

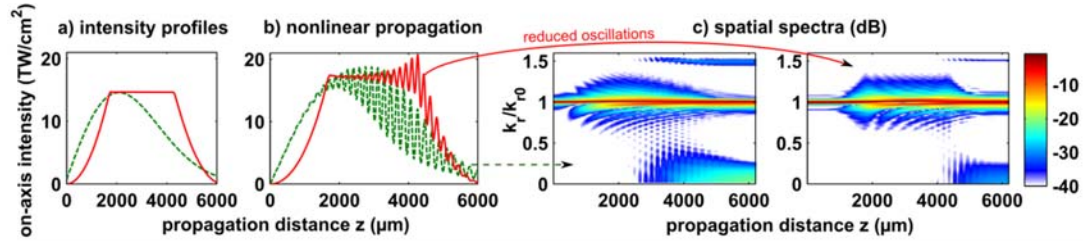
# Four-wave mixing control in the filamentation of ultrafast Bessel beams via longitudinal intensity-shaping

Ismail Ouadghiri-Idrissi, Remo Giust, John M. Dudley, and Francois Courvoisier

FEMTO-ST Institute, University of Bourgogne-Franche-Comté UMR-6174, 25030 Besancon, France

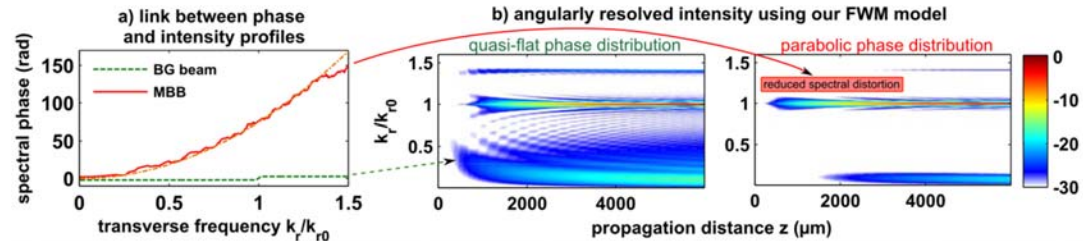
Bessel beams exploit conical energy flow to yield near-uniform intensity along a line focus which has been shown to be extremely attractive for laser processing in dielectrics. At high power, however, the nonlinear Kerr effect is known to induce significant oscillations of the on-axis intensity which is deleterious for machining applications. Here, we show through theory and numerical modelling how this problem can be understood and overcome by appropriate spatial phase shaping of the input profile [1]. Our results also solve the longstanding problem related to the nonlinear Bessel beam dynamics seen at an air-dielectric interface [2].

Our approach is based on the numerical solution of Maxwell's equations with cylindrical symmetry including Kerr nonlinearity. Our results are valid when plasma formation is included. Figure 1 shows simulation results modelling the nonlinear propagation in glass of two 100 fs Bessel beams with different intensity profiles. Figure 1(a) shows the target linear profiles: a Bessel-Gauss beam (BG, green dashed) and a modified Bessel beam (MBB, red) with a smoother profile. Figure 1(b) shows the result of the nonlinear propagation. The Modified Bessel Beam (red curve) leads to significantly reduced nonlinear oscillations compared to the BG beam. Figure 1(c) shows the evolution of the corresponding spatial spectra in dB along propagation (left: BG beam; right: MBB) clearly showing strong attenuation of the spectral components around  $k_r \sim \sqrt{2}k_{r0}$  and  $k_r \sim 0$  associated with the detrimental oscillations.



**Fig 1.** Simulation of the nonlinear propagation of two 100 fs Bessel beams with different a) target on-axis intensity profiles. Evolution of their respective b) on-axis intensities and c) spatial spectra (dB) along propagation. beam waist:  $w_0 = 300 \mu\text{m}$  and cone angle:  $\theta = 4^\circ$ ,

These numerical results are complemented by an analytic model of four wave mixing (FWM) which shows that the input spectral phase (Fig. 2.a) is a key parameter to control the growth of FWM components. The efficiency of the FWM process is much higher (Fig. 2(b), left) if the input beams have a quasi-flat spectral phase, which is the case for the Bessel-Gauss (BG) profile. In contrast, it is greatly reduced for the MBB (Fig. 2(b), right), where the input spatial spectrum has a rapidly varying phase evolving as a parabola.



**Fig 2.** Semi-analytical results using our FWM model: b) input spectral phase distributions corresponding to the target intensity profiles and b) spatial spectra (dB) along propagation in case of a parabolic (middle) and quasi-flat (right) spectral phase distributions.

The results of our reduced analytical model are in good agreement with the full numerical model of Fig. 1. Additional time-resolved numerical simulations including material dispersion and nonlinear plasma generation clearly show the potential of longitudinal beam shaping to control nonlinear intensity. This opens novel perspectives to control nonlinear propagation and filamentation. We acknowledge funding from the European Research Council (ERC) under Horizon 2020 program (GA N° 682032-PULSAR) and the Labex ACTION (ANR-11-LABX-0001-01).

## References

- [1] I. Ouadghiri-Idrissi, R. Giust, L. Froehly, M. Jacquot, L. Furfaro, J. M. Dudley, and F. Courvoisier, "Arbitrary shaping of on-axis amplitude of femtosecond Bessel beams with a single phase-only spatial light modulator," *Opt. Express* **24**, 11495-11504 (2016).
- [2] P. Polesana, A. Couairon, D. Faccio, A. Parola, M. A. Porras, A. Dubietis, A. Piskarskas, and P. Di Trapani, "Observation of Conical Waves in Focusing, Dispersive, and Dissipative Kerr Media" *Phys. Rev. Lett.* **99**, 223902 (2007).