FROM INFORMATIONALLY COMPLETE POVMS TO THE KOCHEN-SPECKER THEOREM

MICHEL PLANAT

Université de Bourgogne/Franche-Comté, Institut FEMTO-ST CNRS
UMR 6174, 15 B Avenue des Montboucons, F-25044 Besançon, France.
michel.planat@femto-st.fr

Abstract of talk

It is possible to conciliate informationally complete (IC) measurements on an unknown density matrix and Kochen-Specker (KS) concepts (which forbid hidden variable theories of a non-contextual type). This was shown in [1] for qutrits and it is continued here for two-qubits (2QB), three-qubits (3QB) and two qutrits (2QT). Non symmetric IC-POVMs have been found in dimensions 3 to 12 starting from permutation groups, the derivation of appropriate non-stabilizer states (magic states) and the action of the Pauli group on them [2, 3]. For 2QB, 3QB and 2QT systems, Kochen-Specker theorem follows.

For $2QB$, the magic state is of type $(0,1,\omega,\omega-1)$, $\omega=\exp(\frac{i\pi}{3})$, and the IC-POVM manifests dichotomic trace products of projectors $\Pi_i=|\psi_i\rangle\langle\psi_i|$ as $\mathrm{tr}(\Pi_i\Pi_j)_{i\neq j}=|\langle\psi_i|\,\psi_j\rangle|^2_{i\neq j}\in\{\frac{1}{3},\frac{1}{3^2}\}$. The triple products of projectors whose trace is $\pm\frac{1}{27}$, and simultaneously equal plus or minus the identity matrix, are organized as a (3×3)-grid. Taking the vertices of the grid as the 2QB Pauli group operators acting on the magic state instead of the corresponding projectors one recovers the standard form of Mermin square -that is used as an operator proof of the KS theorem.

For $3QB$, the Hoggar magic state $(-1\pm i,1,1,1,1,1,1,1)$ leads to a SIC. Within the 4032 triples whose trace of triple products equal $-\frac{1}{27}$ [4], those whose product of projectors equal plus or minus the identity are organized into a geometric configuration $[63_3]$ whose automorphism group $G_2(2)=U_3(3)\rtimes\mathbb{Z}_2$ is of order 12096 and corresponds to the generalized hexagon $GH(2,2)$ (or its dual). These configurations are related to the 12096 Mermin pentagrams that build a proof of the three-qubit Kochen-Specker theorem [5]. From the structure of hyperplanes of our $[63_3]$ configuration, one learns that we are concerned with the dual of G_2.

Finally for 2QT, a magic state such as $(1,1,0,0,0,0,-1,0,-1)$ may be used to generate an IC-POVM with dichotomic pairwise products $|\langle\psi_i|\,\psi_j\rangle|^2_{i\neq j}\in\{\frac{1}{4},\frac{1}{4^2}\}$. Defining lines as triple of projectors with trace $\frac{1}{8}$, one gets a geometric configuration of type $[81_3]$ that split into nine disjoint copies of type $[9_3]$. Each of them can be seen as a 3QT proof of KS theorem since the product law for the eigenvalues of 2QT operators O_i, that is $\nu(\Pi_{i=1}^9 O_i)=\Pi_{i=1}^9[\nu(O_i)]$,

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is violated. The left hand side equals ω_3 while the right hand side equals ± 1 [3]. No non-contextual hidden variable theory is able to reproduce these results.

References

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