Joint optimization of train assignment and predictive maintenance scheduling *

Nathalie Herr [a†], Jean-Marc Nicod [a], Christophe Varnier [a],
Noureddine Zerhouni [a], Malek Cherif [a], Nader Fnaiech [b‡]
[a] FEMTO-ST, AS2M, Univ. Bourgogne Franche-Comté, UFC/CNRS/ENSMM/UTBM
F-25000 Besançon, France
E-mails: [firstname.lastname]@femto-st.fr
[b] SIME, ENSIT/University of Tunis
1008, Tunis, Tunisia
E-mail: fnaiechnader@yahoo.fr

Abstract
In this paper, we propose to jointly optimize the rolling stock assignment and the maintenance scheduling in a Prognostics and Health Management (*PHM*) context. The aim is to determine an appropriate use of the rolling stock considering predefined train timetables and prognostics information. The problem is to associate a rolling stock unit to each train trip and to integrate the necessary maintenance operations in the schedule according to the real state of health of trains. This problem, which falls within the decision part of *PHM*, is proposed to be solved using an optimal approach based on Linear Programming. The use of the proposed linear program is illustrated on a simple use case.

Keywords
Predictive maintenance, PHM, Rolling stock assignment, Train maintenance, Linear programming

1 Introduction and related work

This paper considers the rolling stock assignment and maintenance scheduling problem. Maintenance in the railway domain differs from the traditional maintenance in that vehicles to be maintained are mobile. Their location over time depends directly on train routing. Rail vehicles maintenance is then strongly linked to their routing. In the literature, the routing problem and the maintenance problem are however often treated separately or sequentially. In some cases, the train routing is defined without any maintenance consideration and adapted later to include necessary maintenance operations. Time slots for maintenance tasks can also be included arbitrarily in the train routing, irrespective of which unit needs to be maintained and when. A separate maintenance routing process is in this case carried out very regularly to define which train units have to be maintained in the predefined restricted time periods. Some studies aim finally to optimize the routing and the maintenance

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†Corresponding author – E-mail: nathalie.herr@femto-st.fr
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scheduling jointly. Our contribution falls within this case. Andrés et al. (2015) proposed for instance a mixed integer linear programming model that determines an appropriate train routing and schedules the necessary maintenance operations, with global cost minimization as objective. Giacco et al. (2014) proposed also a mixed integer linear programming model to deal with the interaction between the rolling stock routing and the maintenance planning. The objective considered in this case is to minimize the total number of rolling stock units that are used and the number of empty rides and to maximize the distance traveled by each train between two maintenance operations of the same type.

Rolling stock maintenance is traditionally based on static strategies that schedule maintenance operations in advance according to time and distance criteria. This corresponds to preventive maintenance, which often implies unnecessary maintenance activities and reduces the useful life of rolling stock components due to early replacement. Works cited previously considered this type of maintenance. As stated by Umiliacchi et al. (2011), improvement can be achieved by estimating the time when a failure is likely to occur and by adapting maintenance interventions accordingly. This is known as predictive maintenance, which is a dynamic maintenance policy that makes use, in addition to current degradation information, of predictive information in the form of remaining useful life to optimally schedule maintenance actions (Horenbeek and Pintelon, 2013). Predictive maintenance is based on the prognostics phase of Prognostics and Health Management (*PHM*) and allows to do more than just react to threshold crossings and diagnostics alerts. PHM has been shown to provide many benefits for the health management of systems, such as avoiding failures, minimizing loss of remaining useful life, optimizing resource usage or increasing availability. Studies proposed in the railway domain in a *PHM* context focused so far for the vast majority on the prognostics phase. Some contributions tackled rolling stock prognostics, with for instance the prediction of the remaining useful life of train axle bearings (Fumeo et al., 2015) or rail wagon bearings. Prognostics has also been applied on infrastructure elements, such as railway turnouts (Camci, 2014) or rail tracks (Letot et al., 2016). Very few works addressed the decision part of *PHM* dealing with maintenance optimization. Letot et al. (2016) proposed an adaptive opportunistic predictive maintenance model for railway tracks based on the track geometry observation. They search for the optimum tamping time considering a set of rail tracks sections. Camci (2014, 2015) addressed the problem of predictive maintenance for systems located in various places, which can be applied for the maintenance of railway switches. The general problem has been introduced in (Camci et al., 2012) and resolution methods based on a Genetic Algorithm formulation have been proposed for many variants of the predictive maintenance optimization problem in (Camci, 2014) and (Camci, 2015). These works relate to the maintenance of geographically distributed, but stationary systems, which are part of the railway infrastructure.

In this paper, we focus on the maintenance of trains, which are geographically moving systems and whose maintenance is, as mentioned before, strongly linked to their routing. Then, we propose to jointly optimize the rolling stock assignment and the maintenance scheduling in a *PHM* context. The aim is to determine an appropriate use of trains considering predefined train timetables and prognostics information. The problem is to associate a rolling stock unit to each train trip and to integrate the necessary maintenance operations in the schedule according to the real state of health of trains.

For each train, prognostics information is considered in the form of a degradation level which evolves over time with the use of trains. Compared to traditional preventive approaches, consideration of prognostics results allows to match each degradation level evolution to the real use of trains. It is thus possible to take into account the impact of trips on trains state of health. Each trip can indeed impact the trains wear and tear in various ways, as a function of different criteria such as the difference in height, the rails state of health or the train speed that is authorized. This allows to enhance the decisions made in the maintenance scheduling, as well as in the assignment process, which defines which train unit has to be used for each trip. The knowledge of trains state of health and the prediction of their evolution allows indeed to choose the best train for each trip in the timetable. These decisions, which fall within the decision part of the *PHM* process, are proposed to be optimized using linear programming.

The organization of the paper is as follows: the problem statement is first detailed in Section 2, with the description of the application framework and the optimization problem and with a mathematical formulation of the problem. The proposed resolution method based on linear programming is developed in Section 3. The use of this resolution method is then illustrated on a simple use case in Section 4. This work is finally concluded and some future works are given in Section 5.

2 Problem statement

2.1 Application framework

The application addressed in this paper is based on a set of m trains M_j ($1 \leqslant j \leqslant m$). All the trains are supposed to be of the same type. They are however differentiated by their degradation level provided by the prognostics. Each level, denoted $H_j \in [0, 1]$ (H_j^0 at the beginning of the scheduling process), stands for the state of health of the train M_j. $H_j = 0$ means that the train M_j is as good as new and $H_j = 1$ indicates that the train has reached its end of life and that a maintenance is required. Each degradation level is supposed to remain constant when the corresponding train is not used. In order to avoid failures, corrective maintenance and associated additional costs, the maintenance of a train is triggered when its degradation reaches a certain threshold denoted $\Delta_j \in [0, 1[$. The relation between the two latter variables and the launch of maintenance is illustrated in Figure 1.

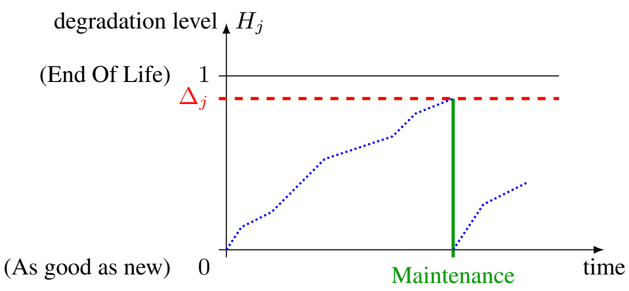

Figure 1: Evolution of a train state of health with maintenance

Each maintenance is supposed to be perfect. Then, once a maintenance operation is performed, the degradation level of the maintained train falls to 0, which means that the train is as good as new (see Figure 1). The number of maintenance operations allowed during the scheduling horizon for each train unit is fixed for each optimization problem instance and denoted K. The duration of each maintenance is supposed to be the same for each train: $pm_{j,k} = pm \ \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K$. Maintenance can furthermore be made at each train station. Then, no additional ride is required before the launch of a maintenance operation. The starting date of the k^{th} maintenance performed on the train M_j is denoted $\tau_{j,k}$ ($\forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K$).

Trains have to be used to perform n trips J_i ($1 \leqslant i \leqslant n$), following a predefined timetable. Each trip is associated to a starting date t_i and to a certain duration p_i. As the trains are supposed to be of the same type, the time needed to perform each trip is the same whatever the train that is used. If its state of health H_j is sufficient, any train can be assigned to any trip. Each trip is supposed to impact each train state of health in the same way. All the trips are however not associated to the same degradation. This degradation, denoted $\delta_i \in [0, 1]$, corresponds to a wear rate and is defined for each trip J_i as its duration p_i divided by the maximal time during which each train could be used to travel on the trip if it was associated to an infinite duration (see Equation (1) and Figure 2). This maximal time $p\text{max}_i$ can be seen as the remaining useful life of each train for the trip J_i.

$$\delta_i = \frac{p_i}{p\text{max}_i} \tag{1}$$

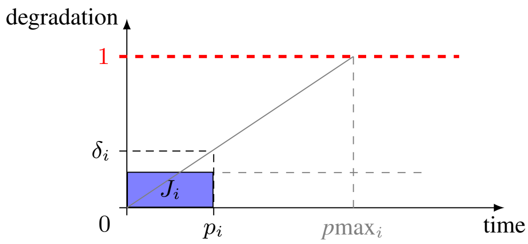

Figure 2: Definition of the degradation level associated to each train trip J_i

2.2 Optimization problem

The problem consists in assigning the appropriate train unit to each train trip, considering the prognostics information, the impact of the trip on the train state of health and the maintenance opportunities. As the assignment of trains to trips impacts directly the trains state of health, the routing problem and the maintenance one are closely related. One important part of the problem is then to maintain the trains when needed in order to avoid failures, while guarantying that as much trains as needed are available at each time to carry out all the trips defined in the considered timetable.

In order to optimize the maintenance, the objective taken into account is the maximization of the use of each train potential in terms of useful life. In other words, the aim is to schedule each maintenance task as closely as possible to the failure while avoiding it. For the considered fleet of trains, the considered objective is to maximize the minimal degradation level among those reached by all the trains before each maintenance. A mathematical expression of this objective function and the constraints associated to the considered optimization problem are detailed in next section.

2.3 Mathematical model

In order to express the objective function and the constraints associated to the optimization problem, some variables need first to be introduced. Let $x_{i,j} \in \{0,1\}$ $(1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m)$ be the binary decision variables used to define the resource assignment such that $x_{i,j} = 1$ if the train M_j is used to process the trip J_i ; $x_{i,j} = 0$ otherwise. Let $z_{i,k} \in \{0,1\}$ $(1 \leqslant i \leqslant n, 0 \leqslant k \leqslant K)$ be the binary decision variables used to schedule the maintenance operations such that $z_{i,k} = 1$ if the trip J_i is performed before the k^{th} maintenance task ; $z_{i,k} = 0$ otherwise. Let $y_{i1,i2} \in \{0,1\}$ $(1 \leqslant i1 < i2 \leqslant n)$ be the binary variable used to express the predefined precedence constraints between trips. Values associated to all the variables $y_{i1,i2}$ are set before the problem resolution according to the following convention: $y_{i1,i2} = 1$ if the trip J_{i1} ends before the starting of the trip J_{i2} ; $y_{i1,i2} = 0$ otherwise.

As mentioned in previous section, the objective is to maximize the degradation of each train before maintenance. The degradation level of each train M_j before each k^{th} maintenance is denoted $\Gamma_{j,k}$. The expression of this degradation is split in two parts for each train. First one, detailed in Equation (2), corresponds to the degradation caused by all the trips performed by the considered train M_j before the first maintenance operation ($k = 1$). This first part allows to take into account the initial degradation level H_j^0 of the train M_j. Second part, detailed in Equation (3), gathers the degradation levels of the train before each maintenance operation excluding the first one (for $2 \leqslant k \leqslant K$). Based on these two equations, the objective function can be expressed as defined in Equation (4).

$$\Gamma_{j,1} = H_j^0 + \sum_{i=1}^{n} \delta_i \cdot x_{i,j} \cdot z_{i,1} \qquad \forall\, 1 \leqslant j \leqslant m \tag{2}$$

$$\Gamma_{j,k} = \sum_{i=1}^{n} \delta_i \cdot x_{i,j} \cdot (z_{i,k} - z_{i,k-1}) \qquad \forall\, 1 \leqslant j \leqslant m, \forall\, 2 \leqslant k \leqslant K \tag{3}$$

$$\max \quad \min_{1 \leqslant j \leqslant m} \left(\Gamma_{j,1} + \sum_{k=2}^{K} \Gamma_{j,k} \right) \tag{4}$$

Constraints defined in following equations allow to take into account characteristics related to the trains, the trips and the maintenance. First set of constraints, detailed in Equation (6), ensures that a train M_j and only one is assigned to each trip J_i in the considered timetable.

$$\sum_{j=1}^{m} x_{i,j} = 1 \qquad \forall\, 1 \leqslant i \leqslant n \tag{5}$$

A train unit M_j can be assigned to two different trips J_{i1} and J_{i2} only if one of these two trips is scheduled before the other one, with no overlapping (see Equation (6)).

$$x_{i1,j} + x_{i2,j} \leqslant 1 + y_{i1,i2} \qquad \forall\, 1 \leqslant i1 < i2 \leqslant n, \forall\, 1 \leqslant j \leqslant m \tag{6}$$

A train M_j can be assigned to a trip J_i only if its state of health is sufficient, that is, if the degradation caused by the trip added to the train actual degradation level does not pass the degradation threshold Δ_j (see Equation (7)).

$$\Gamma_{j,k} \leqslant \Delta_j \qquad \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K \tag{7}$$

As stated by Equation (8), each maintenance operation can be launched on a train M_j only after the end of the trip $J_{i'}$ assigned to this train which directly precedes the maintenance. As the constraint considers all the trips J_i, for all $1 \leqslant i \leqslant n$, the term $U \cdot (1 - z_{i,k})$ has been added to arbitrarily underestimate the lower bound set for the determination of the starting date $\tau_{j,k}$ of each maintenance that has to be launched after the trip $J_{i'}$. The constant $U \in \mathbb{R}^{*+}$ should be greater than the time horizon of the considered timetable.

$$\begin{aligned}(t_i + p_i) \cdot x_{i,j} \leqslant \tau_{j,k} + U \cdot (1 - z_{i,k}) \qquad & \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \\ & \forall 1 \leqslant k \leqslant K, U \in \mathbb{R}^{*+}\end{aligned} \tag{8}$$

The last set of constraints (Equation (9)) sets that trips can start only after the end of maintenance operations. In the same way as for the previous constraint, the term $-U \cdot z_{i,k}$ has been added to arbitrarily overestimate the upper bound set for the determination of the starting date $\tau_{j,k}$ of each maintenance.

$$\begin{aligned}t_i \geqslant (\tau_{j,k} + pm) \cdot x_{i,j} - U \cdot z_{i,k} \qquad & \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \\ & \forall 1 \leqslant k \leqslant K, U \in \mathbb{R}^{*+}\end{aligned} \tag{9}$$

The mathematical program defined by the previously defined objective function and constraints is detailed in the set of Equations (10).

$$\left\{\begin{array}{lll}
\max & \min\limits_{1 \leqslant j \leqslant m} \left(\Gamma_{j,1} + \sum\limits_{k=2}^{K} \Gamma_{j,k}\right) & (10a) \\
 & \Gamma_{j,1} = H_j^0 + \sum\limits_{i=1}^{n} \delta_i \cdot x_{i,j} \cdot z_{i,1} \qquad \forall 1 \leqslant j \leqslant m & (10b) \\
 & \Gamma_{j,k} = \sum\limits_{i=1}^{n} \delta_i \cdot x_{i,j} \cdot (z_{i,k} - z_{i,k-1}) \qquad \forall 1 \leqslant j \leqslant m, \forall 2 \leqslant k \leqslant K & (10c) \\
\text{s.t.} & \sum\limits_{j=1}^{m} x_{i,j} = 1 \qquad \forall 1 \leqslant i \leqslant n & (10d) \\
 & x_{i1,j} + x_{i2,j} \leqslant 1 + y_{i1,i2} \qquad \forall 1 \leqslant i1 < i2 \leqslant n, \forall 1 \leqslant j \leqslant m & (10e) \\
 & \Gamma_{j,k} \leqslant \Delta_j \qquad \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & (10f) \\
 & (t_i + p_i) \cdot x_{i,j} \leqslant \tau_{j,k} + U \cdot (1 - z_{i,k}) \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, & (10g) \\
 & \qquad \forall 1 \leqslant k \leqslant K, U \in \mathbb{R}^{*+} & \\
 & t_i \geqslant (\tau_{j,k} + pm) \cdot x_{i,j} - U \cdot z_{i,k} \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, & (10h) \\
 & \qquad \forall 1 \leqslant k \leqslant K, U \in \mathbb{R}^{*+} & \\
\text{with} & x_{i,j} \in \{0, 1\} \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m & (10i) \\
 & z_{i,k} \in \{0, 1\} \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant k \leqslant K & (10j) \\
 & \tau_{j,k} \in \mathbb{R}^{+} \qquad \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & (10k)
\end{array}\right.$$

3 Resolution method

Linear optimization is proposed to be used to cope with the optimization problem detailed in previous section. The mathematical program previously expressed in set of Equations (10) being not linear, some modifications are mandatory.

The optimization function is first a Max-Min problem. It can be expressed as a linear function by defining an additional constraint which limits the term that has to be minimized. For recall, this term corresponds to the minimal cumulative degradations before maintenance among trains, which depends on the problem solution. Then, a variable denoted $\widetilde{\Gamma}$ is introduced and the optimization function defined by Equation (10a) in the mathematical program is replaced with the Equation (11), associated to the constraint in Equation (12).

$$\max \quad \widetilde{\Gamma} \tag{11}$$

$$\widetilde{\Gamma} \leqslant \sum_{k=1}^{K} \Gamma_{j,k} \qquad \forall 1 \leqslant j \leqslant m \tag{12}$$

The product of the two binary variables $x_{i,j}$ and $z_{i,k}$ has also to be linearized. This can be done by introducing a new variable $s_{i,j,k} \in \mathbb{R}$ associated to the constraints detailed in the set of Equations (13) (Billionnet, 2007).

$$\begin{cases} s_{i,j,k} \leqslant x_{i,j} \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(13a)} \\ s_{i,j,k} \leqslant z_{i,k} \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(13b)} \\ 1 - x_{i,j} - z_{i,k} + s_{i,j,k} \geqslant 0 \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(13c)} \\ s_{i,j,k} \geqslant 0 \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(13d)} \end{cases}$$

Finally, the product of the real variable $\tau_{j,k}$ with the binary variable $x_{i,j}$ is linearized by introducing the variable $e_{i,j,k} \in \mathbb{R}$ with the additional constraints defined in the set of Equations (14) (Billionnet, 2007). In these equations, *tmax* corresponds to an upper bound for each $\tau_{j,k}$: $\tau_{j,k} \in [0, \mathit{tmax}]$ $\forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K$. *tmax* can be expressed as the maximal date in the considered train timetable to which we subtract the duration pm of a maintenance.

$$\begin{cases} e_{i,j,k} \leqslant x_{i,j} \cdot \mathit{tmax} \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(14a)} \\ e_{i,j,k} \leqslant \tau_{j,k} \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(14b)} \\ e_{i,j,k} \geqslant \tau_{j,k} - (1 - x_{i,j}) \cdot \mathit{tmax} \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(14c)} \\ e_{i,j,k} \geqslant 0 \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(14d)} \end{cases}$$

The whole linear program associated to the considered optimization problem is detailed in the set of Equations (15). Solving this linear program with a fixed number of maintenance for each train (K) allows to obtain an optimal assignment of trains to the trips in the considered timetable, associated to an optimal scheduling of maintenance operations. The proposed linear program allows to find solutions in reasonable time only for small size instances of the problem, that is for problem instances considering a limited number of trains and a limited number of trips.

$$
\left\{
\begin{array}{lll}
\max & \widetilde{\Gamma} & \text{(15a)} \\
& \widetilde{\Gamma} \leqslant \sum_{k=1}^{K} \Gamma_{j,k} \qquad \forall 1 \leqslant j \leqslant m & \text{(15b)} \\
& \Gamma_{j,1} = H_j^0 + \sum_{i=1}^{n} \delta_i \cdot s_{i,j,1} \qquad \forall 1 \leqslant j \leqslant m & \text{(15c)} \\
& \Gamma_{j,k} = \sum_{i=1}^{n} \delta_i \cdot (s_{i,j,k} - s_{i,j,k-1}) \qquad \forall 1 \leqslant j \leqslant m, \forall 2 \leqslant k \leqslant K & \text{(15d)} \\
& s_{i,j,k} \leqslant x_{i,j} \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(15e)} \\
& s_{i,j,k} \leqslant z_{i,k} \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(15f)} \\
& 1 - x_{i,j} - z_{i,k} + s_{i,j,k} \geqslant 0 \; \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(15g)} \\
& s_{i,j,k} \geqslant 0 \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(15h)} \\
\text{s.t.} & \sum_{j=1}^{m} x_{i,j} = 1 \qquad \forall 1 \leqslant i \leqslant n & \text{(15i)} \\
& x_{i1,j} + x_{i2,j} \leqslant 1 + y_{i1,i2} \qquad \forall 1 \leqslant i1 < i2 \leqslant n, \forall 1 \leqslant j \leqslant m & \text{(15j)} \\
& \Gamma_{j,k} \leqslant \Delta_j \qquad \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(15k)} \\
& (t_i + p_i) \cdot x_{i,j} \leqslant \tau_{j,k} + U \cdot (1 - z_{i,k}) \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, & \text{(15l)} \\
& \qquad \forall 1 \leqslant k \leqslant K, U \in \mathbb{R}^{*+} & \\
& t_i \geqslant pm \cdot x_{i,j} + e_{i,j,k} - U \cdot z_{i,k} \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, & \text{(15m)} \\
& \qquad \forall 1 \leqslant k \leqslant K, U \in \mathbb{R}^{*+} & \\
& e_{i,j,k} \leqslant x_{i,j} \cdot \mathit{tmax} \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(15n)} \\
& e_{i,j,k} \leqslant \tau_{j,k} \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(15o)} \\
& e_{i,j,k} \geqslant \tau_{j,k} - (1 - x_{i,j}) \cdot \mathit{tmax} \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, & \text{(15p)} \\
& \qquad \forall 1 \leqslant k \leqslant K & \\
& e_{i,j,k} \geqslant 0 \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(15q)} \\
\text{with} & x_{i,j} \in \{0,1\} \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m & \text{(15r)} \\
& z_{i,k} \in \{0,1\} \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant k \leqslant K & \text{(15s)} \\
& s_{i,j,k} \in \mathbb{R} \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(15t)} \\
& e_{i,j,k} \in \mathbb{R} \qquad \forall 1 \leqslant i \leqslant n, \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(15u)} \\
& \tau_{j,k} \in \mathbb{R}^{+} \qquad \forall 1 \leqslant j \leqslant m, \forall 1 \leqslant k \leqslant K & \text{(15v)}
\end{array}
\right.
$$

4 Demonstration of the problem

The use of the proposed linear program is illustrated on a simple use case with $m = 2$ trains, $n = 5$ trips and $K = 1$ maintenance allowed for each train. The characteristics taken into account are detailed in the two next tables. In Table 1, the initial degradation level H_j^0, the degradation threshold Δ_j, the number K of maintenance allowed and the duration of each maintenance are shown for each train M_j ($1 \leqslant j \leqslant 2$).The duration of each maintenance is supposed to be the same for each train: $pm = 1$ unit of time. In Table 2, the starting date t_i, the duration p_i and the degradation rate δ_i are shown for each trip J_i ($1 \leqslant i \leqslant 5$).

Table 1: Trains characteristics

Train M_j	H_j^0	Δ_j	K	pm
M_1	0.2	0.9	1	1
M_2	0.3	0.9	1	1

Table 2: Trips characteristics

Trip J_i	t_i	p_i	δ_i
J_1	8	2	0.4
J_2	9	2	0.6
J_3	12	1	0.19
J_4	15	1	0.06
J_5	16	2	0.2

The solution obtained for this use case with the proposed linear program is depicted in Figure 3. One can see that one maintenance has been scheduled for each train, allowing the respect of the train timetable.

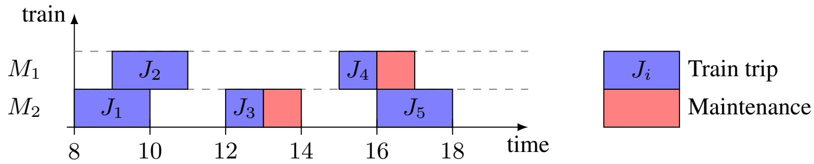

Figure 3: Schedule obtained with the linear program for the considered use case

5 Conclusion

A joint optimization of train assignment and maintenance scheduling has been proposed in a Prognostics and Health Management (*PHM*) context. Advantage is taken from the knowledge of prognostics information in the form of degradation levels to launch maintenance operations only when they are needed, that is when degradation levels have reached a fixed threshold. A mathematical formulation has been detailed, including an objective function which aims to minimize the degradation level reached before each maintenance and several constraints related to the railway application context. Linear programming has been proposed to tackle the considered optimization problem. The use of the proposed linear program has been illustrated on a simple use case with few trains and few trips.

As future work, performance of the proposed optimal approach will be assessed through exhaustive simulations for small size instances of the optimization problem. For more realistic problem sizes, with more trains and more trips, defining scalable heuristics will be mandatory.

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