Optimal design of a unimorph piezoelectric cantilever devoted to energy harvesting to supply animal tracking devices

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Abstract: This paper presents the mechanical structure optimization of a piezoelectric energy harvesters devoted to supply embedded animal tracking devices. The harvester mechanical structure being a bilayer unimorph piezoelectric cantilever is composed of two elements: a piezoelectric layer and a non-piezoelectric layer (i.e, active and passive layers). The suggestion of this paper is to find the optimal ratio between their two thicknesses in order to maximize the output recuperated voltage at the electrodes. For that, from the model that links the mechanical excitation moment and the output electrical charge, a gradient-based optimization is carried-out. Comparison with existing mechanical piezoelectric harvester structure is made and which clearly demonstrates that the proposed structure permits to gain up to five-times in terms of the output charge and a significant gain in terms of output electrical power for the same condition.

Keywords: Energy harvesting, piezoelectric harvester, smart structures, structural optimization

1. INTRODUCTION

Motivated by the increasing trend towards autonomous miniaturized apparatus with low energy consumption, energy harvesting (EH) as power supply has been given many attention within the last twenty years. Vibrational piezoelectric energy harvester (vPEH) is the most studied for that as it suggests several features: utilizable everywhere, miniaturization possibility, low-cost, and ease and rapid to setup relative to other energy harvesters. VPEH consists in harvesting the energy from ambient vibrations and transforming this into electrical energy thanks to a piezoelectric transducer. A vPEH is composed of a mechanical structure, the transducer itself, which provides a sine voltage on its electrodes when vibrated, and an electrical circuit which permits to put this latter into condition for a more usable regulated voltage [1]. The most classical and used transducer is a unimorph piezoelectric structure, composed of two layers (a piezoelectric layer and a non-piezoelectric layer). They are easy to fabricate and are widely available in commerce. Figure 1 depicts such structure with a basic electrical circuit based on a four-diodes rectifier, a smoothing capacitance and a resistive load. Roughly, the output power and the output regulated voltage at the load increase with the vibrations amplitude and frequency, with the quality of the piezoelectric material, and with the quality of the electrical circuit.

Strong efforts have initially been focused on high frequency vibrations in order to maximize the output power but these frequencies are not readily available in the environment [2]. In the search of paradigm in vPEH capable of furnishing sufficient power from ambient vibrations, recent researches permit to reduce the operating frequencies from several

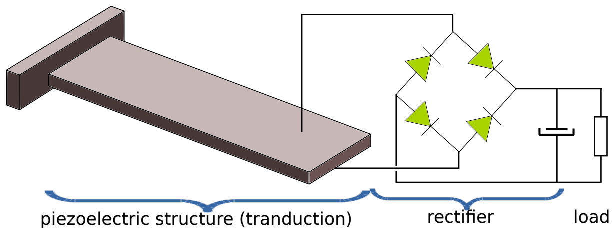

Fig. 1. Scheme of basic electric circuit for the energy harvesting of a cantilever

kiloHertz to a range between 10 and 300Hz. They can be classified into five major categories: i) multilayered vPEH [3-6] which consists in increasing the layers number in the mechanical structure, ii) hybrid vPEH [7-9] which combines vPEH with another harvester technique (electrostatic, magnetic...) within the same harvesting device, iii) electrical circuit improvement [10-13], iv) broadband vPEH [14-17] which adapts the mechanical structure to be sensitive to several vibrations frequencies, v) and novel piezoelectric materials studies [18-20].

Within the project context of elaboration of vPEH for powering animal tracking devices [21], this paper deals with the study and optimization of the mechanical transducer structure in order to obtain maximized voltage at the electrodes of the piezoelectric cantilever. Relative to multilayers and to hybrid approaches referenced above, the advantage here is to maintain purely piezoelectric harvester with two layers (unimorph bilayer) such that we ensure a high level of miniaturization and we ease the fabrication and setting up. The target is to find the optimal thicknesses ratio of the two layers that compose the transducer structure so that the output voltage on the electrodes be maximized. As this voltage is related directly to the charge furnished by the piezoelectric harvester, we focus here on the optimization of this charge. For that, a theoretical model of the harvester is derived following the development and the equations given in [25]. The model links the applied moment that comes from external vibration and the charge caused by the vibration of the harvester. Then a gradient-based optimization technique is applied to maximize the provided charge where the optimization variable is the ratio between the two layers thicknesses. The obtained results clearly show that, face to the existing bilayer structures, we can gain 4-times in terms of the output charge and a significant gain in terms of output power with the same conditions of utilization (excitation frequency and amplitude).

The paper is organized as follows. In section-2 we remind the governing equations of the mechanical structure (the bilayer cantilever) in a vPEH. Section-3 is devoted to the proposed optimization approach as well as simulation comparison with the existing structures. Discussions related to the results are presented in section-4. Finally, section-5 summarizes the paper and presents some perspectives.

2. SIMPLIFIED MODEL OF AN UNIMORPH BI-LAYERED CANTILEVER

2.1 Multimorph model

We first introduced the idea of a n-layered cantilever as shown of figure 2. Each of the layers can be either being made of piezoelectric material, therefore called active layers, or non piezoelectric material, therefore called passive layers. A cantilever having n_p layer, with $n_p < n$, is called a n_p-morph and has a n-layered structure [25]. Our goal is to optimize a given n_p-morph n-layered for an energy harvesting application. In order to amorce this work, we have focus on the optimization of an 1-morph 2-layered cantilever, simply called unimorph bi-layered as shown on figure 2.

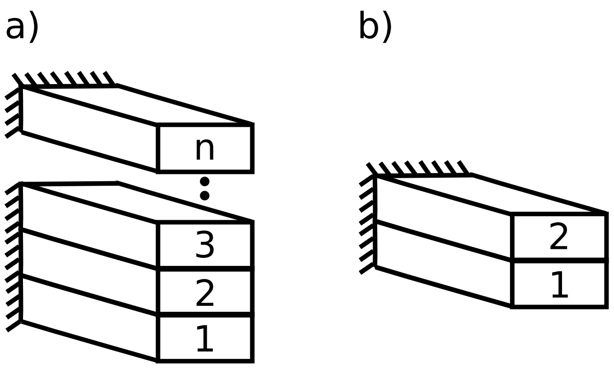

Fig. 2. Representation of cantilever a) n-layered b) unimorph bi-layered

We now consider the unimorph bi-layered cantilever. We note the length l, the different width are imposed constant and equal to e, the thickness of the i^{th} layer is denoted h_i, the figure 3 resuming this all. Furthermore we denote the elastic coefficient of the active (respectively passive) layer s_1 (respectively s_2). The elastic coefficient correspond to $s_i = s_{11,i} = \frac{1}{E_{11,i}}$, $E_{11,i}$ being the axial elasticity or Young modulus along x-axis. Finally we denote the transversal piezoelectric coefficient of the active layer d_{31} (if the layer is passive we have $d_{31} = 0$).

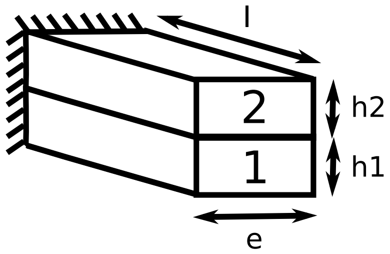

Fig. 3. Dimensions of unimorph bi-layered

2.2 General model

In this subsection we derive the analytical model of the charge $Q(x,t)$ when applying an harmonic moment $M(t)$ in the form of $M(t) = M_0 \cos(\Omega t)$ with M_0 being the amplitude in $N.m$ and Ω the pulsation in rad/s. The model of the piezoelectric cantilever can be derived from the general form in [25]. We choose to focus the analysis only on the first resonnant from the statement that it is where we will obtain the maximal transmittance. This allow us to obtain the following model :

$$Q(x,t) = Z(x,t) M_0 \cos(\Omega t - \Psi) \quad (1)$$

with :

$$Z(x,t) = m_{piezo} \frac{4}{l^2 \mu} \frac{dX}{dx}(x) \frac{(kl)\alpha_M(kl)}{\omega^2 \sqrt{(1-\eta^2)^2 + (2\zeta\eta)^2}} \quad (2)$$

where $\zeta < 1$ is the damping coefficient of the first mode and μ the area mass : $\mu = \frac{m}{l}$. The mass m can be obtained using $m = el(h_1\rho_1 + h_2\rho_2)$ with ρ_i the density of the i^{th} layer. Coefficient η is the ratio between the excitation frequency Ω and the undamped oscillation ω : $\eta = \frac{\Omega}{\omega}$, such that $\omega = \frac{(kl)^2}{l^2}\sqrt{\frac{C}{\mu}}$, where $(kl) \simeq 1.8751$ for the considered first mode. The coefficients C (flexural rigidity) and m_{piezo} can be expressed for our cantilever structure and using the hypothesis mentioned on subsection 2.1 as follows:

$$C = e \frac{h_2^4 s_1^2 + 2h_1h_2(2h_1^2 + 3h_1h_2 + 2h_2^2)s_1s_2 + h_1^4 s_2^2}{12 s_1 s_2 (h_2 s_1 + h_1 s_2)} \quad (3)$$

$$m_{piezo} = -e \frac{d_{31} h_2 (h_1 + h_2)}{2(h_2 s_1 + h_1 s_2)} \quad (4)$$

The remaining coefficients of (2) are described in table 1.

2.3 Simplified model

We shall now make assumptions in order to simplify the previous model. First we replace the expression for the pulsation ω presented above. Then we assume $x = l$, meaning Z represent the transmittance on the whole length of the cantilever. Furthermore for an easy reading we now write $Z(x)$ as Z. One last hypothesis is taken as $\eta = 1$, meaning $\Omega = \omega$. This is only to further simplify the transmittances as if we don't excite the system on it natural frequency we will only obtain a lowered transmittance. All of those assumptions leads to:

$$Z = \frac{2lm_{piezo}}{C}\tilde{X}\frac{\alpha_M(kl)}{(kl)^3\zeta} \tag{5}$$

with

$$\tilde{X} = \tilde{S}(kl) - \tilde{c}(kl)\frac{\tilde{C}(kl)}{\tilde{S}(kl)} \tag{6}$$

Table 1. Parameters for the transmittance

Parameter	Formula	Name
Ψ	$= \arctan(\frac{2\zeta\eta}{1-\eta^2})$	Phase
$X(x)$	$= \tilde{c}(kx) - \tilde{s}(kx)\frac{\tilde{C}(kl)}{\tilde{S}(kl)}$	Eigenmode
$\tilde{c}(x)$	$= \frac{1}{2}(\cosh(x) - \cos(x))$	
$\tilde{s}(x)$	$= \frac{1}{2}(\sinh(x) - \sin(x))$	
$\tilde{C}(x)$	$= \frac{1}{2}(\cosh(x) + \cos(x))$	
$\tilde{S}(x)$	$= \frac{1}{2}(\sinh(x) + \sin(x))$	
$\alpha_M(x)$	$= \frac{\sinh(x)\sin(x)}{\sinh(x)+\sin(x)}$	

3. GEOMETRICAL AND MECHANICAL OPTIMIZATION

In this section we explain the process used to optimize the thickness h_i and the compliance s_i of each layer to provide the optimal charge transmittance.

3.1 Optimization function

Our goal is to optimize the transmittance between the harmonic moment and the charge generation, hence the Z value. The four parameters which can be optimized are the thickness and compliance of each layer, namely $\{h_1, s_1, h_2, s_2\}$. This is the result of the observation that every other parameter is either a constant ($\tilde{X}$, α_M and (kl)), an independent experimental value (ζ) or has a direct linear link to the transmittance (d_{31} and l) and hence no optimization needed. Therefore we separate the optimization parameters, leading to the following expression:

$$Z = G \times (-12ld_{31}\tilde{X}\frac{\alpha_M(kl)}{(kl)^3\zeta}) \tag{7}$$

with G being only a function of $\{h_1, h_2, s_1, s_2\}$:

$$G = \frac{s_1 s_2 h_2(h_1 + h_2)}{h_2^4 s_1^2 + 2h_1h_2(2h_1^2 + 3h_1h_2 + 2h_2^2)s_1s_2 + h_1^4 s_2^2} \tag{8}$$

We simplified the study of this function by working with the ratios of both the thicknesses and compliances. We choose to express those ratios as $\lambda = \frac{h_2}{h_1}$ and $\mu = \frac{s_1}{s_2}$. This allow to express G as follow:

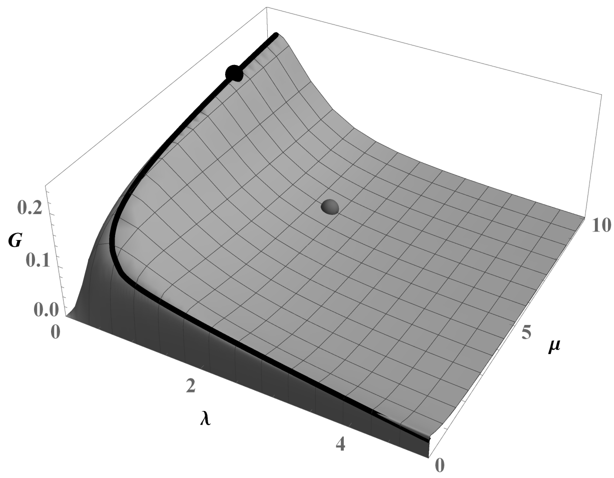

Fig. 4. 3D plot of optimization function G for the λ and μ parameters; the thick black line represent λ_{opt} as a function of μ; the gray sphere in the middle of the surface is the paper configuration, the black one the suggested optimum

$$G = \frac{1}{h_1^2}\frac{\lambda(1+\lambda)\mu}{1 + 4\lambda\mu + 6\lambda^2\mu + 4\lambda^3\mu + \lambda^4\mu} \tag{9}$$

This new expression highlight the direct link between G and the thickness of the first layer h_1. Consequently it can be seen as a parameter without optimization and is removed from the function. This finally lead to the full optimization function :

$$G = \frac{\lambda(1+\lambda)\mu}{1 + 4\lambda\mu + 6\lambda^2\mu + 4\lambda^3\mu + \lambda^4\mu} \tag{10}$$

3.2 optimization process

We show on figure 4 the 3D curve of G from the *Mathematica*© software. We can see two tendencies depending on the observed parameter: when following the λ axis, we observe a potential optimum appearing in the interval $[0; 1]$. But when we follow the μ axis the dependence seems to follow a square root tendency that would lead to no optimization.

In order to verify those conjectures, we use a simple process: searching for the cancellation of the first gradient. To do so we first calculate it and take only the numerator [1]

$$\nabla G = \begin{pmatrix} \frac{\partial G}{\partial \lambda} \\ \frac{\partial G}{\partial \mu} \end{pmatrix} = \begin{pmatrix} \mu(1 - \lambda^2(3+2\lambda)\mu)(1 + \lambda(2+\lambda\mu)) \\ \lambda(1+\lambda)(1 - \lambda^4\mu^2) \end{pmatrix} \tag{11}$$

Trying to cancel the whole gradient lead to no solution, meaning there is no global optimum. Nevertheless when cancelling only the first derivative and searching for a solution for the λ parameter allow to obtain one positive optimal value now denoted λ_{opt}:

[1] The denominator verify $Denominator(G) > 1$ for $\lambda > 0$ and $\mu > 0$ and thus do not provide information for the gradient

$$\lambda_{opt} = \frac{1}{2}(\beta + \frac{1}{\beta} - 1) \quad (12)$$

with

$$\beta = \left(\frac{2 + 2\sqrt{1-\mu} - \mu}{\mu}\right)^{1/3} \quad (13)$$

We show on figure 4 the evolution with ratio μ of this optimum value on the surface of function G.

We now have obtained an optimum ratio λ_{opt} for the thicknesses of the unimorph bi-layered cantilever but we have also proved that there exist no optimum for the ratio of the compliances μ. Nevertheless we suggest to use $\mu = 7.5$ as a maxima for the μ variable, thus now called μ_{max}. This choice can be explained by knowing that $\lim_{\mu\to\infty} G(\mu) = \frac{1}{4}$ and taking in account that $G(\mu_{max}) \simeq 0.197$, meaning we are already close to the upper limit of G. As an example, if we wanted to gain another 10% for the value of G, we would need to have $\mu \simeq 30$, resulting to a value four time higher than μ_{max}.

3.3 Simulation of optimization on existing configuration

In order to show the interest of the optimization we suggest to compare the configuration of a unimorph cantilever and an optimised one by our method. Because we optimised only ratios of parameters, we need to fix two of the four intial parameter in order to obtain a result. We focus on the optimization of the passive layer, hence h_2 and s_2 parameters, as the active layer is generally imposed beforehand. The initial structure parameters from [5] are shown in table 2.
Those values lead to a value of $G \simeq 0.036$, only 18% of the suggested optimal value of 0.2. If we calculate the μ coefficient, hence s_1/s_2, of the initial configuration we obtain $\mu \simeq 6.42$, meaning we are already really close to the μ_{max} value of 7.5. Consequently, and to simplify the study, we choose to maintain the type of material and only calculate the optimal thickness base on this value. This result in $\lambda_{opt}(6.42) \simeq 0.213$. As $\lambda = h_2/h_1$ we finally obtain $h_{2opt} \simeq 21 \times 10^{-6}m$, leading to a value of $G \simeq 0.194$, meaning a increase of +440% for the transmittance, additionnaly with an decreased total thickness of 60%. The position of the initial and optimal design on the surface of function G are shown on figure 4 using spheres, which enhance the fact that we were in the bottom of the surface before the optimization.

Table 2. Initial parameters of cantilever

Parameter	Value
h_1	$100 \times 10^{-6}m$
s_1	$60 \times 10^{-12}Pa^{-1}$
h_2	$200 \times 10^{-6}m$
s_2	$1/107 \times 10^{9}Pa^{-1}$

Moreover we can calculate the natural resonnance frequency $f = \omega/(2\pi)$ of the initial and optimal structure. For that we need the additionnal set of parameters specified in table 3, still taken from the structure of [5]. The active layer is taken as PMN-PT and the passive one as Silicone. We easily obtain with the formulas in section 2.2, $f_{init} \simeq 720Hz$ and $f_{opt} \simeq 190Hz$. This result, roughly a 75% drop of the natural resonnance frequency after optimization, increase the interest of the method, as natural vibration sources are mostly found in lower frequencies [2]. To show this we plot the transmittance of both the initial and optimal structure versus the excitation frequency on figure 5. We can clearly see that for lower frequency we have a higher transmittance, around 5.5 times as stated before, resulting in a maximal transmittance of $Z_{max} = 131 \times 10^{-6}C/(N.m)$ when exciting at the resonnance frequency.

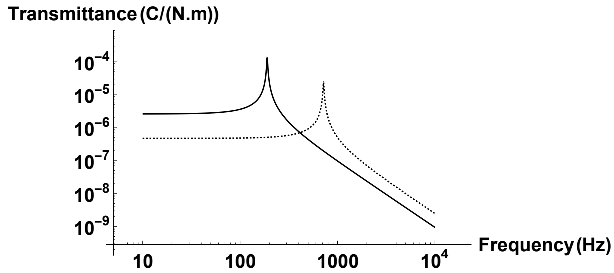

Fig. 5. Plot of the transmittance in C/(N.m) of the initial (dashed line) and optimal (solid line) versus the excitation frequency in Hz; logarithmic scale on both axis

Table 3. Aditionnal parameters of cantilever

Parameter	Value
ζ	0.01
l	$15 \times 10^{-3}m$
e	$2 \times 10^{-3}m$
ρ_1	$8200kg/m^3$
ρ_2	$2330kg/m^3$

4. DISCUSSION

The optimization approach utilized in this paper to find the mechanical structure that provides a maximized voltage is based on the gradient of the moment-to-charge gain G. Another approach that could be used consists in using interval techniques where parameters and variables are written with intervals. The principal advantage of using intervals and related algorithms is the guarantee of the solutions or of non-solutions for given specifications (e.g. for a predefined specified output voltage) and for a range of research (e.g. for a given range of dimensions). In fact, interval techniques [22] have been used and have demonstrated their efficiency [23-24] to design optimal piezoelectric actuator structures. A further work would therefore be to reformulate the same approach for piezoelectric harvesters applications.

Furthermore, we can highlight the effect of the optimization on the moment-to-deflexion transmittance. Indeed it can be easily written using already defined coefficients in section 2.2 :

$$\delta(x,t) = Z_{\delta}(x,t) M_0 \cos(\Omega t - \Psi) \quad (14)$$

with

$$Z_\delta(x,t) = -\frac{4}{l^2\mu}X(x)\frac{(kl)\alpha_M(kl)}{\omega^2\sqrt{(1-\eta^2)^2+(2\zeta\eta)^2}} \qquad (15)$$

Using the same hypothesis as for the charge transmittance, we can calculate it value for the initial configuration and the optimal one. This lead to a transmittance increase of up to 21 times for the optimal configuration, meaning optimising the charge transmittance is thightly linked to the optimization of the deflexion one. Finally this result will be used for future work to estimate the overall power gain by adding the model energy harvester electric circuit shown on figure 1.

5. CONCLUSION

In this paper, the design of a piezoelectric cantilever devoted to energy harvesting applications is presented. It is based on a bilayer structure composed of a piezoelectric layer and non-piezoelectric layer. This simple configuration has been chosen for many raisons: (i) ensuring a high level of miniaturization (ii) easing the fabrication (iii) simplifying the setting up. Basically, this structure provides electrical charge responding to external excitation such as vibration. Seeking to maximize this charge, we completed several steps. First, we derived the theoretical model of the structure, which links the external excitation to the furnished charge. Then, we applied a gradient-based optimization technique to find the optimal thicknesses ratio that increases the output charge. As a result, the structure transmittance is maximized and by the way allows to produce more charge for a reduced size. After that, a comparison is made with existing piezoelectric harvester structures and demonstrates clearly that the proposed structure allows gain up to five-times of charge with a significant gain of electrical power for the same conditions. The structure showed a great promise in several applications, in particular the powering of embedded animal tracking devices.

Ongoing and future woks include the fabrication and the experimental characterization of the proposed structure. Future works will focus also on the design methodologies such as interval techniques [22], based on control theory tools, in order to optimize the structure dimensions and its performances.

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