Predictive maintenance of moving systems

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***Abstract*—In this paper, we propose to optimize both the assignment of missions and the maintenance scheduling of moving systems (e.g. trains) in a Prognostics and Health Management (*PHM*) context. The problem is to associate a system to each mission and to integrate the necessary maintenance operations in the schedule according to the real state of health of systems. This problem, which falls within the decision part of *PHM*, is proposed to be solved using an optimal approach based on Linear Programming. Results based on computational experiments assess the efficiency of the resolution method.**

I. Introduction and related work

Maintenance of moving systems differs from traditional maintenance problems, e.g. in a production context, in that the location of these systems over time depends directly on their use and on the missions they have been assigned to. This is for instance the case for vehicles such as cars, trains or aircrafts. In a Prognostics and Health Management (*PHM*) context, some works have been carried out considering a single moving system [1], [2] or several autonomous systems that have to perform a shared global task [3]. Daigle et al. [1] proposed for instance a model-based prognostics framework to predict the remaining driving time and distance of a planetary rover, which depends upon the amount of power that can be delivered by its batteries and the rover path. Such predictions are used to plan the future operation of the rover. Mission replaning of a planetary rover as a function of its state of health has also been studied by Balaban et al. in [2] and [4]. The proposed decision algorithms were able to manage mechanical failures, electronic defects and insufficient battery charge. Prescott at al. [3] used prognostics on autonomous vehicles working together to predict the probability of mission failure and to make an informed decision on the future of the mission. These works show that the decisions related to the assignment of missions to certain systems can be based on systems real state of health information, provided by the prognostics, allowing to optimize their use. The general idea is the following: if decisions are made with respect to the system health evolution over time, the mission effectiveness can be maximized before energy and health budgets are exceeded.

This idea can be extended for the optimization of the assignment of missions to a fleet of moving systems. As the maintenance of such systems is strongly linked to the missions they have to perform, we propose to jointly optimize the missions assignment and the maintenance scheduling. This has for instance been proposed in the railway domain, in which some studies aim to optimize jointly the routing of trains and their maintenance scheduling. Andres et al. [5] proposed a mixed integer linear programming model that determines an appropriate train routing and schedules the necessary maintenance operations, with global cost minimization as objective. Giacco et al. [6] proposed also a mixed integer linear programming model to deal with the interaction between the rolling stock routing and the maintenance planning. The objective considered in this case was to minimize the total number of rolling stock units that are used and the number of empty rides and to maximize the distance traveled by each train between two maintenance operations of the same type. These works considered however traditional static maintenance strategies that schedule maintenance operations in advance according to time and distance criteria. This corresponds to preventive maintenance, which often implies unnecessary maintenance activities and reduces the useful life of rolling stock components due to early replacement. Studies proposed in the railway domain in a *PHM* context focused so far for the vast majority on the prognostics phase. Some contributions tackled rolling stock prognostics, with for instance the prediction of the remaining useful life of train axle bearings [7] or rail wagon bearings. Prognostics has also been applied on infrastructure elements, such as railway turnouts [8] or rail tracks [9]. Very few works addressed the decision part of *PHM* dealing with maintenance optimization. Letot et al. [9] proposed an adaptive opportunistic predictive maintenance model for railway tracks based on the track geometry observation. They search for the optimum tamping time considering a set of rail tracks sections. Camci et al. [8], [10] addressed the problem of predictive maintenance for systems located in various places, which can be applied for the maintenance of railway switches. The general problem has been introduced in [11] and resolution methods based on a Genetic Algorithm formulation have been proposed for many variants of the predictive maintenance optimization problem in [8] and [10]. These works relate however to the maintenance of geographically distributed, but stationary systems, which are part of the railway infrastructure.

We focus in this paper on the joint optimization of the mis-

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sion assignment to moving systems and the predictive maintenance scheduling of such systems, in a *PHM* context. The aim is to determine an appropriate use of systems considering predefined missions and prognostics information. The problem is to associate a system to each mission and to integrate the necessary maintenance operations in the schedule, according to the real state of health of systems and its evolution.

For each system, prognostics information is considered in the form of a degradation level which evolves over time with the use of systems. Compared to traditional preventive approaches (time-based and distance based scheduled maintenance), consideration of prognostics results allows to match each degradation level evolution to the real use of systems. It is thus possible to take into account the impact of missions on systems state of health. Each mission can indeed impact the systems wear and tear in various ways, as a function of different criteria such as the state of roads (or rail tracks), the difference in height or the moving systems speed that is authorized. This allows to enhance the decisions made in the maintenance scheduling, as well as in the assignment process, which defines which moving system has to be used for each mission. The knowledge of systems state of health and the prediction of their evolution allows indeed to choose the best system for each mission. These decisions, which fall within the decision part of the *PHM* process, are proposed to be optimized using linear programming, considering a certain number of constraints.

The organization of the paper is as follows: the problem statement is first detailed in Section II, with the description of the application framework and the optimization problem, followed by a mathematical formulation of the problem. A solution example is given in Section III to illustrate the use of the mathematical model. The proposed resolution method based on linear programming is then developed in Section IV and simulation results are detailed in Section V. This work is finally concluded and some future works are given in Section VI.

II. Problem Statement

A. Application framework

The application addressed here is based on a set of m systems M_j $(1 \leqslant j \leqslant m)$, which could for instance be trains. All the systems are supposed to be of same type. They are however differentiated by their global degradation level provided by the prognostics. Each level, denoted $H_j \in [0,1]$ (H_j^0 at the beginning of the scheduling process), stands for the state of health of the system M_j. $H_j = 0$ means that M_j is as good as new and $H_j = 1$ indicates that M_j has reached its end of life and that a maintenance is required. Each degradation level is supposed to remain constant when the system is not used. In order to avoid failures, corrective maintenance and associated additional costs, the maintenance is triggered when the degradation reaches a certain threshold denoted $\Delta_j = \Delta \in [0,1[$. This threshold can be defined as part of the decision process, or provided by the prognostics in the form of a Remaining Useful Life (*RUL*) value. The relation between the variable H_j, the threshold Δ and the launch of a maintenance operation is illustrated in Figure 1.

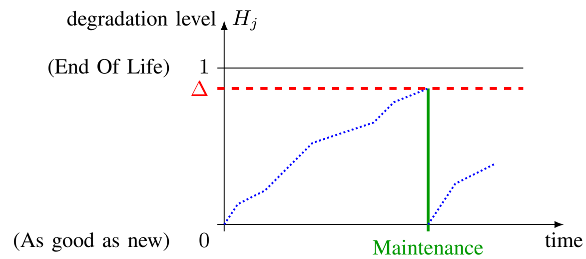

Fig. 1. Evolution of a system state of health with maintenance

The scheduling is supposed to be done for a fixed horizon $H = K\Delta T$, with ΔT the time unit, which can be defined as a day. In this case, K stands for the number of days in the schedule. Each day is denoted k, with $1 \leqslant k \leqslant K$. Each day k, a certain number of missions, denoted P, has to be fulfilled. Each mission T_p $(1 \leqslant p \leqslant P)$ corresponds to a set of rides assigned to a certain system, with the departure and arrival at the maintenance center, and is associated to a degradation $\delta_p \in [0,1[$, which corresponds to a wear rate. For simplicity's sake, missions are assumed to be the same each day and to induce the same degradation on each system: $\delta_p = \delta \;\; \forall p$. If its state of health H_j is sufficient (such that $H_j + \delta \leqslant \Delta_j$), any system M_j can be assigned to any mission T_p.

The number of maintenance allowed during the scheduling horizon H is limited for each optimization problem instance to one operation for each system. This implies that the considered number of time periods K has to be small enough and fixed in compliance with the systems' states of health. Each maintenance is supposed to be perfect. Then, once a maintenance operation is performed, the degradation level of the maintained system falls to 0, which means that it is as good as new (see Figure 1). The duration of each maintenance is supposed to be one day (ΔT) for each system, whatever the type of operation that needs to be done. This duration does not necessarily correspond to the actual length of a maintenance operation, but signifies that when a system needs a maintenance, it is unavailable during the whole day k. The capacity of the maintenance center is supposed to be limited. This capacity, denoted c, is defined as the maximal number of systems that can be maintained each day.

B. Optimization problem

The problem consists in assigning the appropriate system to each mission, considering the prognostics information, the impact of the mission on the system state of health and the maintenance opportunities. The assignment problem and the maintenance one are closely related. Indeed, the assignment of systems to missions impacts directly the systems' state of health. One important part of the problem is then to maintain the systems when needed in order to avoid failures, while guarantying that as much systems as needed are available at each time to carry out all the missions.

In order to optimize the maintenance, the objective taken into account is the maximization of the use of each system potential in terms of useful life. In other words, the aim is to schedule each maintenance task as closely as possible to the failure while avoiding it. For the considered set of systems, the considered objective is to maximize the minimal degradation level among those reached by all the systems before each maintenance. A mathematical expression of this objective function and the constraints associated to the considered optimization problem are detailed in next section.

C. Mathematical formulation

A mathematical formulation of the optimization problem detailed before is proposed in set of Equations (1). In order to express the objective function and the constraints associated to the problem, some variables need first to be introduced. Let $x_{j,k} \in \{0,1\}$ $(1 \leqslant j \leqslant m, 1 \leqslant k \leqslant K)$ be the binary decision variables used to define the resource assignment such that $x_{j,k} = 1$ if the system M_j is used to process a mission during the day k ; $x_{j,k} = 0$ otherwise. Let $y_{j,k} \in \{0,1\}$ $(1 \leqslant j \leqslant m, 1 \leqslant k \leqslant K)$ be the binary decision variables used to schedule the maintenance operations such that $y_{j,k} = 1$ if the system M_j has been maintained at day k ; $y_{j,k} = 0$ otherwise. Each variable $y_{j,k}$ follows the trend depicted in Figure 2.

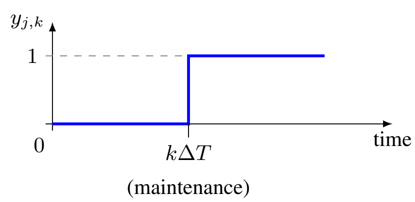

Fig. 2. Evolution trend followed by each binary variable $y_{j,k}$

The considered objective function maximizes the degradation of each system before maintenance. This is consistent with a predictive approach, which aims at maintaining the resources only when needed. This objective can be expressed through the minimization for each system M_j of the difference between the number of time periods during which the system is in revenue service to a maximal number of time periods during which it can be used without failure, namely the *RUL* value (see Equation (1a)).

Constraints defined in the mathematical program allow to take into account characteristics related to the systems, the missions and the maintenance. First set of constraints, detailed in Equation (1b), ensures that, each day k, all the P missions are assigned to a system M_j. A system M_j can be assigned to a mission in a day k only if its state of health is sufficient, that is, if the degradation caused by the mission added to its actual degradation level does not pass the degradation threshold Δ (see Equation (1c)). The set of constraints defined in Equation (1d) ensures that systems are not assigned to missions the day k during which they are maintained. Equation (1e) ensures that the capacity of the maintenance center is observed each day. Equation (1f) limits the number of maintenance operations per system to one during the scheduling horizon H. The respect of the unit step function shape for the binary variables $y_{j,k}$ (see Figure 2) is finally ensured by Equation (1g).

$$
\begin{cases}
\min & \displaystyle\sum_{j=1}^{m} \left(RUL_j - \sum_{k=1}^{K} \delta \cdot x_{j,k}(1 - y_{j,k}) \right) & & \text{(1a)} \\
& \displaystyle\sum_{j=1}^{m} x_{j,k} = P & \forall k & \text{(1b)} \\
& \displaystyle\sum_{k=1}^{K} \delta \cdot x_{j,k}(1 - y_{j,k}) \leqslant RUL_j & \forall j & \text{(1c)} \\
\text{s.t.} & y_{j,k} - y_{j,k-1} \leqslant 1 - x_{j,k} & \forall j,\ \forall k & \text{(1d)} \\
& \displaystyle\sum_{j=1}^{m} (y_{j,k} - y_{j,k-1}) \leqslant c & \forall k & \text{(1e)} \\
& \displaystyle\sum_{k=1}^{K} (y_{j,k} - y_{j,k-1}) \leqslant 1 & \forall j & \text{(1f)} \\
& y_{j,k} \geqslant y_{j,k-1} & \forall j,\ \forall k & \text{(1g)} \\
\text{with} & x_{j,k} \in \{0,1\} & \forall j,\ \forall k & \text{(1h)} \\
& y_{j,k} \in \{0,1\} & \forall j,\ \forall k & \text{(1i)} \\
& y_{j,0} = 0 & \forall j & \text{(1j)}
\end{cases}
$$

III. Solution example

The use of the proposed mathematical model is illustrated on a simple use case, with $m = 5$ systems, $P = 3$ missions for each day and a maintenance capacity $c = 1$. *RUL* values taken into account for the systems are the following: $RUL_1 = 7$, $RUL_2 = 4$, $RUL_3 = 1$, $RUL_4 = 5$ and $RUL_5 = 1$ day(s). A solution that complies with all the constraints defined in the mathematical model is depicted in Figure 3 for a scheduling horizon $H = 10$ days.

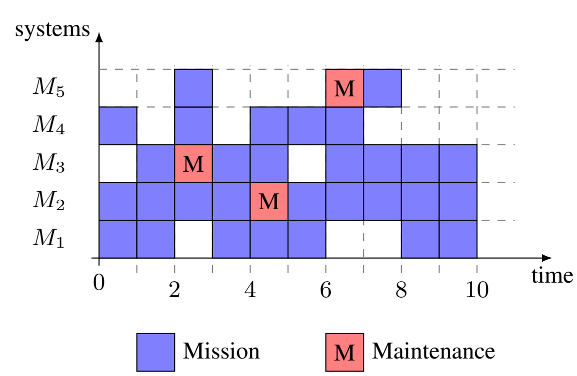

Fig. 3. Schedule obtained with the mathematical program for the considered use case – $m = 5$ systems, $P = 3$ missions per day, maintenance capacity $c = 1$

One can first see that, each day, all the 3 missions are assigned to a system. The maintenance capacity is then well observed, as maximum one maintenance has been scheduled each day. In this case, maintenance operations are moreover launched when systems' *RUL* are equal to 0, which is consistent with the considered objective that maximizes the degradation before maintenance.

IV. Resolution based on Linear Programming

The mathematical program previously expressed being not linear, some modifications are mandatory to solve the considered optimization problem with linear optimization. The product of the two binary variables $x_{j,k}$ and $y_{j,k}$ has to be linearized. This can be done by introducing a new variable $e_{j,k} \in \mathbb{R}$ associated to the constraints detailed in the set of Equations (2) [12].

$$\begin{cases} e_{j,k} \leqslant x_{j,k} \quad \forall j,\ \forall k & \text{(2a)} \\ e_{j,k} \leqslant y_{j,k} \quad \forall j,\ \forall k & \text{(2b)} \\ 1 - x_{j,k} - y_{j,k} + e_{j,k} \geqslant 0 \quad \forall j,\ \forall k & \text{(2c)} \\ e_{j,k} \geqslant 0 \quad \forall j,\ \forall k & \text{(2d)} \end{cases}$$

The linear program associated to the considered optimization problem is detailed in the set of Equations (3).

$$\begin{cases} \min & \sum_{j=1}^{m} \left(RUL_j - \sum_{k=1}^{K} \delta \cdot (x_{j,k} - e_{j,k}) \right) & \text{(3a)} \\ & e_{j,k} \leqslant x_{j,k} \quad \forall j,\ \forall k & \text{(3b)} \\ & e_{j,k} \leqslant y_{j,k} \quad \forall j,\ \forall k & \text{(3c)} \\ & 1 - x_{j,k} - y_{j,k} + e_{j,k} \geqslant 0 \quad \forall j,\ \forall k & \text{(3d)} \\ & e_{j,k} \geqslant 0 \quad \forall j,\ \forall k & \text{(3e)} \\ & \sum_{j=1}^{m} x_{j,k} = P \quad \forall k & \text{(3f)} \\ \text{s.t.} & \sum_{k=1}^{K} \delta \cdot (x_{j,k} - e_{j,k}) \leqslant RUL_j \quad \forall j & \text{(3g)} \\ & y_{j,k} - y_{j,k-1} \leqslant 1 - x_{j,k} \quad \forall j,\ \forall k & \text{(3h)} \\ & \sum_{j=1}^{m} (y_{j,k} - y_{j,k-1}) \leqslant c \quad \forall k & \text{(3i)} \\ & \sum_{k=1}^{K} (y_{j,k} - y_{j,k-1}) \leqslant 1 \quad \forall j & \text{(3j)} \\ & y_{j,k} \geqslant y_{j,k-1} \quad \forall j,\ \forall k & \text{(3k)} \\ \text{with} & x_{j,k} \in \{0,1\} \quad \forall j,\ \forall k & \text{(3l)} \\ & y_{j,k} \in \{0,1\} \quad \forall j,\ \forall k & \text{(3m)} \\ & y_{j,0} = 0 \quad \forall j & \text{(3n)} \\ & e_{j,k} \in \mathbb{R} \quad \forall j,\ \forall k & \text{(3o)} \end{cases}$$

V. Simulation results

The resolution method proposed in previous section has been evaluated through simulations on random problem instances. The problem generation is first described. Efficiency of the approach is then discussed.

A. Problem generation

Random problem configurations have been generated using a simulator and configured with many parameters. Each system M_j has first been associated to a global *RUL* which stands for the remaining time during which it can be used before maintenance is needed. Only one *RUL* value being defined for each system in this study, it can be seen as the *RUL* of the limiting component, i.e., the minimal *RUL* of all the components part of the system. Each system may have a different state of health at the beginning of the decision process, due to a different use before the time $t = 0$. *RUL* values have been randomly selected in the range $[0, 90]$ days.

A problem configuration corresponds to a specific set of m systems with *RUL* values generated as defined before, associated to a set of P rides per day and to a maintenance capacity c. Several problem sizes have been considered, with $m = 20, 50, 100$ and 200 systems. The number of missions P per day and the maintenance capacity c have been adapted to these problem sizes in order to obtain problems that can be compared among themselves. For $m = 20$ systems, P has been set to 15 missions and $c \in \{1, 2, 3, 4, 5\}$ systems that can be maintained each day. The consideration of several maintenance capacities allows to a certain extent to study the impact of the maintenance center capacity on the maintenance schedules obtained with the proposed linear programming based approach. Values for the other problem sizes are detailed in Table I.

TABLE I
PROBLEM CONFIGURATIONS

nb. of systems m	nb. of missions P	maintenance capacity c
20	15	$\{1, 2, 3, 4, 5\}$
50	37	7
100	75	15
200	150	30

For each problem configuration, several decision horizons have finally been tested ($H = K\Delta T$, with $K \in \{20, 30, 40, 50, 60, 70, 80, 90\}$ the number of days), each being associated to a specific solution. Following results are represented as a function of this number of days K in the decision horizon. Each result depicted in the following figures is the average of 20 random instances of one problem configuration, for which the number m of systems, the number P of missions and the maintenance capacity c is fixed. For readability reasons, points have been scattered around the corresponding horizon value on the abscissa.

B. Results

Figure 4 shows the mean maximum number of maintenance operations per day for each problem configuration with $m = 20$ systems. One can first see that the maintenance capacity is well observed for all the cases. The number of maintenance operations scheduled each day is moreover below 3 whatever the maintenance capacity allowed. This indicates that a great maintenance capacity might not be necessary for the predictive maintenance and that the proposed scheduling method naturally balances the maintenance load over the scheduling horizon. This remark is however valid only under the previously detailed limiting assumptions and strongly linked with the values of the problem parameters. The only difference between solutions with different maintenance capacities might be the distribution of maintenance operations over the scheduling horizon (i.e., the days during which maintenance operations are scheduled).

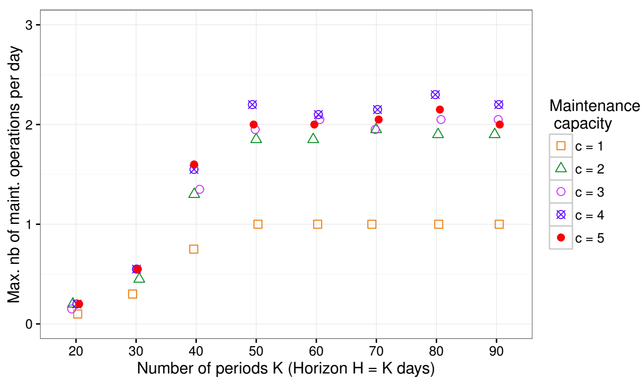

Fig. 4. Maximum number of maintenance operations per day for each maintenance capacity $c = 1, 2, 3, 4$ and 5 – $m = 20$ systems, $P = 15$ missions

Figure 5 shows the total number of maintenance operations scheduled over the whole scheduling horizon for each problem instance with $m = 20, 50, 100$ and 200 systems, with associated number of missions and maintenance capacities (see Table I). Only one maintenance capacity has been associated to problem instances with $m = 20$ systems, namely $c = 3$. One can see that for large scheduling horizons, maintenance has been scheduled for all the systems.

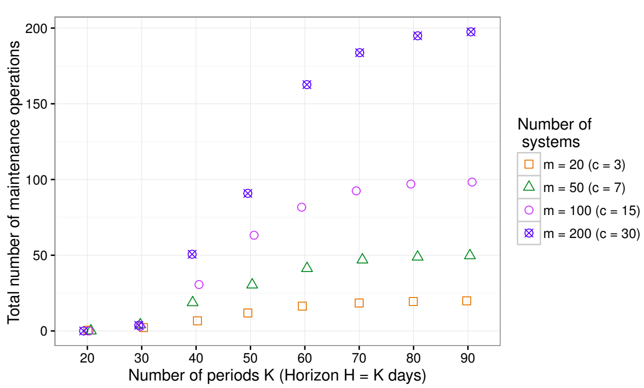

Fig. 5. Total number of maintenance operations for each number of systems $m = 20, 50, 100$ and 200, with associated number of missions and maintenance capacities (see Table I)

Computation times[1] for the problem instances with $m = 20, 50, 100$ and 200 systems are depicted in Figure 6. Logically, computation times increase both with the scheduling horizon and with the number of systems. This is due to the increase of the number of variables in the linear program as a function of K and m. Problems with $m \leqslant 200$ systems can be solved in less than 3 minutes. This time is reasonable and consistent with a predictive maintenance scheduling process, which should be quickly modifiable based on updated prognostics data. The use of linear programming is however not suitable for large problem instances, with a huge number of system and/or long scheduling horizons. Some computation time values are provided in Table II for several numbers of systems m ($m = 20, 50, 100, 200,$ $500, 1000$ and 2000), considering a scheduling horizon $H = 90$ days.

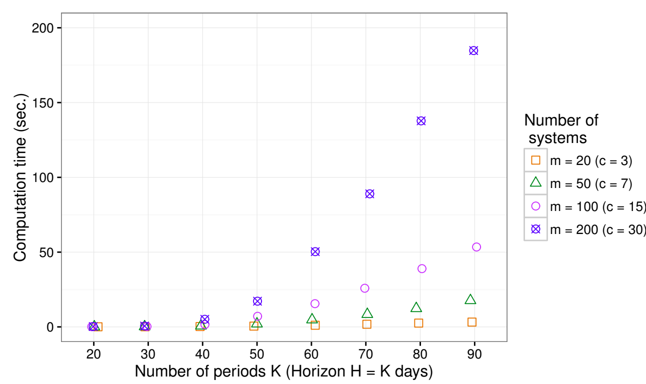

Fig. 6. Computation times for each number of systems $m = 20, 50, 100$ and 200, with associated number of missions and maintenance capacities (see Table I)

TABLE II
COMPUTATION TIMES FOR SEVERAL NUMBER OF SYSTEMS m, FOR A SCHEDULING HORIZON $H = 90$ DAYS

nb. of systems m	compututation time
20	3.23 s.
50	17.66 s.
100	53.43 s.
200	3.08 min
500	12 min.
1000	59.47 min.
2000	2.6 h.

VI. CONCLUSION AND FUTURE WORK

Maintenance scheduling of moving systems has been proposed in a Prognostics and Health Management (*PHM*), allowing to launch maintenance operations only when they are needed. A mathematical formulation of the joint mission assignment and maintenance scheduling problem has been detailed, including an objective function which aims to minimize the degradation level reached before each maintenance and several constraints related to the application context. Linear programming has been proposed to tackle the considered optimization problem. Performance of this optimal approach has been assessed through numerous simulations. First results show that the proposed linear program provides satisfying

[1]Simulations have been launched using MATLAB® and the solver for linear programming Gurobi® (Computation parameters: Processor Intel Core^{TM} i5-3550 CPU 3.30GHz×4, 15.6 Gio, 64 bits)

schedules in limited time for problem sizes that are for instance consistent with a fleet of cars or trains.

As future work, simulations will be performed by considering missions with different associated degradations δ_p. Enhancement of the mathematical model will also be performed by introducing more realistic constraints, in particular constraints related to the maintenance capacity. For more realistic problem sizes, with more systems and more missions, and considering additional constraints, defining scalable heuristics will finally be mandatory.

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