator result in term of phase noise.

Theory and discussion

The fluctuation-dissipation theorem (FDT) as formulated in [1] is used to estimate the power spectral density of thermal noise coming from fluctuations in the thickness ($2h$) of quartz resonators (Fig. 6). An internal friction term, φ, is added in the formulation, in order to obtain a 1/*f* spectrum at low frequencies. Indeed, for this mode characterized by the mechanical displacement inside the resonator the strain S_2 and stress T_2 are then respectively given by:

$$S_2 = \frac{\partial u_2}{\partial x_2}$$

$$T_2 = c_{22}(1 + j\varphi)S_2 + \eta_{22}\frac{\partial S_2}{\partial t}$$

$$\rho\frac{\partial^2 u_2}{\partial t^2} = c_{22}(1 + j\varphi)\frac{\partial^2 u_2}{\partial x_2^2} + \eta_{22}\frac{\partial^3 u_2}{\partial x_2^2 \partial t}$$

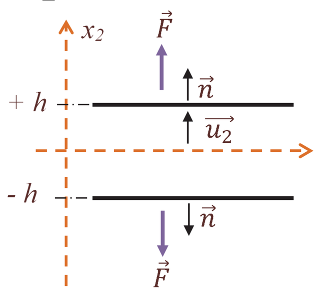


Fig. 6: Resonator design.

with c_{22} the elastic constant and η_{22} the viscoelastic damping constant of quartz crystal. φ is an internal friction coefficient,

Searching for solutions of the type:

$$u_2(x_2, t) = (a\sin(kx_2) + b\cos(kx_2))e^{j\omega t}$$

with limit condition given by: $T_2(\pm h, t) = \frac{F}{S}e^{j\omega t}$

where F is the modulus of the harmonic mechanical force applied to the surface S of the electrodes.

We have: $$k^2 = \frac{\rho\omega^2}{(c_{22} + j(c_{22}\varphi + \eta_{22}\omega))}$$

$$a = \frac{F/S}{(k\cos(kh))(c_{22} + j(c_{22}\varphi + \eta_{22}\omega))} \text{ and } b = 0$$

The complex mechanical admittance of the system is defined by:

$$\bar{Y}(\omega) \equiv \frac{\frac{\partial u_2(\pm h, t)}{\partial t}}{\pm F.e^{j\omega t}} = \frac{j\omega a\sin(kh)}{F}$$

The FDT then states that the spectral power density of the thickness fluctuations is computed by:

$$\frac{u_2^2(\pm h, \omega)}{BW} = \frac{4k_BT}{\omega^2}Re(\bar{Y}(\omega))$$

with T the absolute temperature (in K), k_B the Boltzmann constant (in J/K) and BW bandwidth.

With the assumptions $\varphi \ll 1$ and $\omega \ll c_{22}/\eta_{22}$ and circular frequency at resonance $\omega_r \sim 1/h$, we get:

$$S_y(\omega) \equiv \frac{(\delta\omega_r)^2}{\omega_r^2 BW} = \frac{u_2^2(\pm h, \omega)}{(2h)^2 BW} \approx \frac{1}{\omega} \times \frac{2k_BT}{Vc_{22}}\left(\frac{\eta_{22}}{c_{22}}\omega + \varphi\right) \quad (1)$$

with V vibrating volume.

For 1/*f* (flicker) noise, the standard deviation of the difference of the average fractional frequencies measured for two consecutive samples is given by the expression (Allan deviation):

$$\sigma_{y\_flicker} = \sqrt{2\ln(2)\,S_y(1Hz)}$$

For 5 MHz SC-cut quartz crystal resonator:

c_{22} = 115 GPa, η_{22} = 1.36·10^{-3} Pa·s, T = 350 K and V = 0.104 cm^3.

$\sigma_{y\_flicker}$ would be given by:

$$\sigma_{y\_flicker} = \sqrt{2\ln(2)\frac{2k_BT}{Vc_{22}}\varphi} \approx 1.06 \cdot 10^{-12}\sqrt{\varphi} \quad (2)$$

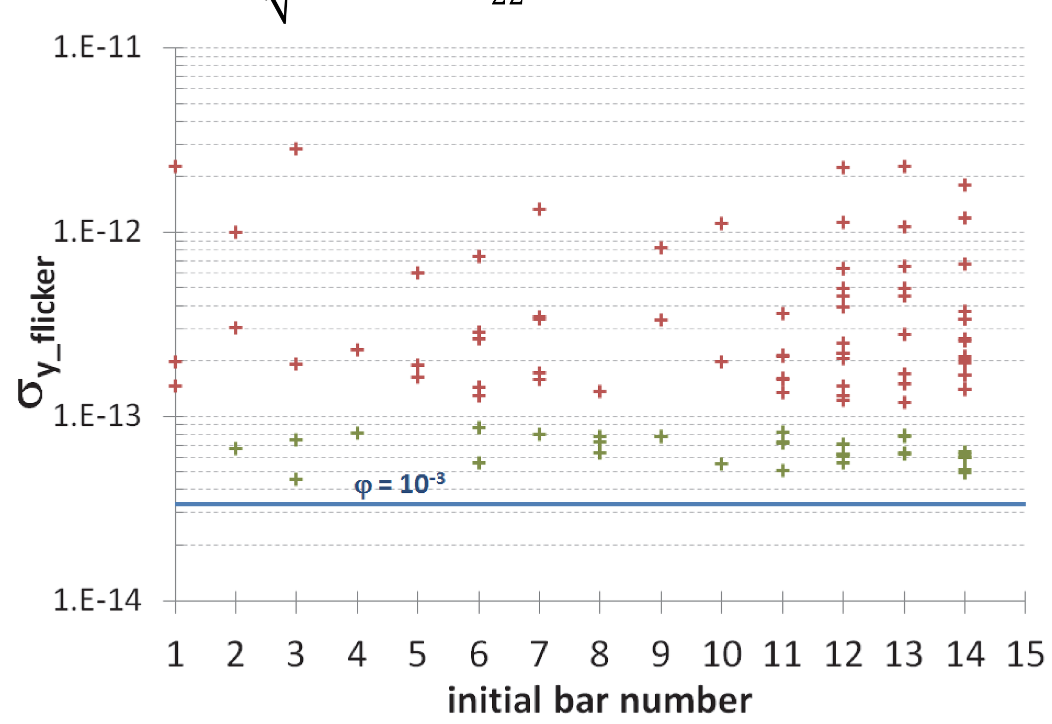
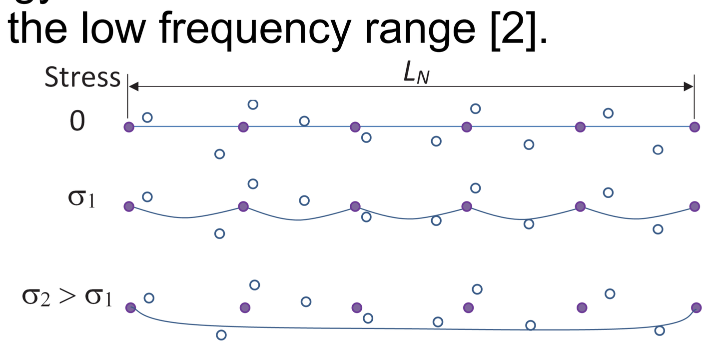


Fig. 7: Comparison between theoretical and experimental $\sigma_{y\_flicker}$.

We try to evaluate φ by the modified Granato-Lücke theory of the energy loss due to some kinds of dislocation motion in the low frequency range [2].

Fig. 8: Dislocation under stress.



The authors found an expression of the logarithmic decrement for the impurity spacing controlled dislocation motion that, in the small stress amplitude limit, is given by:

$$\Delta = \frac{\beta N b L_N}{\pi c^{1/3}\epsilon} \quad (3)$$

where β is a parameter having approximate value of 1.5. N is the total length of dislocation line in a unit volume of material ≈×2 surface dislocation density. This value is of the order of 6 cm/cm^3 judging from the X-ray image of the surface of one of the resonator (Fig. 3). β is the mean length of a Burger's vector ≈ 3×10^{-8} cm. L_N is the network length (√(3/N)). c is the atom fraction of impurity which must be lower than 1 ppm to get Q values as high as a few 10^6. ϵ is the fractional difference between the radius of impurity and host atoms taken to be of the order of 20 % (in quartz, Li and Si) .

From (1), we can deduce that:

$$\frac{1}{Q_{eff}} = \frac{1}{Q_{viscous}} + \varphi \text{ avec } 1/Q_{viscous} = \frac{\eta_{22}\omega}{C_{22}}$$

Therefore at low frequencies $1/Q_{eff} \approx \varphi$. Hence, we attempt to identify Δ with $\pi\varphi$ at low frequencies, in a first approximation in spite of the fact that we are not in the dominantly viscous regime. This would give $Q_{eff} \approx 10^5$ and $\varphi \approx 10^{-5}$ in the low frequency regime, which would be an interesting order of magnitude to attribute at least some non-negligible part of the 1/*f* noise to the fluctuations of thickness. However, this would also mean that at resonance:

$$\frac{1}{Q_{eff}} = \frac{1}{Q_{viscous}} + \varphi \approx 4 \cdot 10^{-7} + 10^{-5} \approx 10^{-5} = \varphi$$

Hence the viscous damping would not be dominant at resonant frequency which is contradictory to experimental facts.

Conclusion

It is possible to find 1/f noise through the fluctuation-dissipation theorem, by adding a constant complex part to the elastic constant in the usual differential equation characteristic of a viscously damped harmonic oscillator. This corresponds to a frequency independent energy loss in the limit of small frequencies. The hysteretic motion of the dislocations described by a modified Koehler-Granato-Lücke model could a priori describe such a loss mechanism. Indeed, it could provide an explanation for the experimental observations that the logarithmic decrement generally decreased when the dislocation density decreased when quartz were not as good as now and that sometimes a slightly higher concentration of impurity could improve the quality factor. However, numerical estimations seem to provide values that are at least an order of magnitude too high.

Hence the physical origin of 1/f noise in quartz crystal ultra-stable oscillators still remains an open question. We therefore plan to study another approach based on thermally activated nucleation and motion of kink-antikink pairs along dislocations, with possibly several different activation energies. This could lead to 1/f noise by the mechanism of Lorentzian summation.

We are looking for physicist partners to explore this phenomenon, please leave your visit card!

[1] S. Ghosh & al., "Theoretical and experimental investigations on 1/f noise of quartz crystal resonators," Proceedings IEEE Joint UFFC, EFTF and PFM Symp., Prague, Czech Republic, 21-25 July, 2013 , pp. 737-740.

[2] J. C. Swartz and J. Weertman, "Modification of Koehler-Granato-Lucke damping theory", J. Appl. Phys, vol. 32, no. 10, pp. 1860-1865, 1961.

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