Dealing with redundancy of a multiple mobile coil magnetic manipulator: a 3RPR magnetic parallel kinematics manipulator

Baptiste Véron, Arnaud Hubert, Joel Abadie and Nicolas Andreff

Abstract This paper presents a magnetic manipulation system composed of three mobile electromagnets. This system is used to control the position and the orientation of a capsule embedding a small permanent magnet in the horizontal plane. The kinematico-magnetic redundancy of the system is dealt with by imposing the planar $3R\underline{P}R$ parallel kinematics constraints. The resulting controller is demonstrated *in silico*.

Key words: Magnetic manipulation, Mobile electromagnets, Redundancy, Control, Parallel kinematics.

1 INTRODUCTION

The main existing systems dedicated to contactless manipulation of a magnetic object can be divided in several categories. In [14], we introduced a kinematic criterion which clusters most of the existing systems into two categories: those using *static electromagnets* [2, 7, 8, 11, 15] and those using *mobile permanent magnet(s)* [1, 3–5, 9, 10].

For the remaining systems, a third category emerges: systems using *mobile electromagnets*. This category has been very little studied so far. More, most of the systems that belong to this category have a limited number of degrees of freedom per electromagnet and use a classical architecture with coils in Helmholtz and Maxwell configuration [16, 17].

We propose here to study a system with 3 mobile electromagnets used to control motion of magnetic capsule in the plane. Unlike what is done on most of the systems found in the literature, both movements and supplied currents of the coils are controlled here, which results in a complex non-linear control

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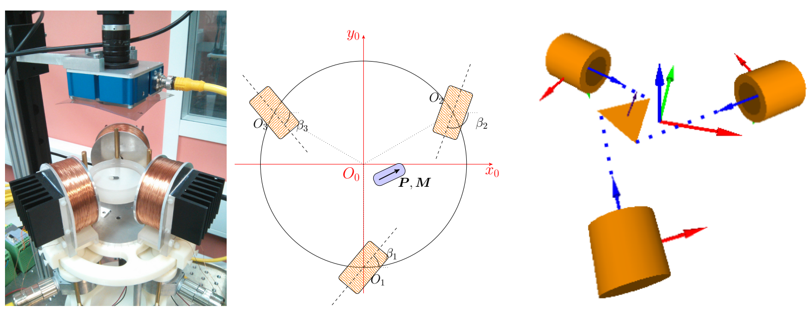
problem. Specifically, the system is *kinematico-magnetically* redundant, because it possesses 6 inputs (3 currents + 3 electromagnet orientations in the horizontal plane) for only 3 outputs (position and orientation of the magnetic capsule in the plane). One way to deal with this redundancy, presented here for the first time, is to impose a kinematic constraint and convert this system into a $3R\underline{P}R$ magnetic parallel manipulator, where mechanical prismatic actuators are replaced by magnetic contactless actuators. Potential interests for such an architecture are: i) it can work in a cluttered environment without the arms sweeping the workspace and ii) it reduces the ratio between the displaced mass and the manipulator masses.

The system studied is described in Section 2. Then, the system model and control law is explained in Sections 2.1 and 3. Finally, results obtained in simulation are shown in Section 4 with emphasis on kinematic issues.

2 SYSTEM DESCRIPTION

Our system is composed of a permanent magnet placed inside a capsule which is controlled in the horizontal plane (3 degrees of mobility). The control is performed by three electromagnets ($n = 3$) placed in an original architecture presented in [12]. As shown in Fig. 1, each electromagnet has one kinematic degree of freedom: a rotation around the vertical axis.



(a) Real system. (b) Schematic description (top-view). (c) A $3R\underline{P}R$ magnetic parallel manipulator.

Fig. 1 System description.

The system control diagram is presented in Fig. 2. It is a closed loop control composed with a *Perception* block for detecting the capsule current position and orientation. This data is provided to a *Trajectory* block where it is compared with the time-varying desired position to determine the desired accelerations for following this trajectory. The $\begin{bmatrix} m_c \\ \underline{\underline{I_c}} \end{bmatrix}$ block computes the efforts to be applied to the capsule thanks to Newton's law. Finally, the

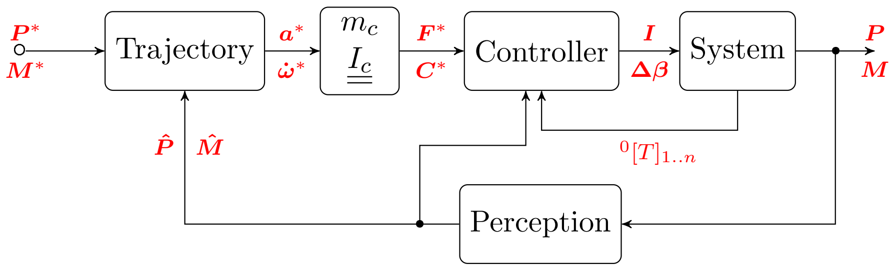


Fig. 2 Control diagram of the system.

Controller block computes the system inputs (currents in the coils $\mathbf{I}$, motion of the coils $\boldsymbol{\Delta\beta}$).

This control law relies on the direct electromagnetic model (which is established following the methodology explained in [8] while complementing the model with the coils mobility) and deals with the redundancy.

2.1 DIRECT ELECTROMAGNETIC MODEL

Most of the literature assumes that the magnetic field ${}^i\boldsymbol{B}_i$ produced at the capsule position ${}^i\boldsymbol{P}$ by the ith electromagnet is proportional to the current I_i flowing through the electromagnet:

$$ {}^i\boldsymbol{B}_i = I_i \cdot {}^i\boldsymbol{b}_i({}^i\boldsymbol{P}) \tag{1} $$

with ${}^i\boldsymbol{b}_i({}^i\boldsymbol{P})$ the magnetic field per current unit created by electromagnet i.

A global reference frame $\mathcal{F}_0$ is defined at the system centre (see Fig. 1(b)). Each electromagnet orientation is defined by an angle β_i, thus the rotation matrix ${}^0\mathbf{R}_i = Rot(\beta_i, \boldsymbol{z})$ represents the transformation between the local reference frame $\mathcal{F}_i$ and $\mathcal{F}_0$. As a result, the magnetic field ${}^0\boldsymbol{B}_i$ produced by an electromagnet is computed in the global frame as:

$$ {}^0\boldsymbol{B}_i({}^0\boldsymbol{P}, \beta_i) = I_i \cdot {}^0\mathbf{R}_i \cdot {}^i\boldsymbol{b}_i({}^0\mathbf{R}_i^T\, {}^0\boldsymbol{P} + {}^0\boldsymbol{t}_i) \tag{2} $$

with ${}^0\boldsymbol{t}_i = \boldsymbol{O_0 O_i}$, the translation vector defining the origin of $\mathcal{F}_i$.

Unlike the model used in [8], equation (2) clearly shows the dependence of the magnetic field to the coil variable poses, thanks to ${}^0\mathbf{R}_i = Rot(\beta_i, \boldsymbol{z}_0)$. The notation ${}^0\boldsymbol{B}_i({}^0\boldsymbol{P}, \beta_i)$ is simplified by ${}^0\boldsymbol{B}_i$ in the sequel.

The interaction between this field and a magnetic capsule creates efforts on this capsule given by [6]:

$$ {}^0\boldsymbol{F}_i = V \cdot \boldsymbol{\nabla}({}^0\boldsymbol{M} \cdot {}^0\boldsymbol{B}_i) \tag{3} $$

$$ {}^0\boldsymbol{C}_i = V \cdot {}^0\boldsymbol{M} \wedge {}^0\boldsymbol{B}_i \tag{4} $$

with $\boldsymbol{\nabla}$ the gradient operator, $\wedge$ the cross product, V the volume of the magnet inside the capsule, and ${}^0\boldsymbol{M}$ its magnetisation.

It is interesting to note that the permanent magnet moment is fixed inside the capsule, thus its magnetisation ${}^0\boldsymbol{M}$ is a good indication of the capsule orientation. Moreover, we assume that the magnetic fields produced by the system are not powerful enough to modify this magnetisation. More, the Jacobian matrix of the magnetic vector ${}^0\boldsymbol{B}_i$ is defined as:

$$ {}^0\mathbf{J}_{Bi} = \begin{bmatrix} \frac{\partial\, {}^0\boldsymbol{B}_i}{\partial\, x} & \frac{\partial\, {}^0\boldsymbol{B}_i}{\partial\, y} & \frac{\partial\, {}^0\boldsymbol{B}_i}{\partial\, z} \end{bmatrix} \tag{5} $$

Thus, equations (3) and (4) can be expressed in a matrix form as:

$$ {}^0\boldsymbol{F}_i = V \cdot {}^0\mathbf{J}_{Bi}^T \cdot {}^0\boldsymbol{M} \tag{6} $$
$$ {}^0\boldsymbol{C}_i = V \cdot [{}^0\boldsymbol{M}]_\wedge \cdot {}^0\boldsymbol{B}_i \tag{7} $$

with $[{}^0\boldsymbol{M}]_\wedge$ the skew-symmetric matrix associated with the vector cross-product. On our system, the electromagnets are considered far enough from each other so that the coupling between them can be neglected. Thus, air and water being linear mediums for magnetic fields, the superposition principle applies and the overall magnetic field ${}^0\boldsymbol{B}(\boldsymbol{\beta}, {}^0\boldsymbol{P}, \boldsymbol{I})$ produced by the system is the sum of the magnetic fields produced by each electromagnet:

$$ {}^0\boldsymbol{B}(\boldsymbol{\beta}, {}^0\boldsymbol{P}, \boldsymbol{I}) = \sum_{i=1}^{3} I_i \cdot {}^0\mathbf{R}_i \cdot {}^i\boldsymbol{b}_i({}^0\mathbf{R}_i^{-1}\, {}^0\boldsymbol{P} + {}^0\boldsymbol{t}_i) \tag{8} $$

with $\boldsymbol{\beta} = \begin{pmatrix}\beta_1 \ \beta_2 \ \beta_3\end{pmatrix}^T$ the vector representing the electromagnets configuration and $\boldsymbol{I} = \begin{pmatrix}I_1 \ I_2 \ I_3\end{pmatrix}^T$ the vector gathering the supplied currents.

Similarly, the gradient of the overall magnetic field is the sum of the gradients produced by each electromagnet:

$$ {}^0\mathbf{J}_B(\boldsymbol{\beta}, {}^0\boldsymbol{P}, \boldsymbol{I}) = \sum_{i=1}^{3} {}^0\mathbf{J}_{Bi} = \sum_{i=1}^{3} I_i \cdot {}^0\mathbf{J}_{bi} \tag{9} $$

with ${}^0\mathbf{J}_{bi}$, the Jacobian matrix of the magnetic field per current unit ${}^0\boldsymbol{b}_i$. The total efforts produced on the capsule are thus given by:

$$ {}^0\boldsymbol{F} = V \sum_{i=1}^{3} I_i \cdot {}^0\mathbf{J}_{bi}^T \cdot {}^0\boldsymbol{M} \tag{10} $$

$$ {}^0\boldsymbol{C} = V\, [M]_\wedge \sum_{i=1}^{3} I_i\, {}^0\mathbf{R}_i\, {}^i\boldsymbol{b}_i({}^0\mathbf{R}_i^{-1}\, {}^0\boldsymbol{P} + {}^0\boldsymbol{t}_i) \tag{11} $$

Introducing $\mathcal{B} = \left[\,{}^0\mathbf{R}_1{\cdot}\boldsymbol{b}_1\;\, {}^0\mathbf{R}_2{\cdot}\boldsymbol{b}_2\;\, {}^0\mathbf{R}_3{\cdot}\boldsymbol{b}_3\,\right]$ and $\mathcal{J} = \left[\,{}^0\mathbf{J}_{b1}^T{\cdot}{}^0\boldsymbol{M}\;\, {}^0\mathbf{J}_{b2}^T{\cdot}{}^0\boldsymbol{M}\;\, {}^0\mathbf{J}_{b3}^T{\cdot}{}^0\boldsymbol{M}\,\right]$, leads to express (10) and (11) in matrix form as:

$$
\begin{aligned}
{}^0\boldsymbol{F} &= V \cdot \mathcal{J} \cdot \boldsymbol{I} && \triangleq \mathcal{A}_{\mathcal{F}}(\boldsymbol{\beta}, {}^0\boldsymbol{P}, {}^0\boldsymbol{M}) \cdot \boldsymbol{I} \qquad (12)\\
{}^0\boldsymbol{C} &= V \cdot [M]_\wedge \cdot \mathcal{B} \cdot \boldsymbol{I} && \triangleq \mathcal{A}_{\mathcal{C}}(\boldsymbol{\beta}, {}^0\boldsymbol{P}, {}^0\boldsymbol{M}) \cdot \boldsymbol{I} \qquad (13)
\end{aligned}
$$

Matrices $\mathcal{A}_{\mathcal{C}}$ and $\mathcal{A}_{\mathcal{F}}$, which depend on the capsule position $\boldsymbol{P}$ and magnetisation ${}^0\boldsymbol{M}$, are computed from the magnetic fields ${}^0\boldsymbol{B}_i$ created by each coil. As shown in (2), these magnetic fields depend on the orientation of each coil $\boldsymbol{\beta}$.

Finally, these equations can be gathered into the direct electromagnetic model (D_{EM}):

$$
\begin{pmatrix} {}^0\boldsymbol{F} \\ {}^0\boldsymbol{C} \end{pmatrix} = \begin{bmatrix} \mathcal{A}_{\mathcal{F}} \\ \mathcal{A}_{\mathcal{C}} \end{bmatrix} \cdot \boldsymbol{I} = \mathcal{A}(\boldsymbol{\beta}, {}^0\boldsymbol{P}, {}^0\boldsymbol{M}) \cdot \boldsymbol{I} \qquad (14)
$$

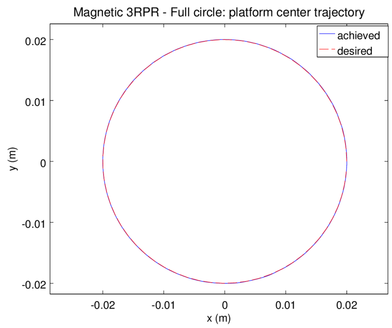
This equation enlightens that the magnetic efforts linearly depend on the currents applied in the electromagnets. Each current modifies the efforts applied on the capsule. This model also highlights the non-linear dependence of the matrix $\mathcal{A}(\boldsymbol{\beta}, {}^0\boldsymbol{P}, {}^0\boldsymbol{M})$ to the capsule position and orientation, but also to the orientation of each electromagnet, the total magnetic field depending on the coils configuration. To simplify notations, we write: $\mathcal{A}(\boldsymbol{\beta}, {}^0\boldsymbol{P}, {}^0\boldsymbol{M}) = \mathcal{A}$ in the sequel and we note that this matrix is of size 6×3.

3 CONTROL

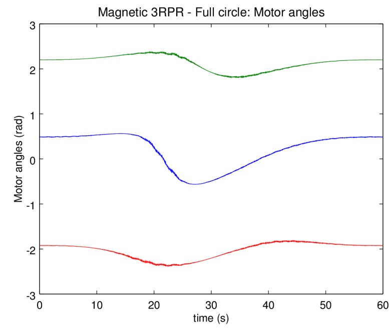
Because of the actuation redundancy of the system, several control laws are admissible to control the capsule. The simplest way would be to keep the electromagnets static and to focus on the currents to apply the efforts allowing the capsule to follow a defined trajectory, as in most of the literature. But here, our aim is to optimize the capsule manipulability and to avoid singularities such as those shown in [13].

To find how to move the electromagnets, several strategies are possible because of the system redundancy (3 degrees of mobility in the plane *vs.* 3 electromagnetic degrees of freedom plus 3 kinematic degrees of freedom). In this paper, we present a funny way to handle this redundancy by applying a kinematic constraint. Instead of aiming at the capsule center as in [12], we chose to mimic a planar $3R\underline{P}R$ parallel kinematics mechanism. Thereby, the coil axes must always aim at a virtual corner of the $3R\underline{P}R$ platform (Fig. 1(c)), replacing mechanical prismatic actuators by magnetic contactless actuators.

Once the angular errors $\boldsymbol{\Delta\beta}_{|k}$ between the current coil axis orientation $\boldsymbol{\beta}_{|k-1}$ and the desired one computed from the 3RPR kinematic constraint



(a) Capsule trajectory.



(b) Motor angles.

Fig. 3 Simulation result: (a) trajectory of the capsule and (b) evolution of the motor angles (right).

are known, a proportional control (with gain λ_β) is used to bring the motors to their next configuration:

$$\boldsymbol{\beta}_{|k} = \boldsymbol{\beta}_{|k-1} + \lambda_\beta \boldsymbol{\Delta\beta}_{|k} \tag{15}$$

The D_{EM} is updated afterwards and the currents are computed by taking the pseudo-inverse of $\mathcal{A}$:

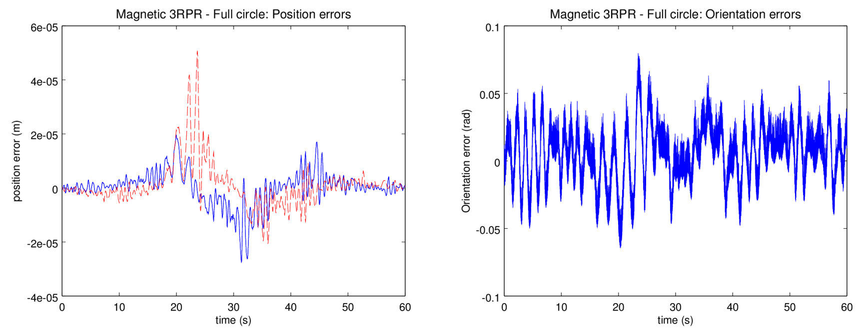
$$\boldsymbol{I} = \mathcal{A}(\boldsymbol{\beta}_{|k}, {}^0\boldsymbol{P}, {}^0\boldsymbol{M})^\dagger \cdot \begin{pmatrix} {}^0\boldsymbol{F}^* \\ {}^0\boldsymbol{C}^* \end{pmatrix} \tag{16}$$

Thus, the modification of the coils orientation allows first to have a better system configuration to realise the requested efforts, second to minimise the supplied current variations. This second point is important, especially if coils have a large number of turns, since it minimises the impact of the coil inductance on current control.

4 RESULTS

This control law was implemented on our C++/OpenGL simulator. To make the simulation more realistic, noise on the capsule position detection (± 0.5 mm, $\pm 1^\circ$), the currents flowing in the coils (5%) and the coils orientation ($\pm 1^\circ$) was added.

In this simulation, the capsule follows a circle with its magnetisation tangent to the circle (Fig. 3(a)), while moving the coils according to the kinematic constraint (Fig. 3(b)). The trajectory is well performed, with a position error less than 0.3 mm (Fig. 4(a)) and an orientation error less than 0.3° (Fig. 4(b)).



(a) Position errors. (b) Orientation error.

Fig. 4 Simulation result: (a) position errors and (b) orientation errror along the trajectory.

5 CONCLUSIONS AND PERSPECTIVES

A magnetic manipulation system with mobile electromagnets was presented in this article. A model of the system was established to compute the magnetic efforts. Unlike the models found in the literature, this one takes into account the mobility of each electromagnet composing the system. This yields a highly non-linear control problem, with several additional difficulties: redundancy, kinematico-magnetic couplings, among others.

This opens to the development of new control laws as the one presented here, where redundancy was handled by imposing virtual kinematic linkage. The coil motors thus behave as the first passive joints of a planar $3R\underline{P}R$ parallel kinematics manipulator, whereas the magnetic field created by each coils plays roughly the role of the prismatic actuators. Thereby, kinematic control and magnetic control are decoupled, and magnetic control reuses the literature results.

The effectiveness of this strategy was implemented and tested in simulation. Of course, singularities might occur in the system, but surely in different locations than the kinematic singularities. For instance, the system does not loose torsional rigidity in the kinematic singularity where all legs intersect, because in that case, the magnetic field always produces a torque. This opens up to new kinematico-magnetic analyses, and wider, to new possibilities in the design of manipulation systems. Also, such coupled systems deserve and support research in non-linear and redundant control.

Finally, an important hypothesis was made while developing the direct electromagnetic model: the electromagnets were considered far enough from each other so that coupling between them were neglected. In practice, this hypothesis might not always be true and opens research paths related to electromagnetics. But this is pretty far away from ARK!

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