A Model-Based Feedforward Hysteresis Compensator for Micropositioning Control

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***Abstract*— Further results on hysteresis compensation using the inverse multiplicative structure of the rate-dependent Prandtl-Ishlinskii (RDPI) model are presented. The proposed model-based feedforward controller is used to compensate for the hysteresis nonlinearities at different operating conditions without formulating the inverse model. The study investigates the compensation error and parameters uncertainties when the proposed compensator is applied for compensation of the rate-dependent hysteresis nonlinearities. The compensation results show that better performance can be achieved with low sampling time. The proposed compensator is further examined in a closed-loop control system to improve the tracking performance of a piezoelectric cantilevered actuator.**

I. INTRODUCTION

Several control methodologies have been suggested to cancel out the hysteretic effects of smart material-based actuators, these include adaptive control [1]–[5], energy-based control [6], sliding mode control [5], H_∞ control [7], [8], and inverse hysteresis models [9]. Among the available control methodologies, using the inverse of the hysteresis model is considered an attractive approach allows compensation of hysteresis nonlinearities of smart material-based actuators in an open-loop manner. [2], [9], [10]. However, this approach necessitates selecting an appropriate hysteresis model that permits the formulation of an inverse compensator.

The Prandtl-Ishlinskii model is considered an attractive choice for modeling and compensation of hysteresis nonlinearities due to its simplicity and suitability for real-time applications [11]. The classic version of this model has been extended in [12], [13] to account for the rate effect of the applied input. The RDPI model can describe the hysteretic behaviour of smart material-based actuators which incorporates rate-independent hysteresis at shallow levels of excitation frequency, as well as rate-dependent hysteresis nonlinearities at high rates of input [14]. In addition, the RDPI model is analytically invertible which allows compensation of rate-dependent hysteresis nonlinearities. However, the inverse RDPI model is available under mathematical conditions restricts formulating an accurate model for describing hysteresis nonlinearities.

Recently [11], a new methodology that considers restructuring the model itself in an inverse multiplicative scheme has been suggested for compensation of hysteresis nonlinearities of smart material-based actuators. The inverse multiplicative scheme has been successfully applied to reduce the hysteresis nonlinearities characterized by the classical Bouc-Wen model [15] and the Prandtl-Ishlinskii model [11]. This approach has been extended in [16] for compensation of rate-dependent hysteresis characterized by RDPI model. Although this methodology can effectively compensate for the rate-dependent hysteresis without formulating rate-dependent inverse model as in [16], investigating the boundedness of compensation error is essential for applying feedback control designs. Thus, synthesizing robust H_∞ and internal model-based feedback control designs for example to enhance the performance of a dynamic plant necessitates adequate consideration for the boundedness of the compensation error when the inverse multiplicative scheme is applied.

In this paper, the boundedness of the tracking error when the inverse multiplicative structure of the RDPI model is applied as a compensator is investigated. This aims at facilitating the implementation of the feedback control design to enhance the performance of a hysteretic dynamic plant that is associated with parameters uncertainty. A piezoelectric cantilevered actuator that exhibits rate-dependent hysteresis nonlinearities is employed to examine the effectiveness of the proposed control design.

II. THE RATE-DEPENDENT PRANDTL-ISHLINSKII MODEL

The proposed compensator is a model-based controller that is constructed using the RDPI model without formulating an inverse model. In this section, we remind the mathematical formulation of the RDPI model that is used to describe the rate-dependent hysteresis nonlinearities. The model is presented in this section along with the associated discrete form. The RDPI model [12] has been suggested to describe the rate-dependent hysteresis loops between input (voltage or current) and output displacement of piezoelectric and magnetostrictive actuators.

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The discrete form of the RDPI model can be represented with the sampling time T_s, where $T_s = t_k - t_{k-1}$, $k = 1, 2, \ldots, K$, and $K \in \mathbb{N}$ is an integer. With the applied input $z(k)$ and the rate of the input $v(t)$, the output of the discrete RDPI model is

$$y(k) = \Gamma[z](k) := \rho_0 z(k) + \Omega[z](k)\,, \tag{1}$$

where

$$\Omega[z](k) = \sum_{i=1}^{n} \rho_i \Phi_{r_i(v(k))}[z](k),$$

ρ_0 and ρ_i represent the weights, while the discrete rate-dependent play (RDP) operator $\xi_i(k) = \Phi_{r_i(v(k))}[z](k)$ and

$$\xi_i(k) = \max\{z(k) - r_i(v(k)), \min\{z(k) + r_i(v(k)), \xi_i(k-1)\}\},$$

where

$$r_i(v(k)) = \delta_1 i + \delta_2 |v(k)|,$$

where δ_1 and δ_2 are positive constants.

III. An Inversion-Free Feedforward Rate-Dependent Compensator

A. The proposed compensator and the maximum compensation positioning error

A new rate-dependent feedforward compensator based on the inverse multiplicative scheme is presented in this section for compensation of rate-dependent hysteresis nonlinearities of the RDPI model without formulating an inverse model. The major advantage of the proposed methodology is that no additional calculations are required to obtain the compensator parameters. Thus, as long as the model is identified, the compensator is yielded. Furthermore, no condition has to be satisfied in order to ensure the invertibility of the RDPI model. The output of the proposed compensator is

$$u(k) = \Pi[z](k) = \rho_0^{-1}\Big(z(k) - \Omega[u](k-1)\Big). \tag{2}$$

Applying the output of the proposed compensator as an input to the discrete RDPI model in (1) yields

$$y(k) = \Gamma\Big[\rho_0^{-1}\Big(z(k) - \Omega[u](k-1)\Big)\Big](k). \tag{3}$$

Let $\eta(k) = z(k) - \Omega[u](k-1)$. Then $y(k) = \rho_0\Big[\rho_0^{-1}\Big(\eta(k)\Big)\Big] + \Omega\Big[\rho_0^{-1}\Big(\eta(k)\Big)\Big]$ and

$$y(k) = z(k) + \Omega[u](k) - \Omega[u](k-1). \tag{4}$$

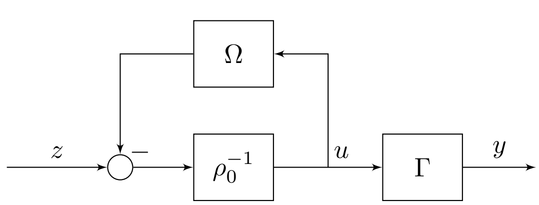

Fig. 1: The structure of the proposed compensator in cascade arrangement with the RDPI model Γ.

Then the error of compensation is $e(k) = \Omega[u](k) - \Omega[u](k-1)$, and

$$e(k) = \sum_{i=1}^{n} \rho_i \Big(\Phi_{r_i(v(k))}[u](k) - \Phi_{r_i(v(k-1))}[u](k-1)\Big). \tag{5}$$

To show the maximum compensation positioning error when the proposed compensator is applied for compensation of hysteresis, we obtain

$$|e(k)| = |\Omega[u](k) - \Omega[u](k-1)|. \tag{6}$$

(i) When the input $u(k)$ increases, $u(k) > u(k-1)$, and for $A, B, C, D \in \mathbb{R}$, we have [17]

$$|\max\{A, B\} - \max\{C, D\}| \le |\max\{|A - C|, |B - D|\}. \tag{7}$$

Then

$$|e(k)| \le \sum_{i=1}^{n} \rho_i \max\{|u(k) - u(k-1) + r_i(v(k)) - r_i(v(k-1))|, |\xi_i(k) - \xi_i(k-1)|\}, \tag{8}$$

and $r_i(v(k)) - r_i(v(k-1)) = \delta_2(|v(k)| - |v(k-1)|)$. Let $u(k) - u(k-1) = \epsilon_u(k)$, $|v(k)| - |v(k-1)| = \epsilon_v(k)$, and $|\xi_i(k) - \xi_i(k-1)| = \epsilon_{\xi i}(k)$, then

$$|e(k)| \le \sum_{i=1}^{n} \rho_i \max\{|\epsilon_u(k)| + \delta_2|\epsilon_v(k)|, |\epsilon_{\xi i}|(k)\}, \tag{9}$$

Let $\max|\epsilon_u(k)| = \epsilon_1$, $\max|\epsilon_v| = \epsilon_2$, and $\max|\ |\epsilon_{\xi i}|(k) = \epsilon_{3i}(k)$, where ϵ_1, ϵ_2, and ϵ_{3i} are small positive constants, and $\rho_{\max} = \max\{\rho_i\}$. Let $\epsilon_3 = \max \epsilon_{3i}$. Then we conclude

$$|e(k)| \le n\rho_{\max} \max\{\epsilon_1 + \delta_2\epsilon_2, \epsilon_3\}. \tag{10}$$

Let $\epsilon = \max\{\epsilon_1 + \delta_2\epsilon_2, \epsilon_3\}$, then $|e(k)| \le n\rho_{\max}\epsilon$. The error bound is $\mathcal{E} = n\rho_{\max}\epsilon$. For very small ϵ, the proposed compensator yields $y(k) \cong z(k)$. (ii) When the input $u(k)$ decreases, $u(k) < u(k-1)$, we have $|\min\{A, B\}$-$\min\{C, D\}| = |\max\{-C, -D\} - \max\{-A, -B\}|$, and

$$|\max\{-C, -D\} - \max\{-A, -B\}| \le |\max\{|A - C|, |B - D|\}. \tag{11}$$

Then, we conclude that the error bound is $\mathcal{E} = n\rho_{\max}\epsilon$ and for very small ϵ, $y(k) \cong z(k)$. (iii) For non-decreasing or non-increasing input $u(k)$, $u(k) = u(k-1)$, we have $\Omega[u](k) = \Omega[u](k-1)$ and the error $e(k) = 0$. Then we conclude $y(k) = z(k)$.

Then, it can be concluded that

$$\Gamma \circ \Pi[z](k) \cong z(k). \tag{12}$$

Thus, the proposed rate-dependent compensator can be applied to ensure the tracking performance of system that exhibits rate-dependent hysteresis nonlinearities.

IV. Parameters Uncertainty and Numerical Example

Since this study suggests the RDPI model as a model and compensator for describing and compensation of rate-dependent hysteresis nonlinearities of smart material-based actuators, then it is essential to explore the effectiveness of the proposed compensator in the presence of the characterization errors. This section presents the parameters uncertainty of RDPI model with the proposed compensator.

A. Parameters Uncertainty

The output of the estimated discrete RDPI model is

$$\hat{y}(k) = \hat{\Gamma}[u](k) := \hat{\rho}_0 u(k) + \hat{\Omega}[u](k)\,, \tag{13}$$

where $\hat{\rho}_0$ is a positive constant and

$$\hat{\Omega}[u](k) = \sum_{i=1}^{n} \hat{\rho}_i \Phi_{\hat{r}_i(v(k))}[u](k), \tag{14}$$

where $\hat{\rho}_i$ are positive constants. The output of the proposed compensator constructed using estimated RDPI model is

$$u(k) = \hat{\rho}_0^{-1}\Big(z(k) - \hat{\Omega}[u](k-1)\Big). \tag{15}$$

Then, the output of the compensation can be expressed as

$$y(k) = \rho_0 \hat{\rho}_0^{\,-1} z(k) - \rho_0 \hat{\rho}_0^{-1} \hat{\Omega}[u](k-1) + \Omega[u](k), \tag{16}$$

and the error is

$$e(k) = z(k)(1 - \rho_0 \hat{\rho}_0^{-1}) + \rho_0 \hat{\rho}_0^{\,-1} \hat{\Omega}[u](k-1) - \Omega[u](k). \tag{17}$$

Let $\rho_0 \hat{\rho}_0^{\,-1} = \tau$ and $\mathcal{P}[k] = \rho_0 \hat{\rho}_0^{-1}\ \hat{\Omega}[u](k-1) - \Omega[u](k)$, then

$$\mathcal{P}[k] = \sum_{i=1}^{n} \Big(\tau \hat{\rho}_i \Phi_{\hat{r}_i(v(k-1))}[u](k-1) - \rho_i \Phi_{r_i(v(k))}[u](k)\Big). \tag{18}$$

and

$$\mathcal{P}[k] = \sum_{i=1}^{n} \max\{|\tau \hat{\rho}_i u(k-1) - \rho_i u(k) + \tau \hat{\rho}_i r_i(v(k-1)) - \rho_i r_i(v(k))|, |\xi_i(k) - \xi_i(k-1)|\}. \tag{19}$$

For $\hat{\rho}_i = \rho_i + \lambda_i$

$$\mathcal{P}[k] = \sum_{i=1}^{n} \max\{|(\tau\rho_i + \tau\lambda_i)u(k-1) - \rho_i u(k) + (\tau\rho_i + \tau\lambda_i) r_i(v(k-1)) - \rho_i r_i(v(k))|, |\xi_i(k) - \xi_i(k-1)|\} \tag{20}$$

and

$$|\mathcal{P}[k]| \le \sum_{i=1}^{n} \max\{|\tau\rho_i u(k-1) - \rho_i u(k) + \tau\rho_i r_i(v(k-1)) - \rho_i r_i(v(k))| + \tau\lambda_i u(k-1) + \tau\lambda_i r_i(v(k-1)), |\xi_i(k) - \xi_i(k-1)|\} \tag{21}$$

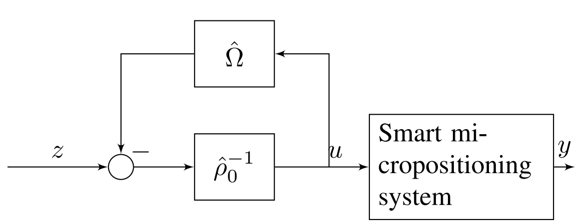

Fig. 2: The application of the model-based feedforward compensator to a smart micropositioning system.

Let $\max\{|\tau u(k) - u(k-1)|\} = \bar{\epsilon}_1$, $\max\{|u(k-1) + r_i(v(k-1))|\} = \kappa$, $\max|\tau r_i(v(k-1)) - r_i(v(k))| = \bar{\epsilon}_2$, and $\lambda_{\max} = \max\{|\lambda_i|\}$, where $\bar{\epsilon}_1$, $\bar{\epsilon}_2$, and κ are positive constants. Then the error is

$$|e(k)| \le |z(k)||(1-\tau)| + n\rho_{\max}\{\bar{\epsilon}_1 + \bar{\epsilon}_2 + \lambda_{\max}\kappa, \epsilon_3\}. \tag{22}$$

It can be concluded that the tracking error is bounded in the presence of the RDPI model parameters uncertainty when the proposed feedforward compensator is applied for compensation of rate-dependent hysteresis nonlinearities. This can be achieved with very small sampling time relative to the period of the signal involved. The following conditions can be suggested to obtain low compensation error

$$T_s < \frac{1}{n(\rho_0 + 1)(\sum_{i=1}^{n} \rho_i + 1)(f_{\max} + 1)} \tag{23}$$

considering that

$$\rho_{\max} n < 1 \tag{24}$$

B. Numerical example

Consider a reference input $z(k) = 20\sin(2\pi f k T_s)$ μm applied to a hysteretic positioning actuator under $f = 1$ Hz, 50 Hz and 100 Hz excitations of frequency. A RDPI model is formulated using $n = 5$ play operators and the parameters $\rho_0 = 0.6295$, $\rho_1 = 0.1617$, $\rho_2 = 0.1137$, $\rho_3 = 0.0611$, $\rho_4 = 0.0328$, $\rho_5 = 0.0716$, $\delta_1 = 1.7504$, and $\delta_2 = 3.7627 \times 10^{-4}$. The response of the model under these excitations is presented in Figure 3 (a). The proposed compensator was subsequently applied for compensation of hysteresis nonlinearities of the RDPI model at sampling time of 1×10^{-6} Sec considering that $f_{max} = 500$ Hz. Figure 3 (b) shows the input-output of the RDPI model when the proposed compensator is applied in a feedforward manner at 1 Hz, 50 Hz and 100 Hz excitations of frequency. Figure 3 (c) illustrates the time history of the error e and Δe when the proposed compensator is applied at input frequency of 50 Hz and sampling time of $T_s = 1 \times 10^{-6}$ Sec. Both the sampling time and weights are selected to satisfy the suggested conditions (23) and (24).

The effectiveness of the proposed compensator is further examined at same excitation frequency $f = 50$ Hz but with a sampling time of $T_s = 5 \times 10^{-4}$ Sec, which ignores the suggested condition (23). The time history of e and Δe are shown in Figure 3 (d), which illustrates that increasing the sampling time contributes substantial compensation error. The compensation error signals e and Δe are further examined in Figure 3 (e) under excitation frequency of 50 Hz with $T_s = 1 \times 10^{-6}$ Sec sampling time. The weights of the RDPI model are selected to ignore the suggested condition (24). As the results demonstrate, ignoring condition (24) contributes a dense noisy output signal that might effect the tracking performance of control systems in real-time. The maximum error is finally calculated and presented as a function of the sampling time under input frequency of 50 Hz in Figure 3 (f). The figure shows that decreasing the sampling time yields lower compensation error.

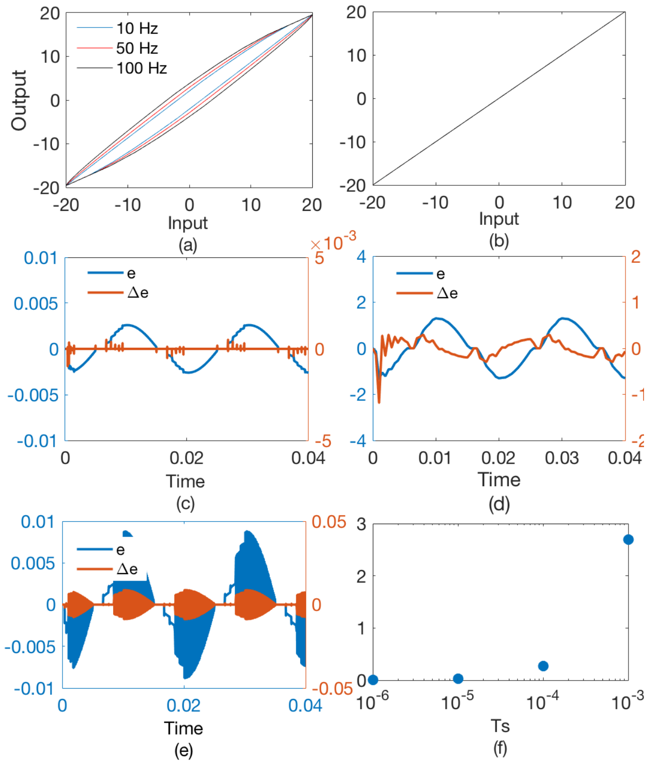

Fig. 3: (a) Input-output rate-dependent hysteresis nonlinearities of RDPI model at excitation frequency of 1 Hz, 50 Hz and 100 Hz, (b) input-output of RDPI when the proposed compensator is applied at 1 Hz, 50 Hz and 100 Hz excitation of frequency, time-history of e and Δe at (c) excitation frequency of 50 Hz and sampling time $T_s = 1 \times 10^{-6}$ Sec, (d) excitation frequency of 50 Hz excitation frequency and $T_s = 5 \times 10^{-4}$ Sec, (e) excitation frequency of 50 Hz and $T_s = 1 \times 10^{-6}$ Sec with violating condition (24) and (f) the maximum compensation error at different samplings of time.

V. Application to a Piezoelectric Cantilevered Actuator

A. The experimental study

The effectiveness of proposed rate-dependent compensator was further examined in real-time system. A piezoelectric cantilevered actuator was considered for the experimental study to evaluate the effectiveness of the proposed compensator to compensate for the rate-dependent hysteresis nonlinearities. This actuator exhibits output displacement (deflection) $y[\mu\text{m}]$ in response to input voltage $u[\text{V}]$.

The piezoelectric cantilevered actuator was subjected to a sinusoidal harmonic input voltage of 8 V amplitude at different excitations of frequency. Figure 4 shows the input voltage-output displacement rate-dependent hysteresis loops measured at 1 Hz, 90 Hz, 130 Hz and 190 Hz excitation frequency. As the figure shows, increasing the excitation frequency of the applied input yields a significant increase in the hysteresis nonlinearities of the actuator.

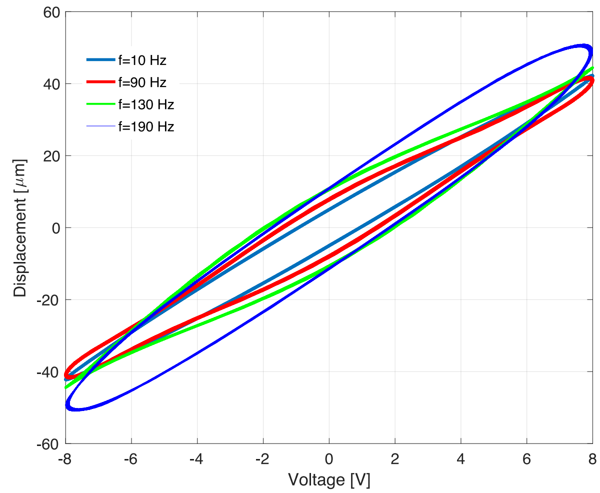

Fig. 4: Measured hysteresis loops at 8 V amplitude applied under 1 Hz, 90 Hz, 130 Hz and 190 Hz excitation frequency.

B. Open-loop compensation

The measured experimental results in Figure 4 were employed to identify the parameters of a RDPI model. The model was formulated on the basis of the discrete version presented in (1) and considering conditions (23) and (24). The identified RDPI model was afterwards restructured to obtain the associated inverse multiplicative scheme as illustrated in Figure 2. Subsequently, model-based compensator was applied for compensation of rate-dependent hysteresis nonlinearities of the piezoelectric cantilevered actuator. The effectiveness of the proposed compensator was examined at different excitations of input frequency differs than those selected to identify the RDPI model. The response of the actuator when the proposed rate-dependent compensator is applied as a feedforward compensator are given in Figure 5. The figure shows the desired output displacement $y[\mu\text{m}]$ versus the resulting output displacement $y[\mu\text{m}]$ at different frequencies in the 0.1 Hz - 220 Hz range. The compensation results demonstrate that the suggested rate-dependent compensator can be employed to cancel out the rate-independent and rate-dependent hysteresis nonlinearities of the piezoelectric cantilevered actuator. However, at very higher excitations, the compensation results revealed slight error e that is mostly attributed to the other dynamics that could not be represented by the RDPI model.

VI. Application to a Closed-Loop Control

A. Internal model-based control design

The previous sections dealt with the feedforward compensation of the rate-dependent hysteresis of the initial system. The compensator was formulated on the basis of the RDPI model that can describe rate-dependent hysteresis nonlinearities. The compensation error $e(k)$ can be considered as an internal perturbation of a linear system. Thus, the

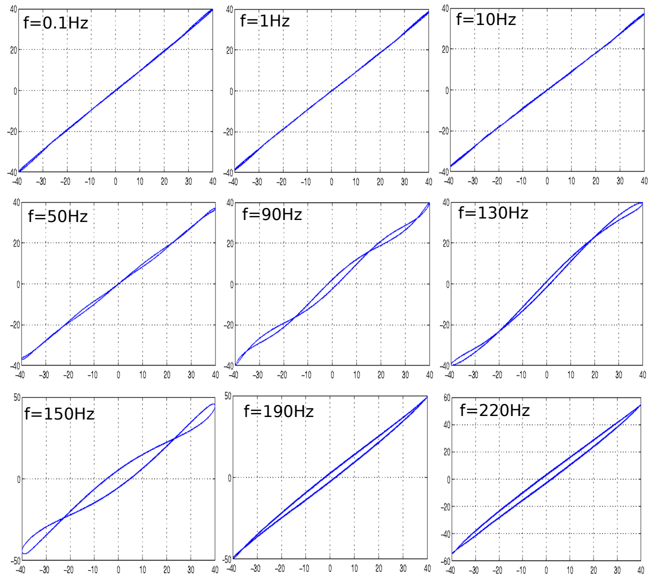

Fig. 5: The hysteresis loops of the piezoelectric cantilevered actuator when the proposed compensator is applied for compensation of hysteresis nonlinearities.

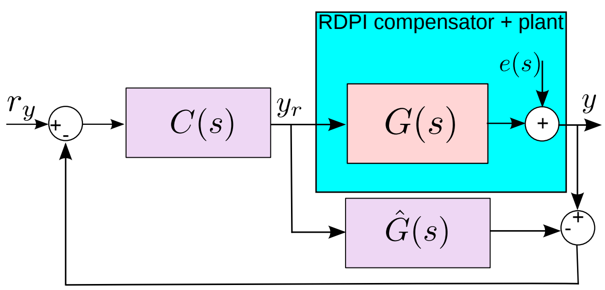

Fig. 6: Feedforward control augmented by an internal model feedback scheme.

system can be represented using a linear model subjected to a disturbance:

$$y(s) = G(s)y_r(s) + e(s) \tag{25}$$

where $G(s) = 1$, and y_r is the reference displacement input. Since there are always high dynamics that could not be represented by the RDPI and hence were not compensated then $G(s) \neq 1$. In the sequel, we leave $G(s)$ in order to consider such more general case. The identification of $G(s)$ can be done with the new system having as input y_r and output y, see Figure 1.

In this section, we augment the previous feedforward scheme with a feedback controller in order to add robustness and improve the tracking objective. In particular, we construct the feedback control law from the error analysis of the previous sections. The internal model scheme will be employed as a feedback control, which permits the consideration of the perturbation and modeling error in $G(s)$. Figure 6 shows the feedforward scheme augmented by the internal model based feedback control. In the figure, r_y is the exogenous reference input, while $C(s)$ and $\hat{G}(s)$ represent the feedback controller to be calculated. In fact, the objective is to obtain a $\hat{G}(s)$ as equal as possible to the real behavior $G(s)$. $\hat{G}(s)$ is therefore an approximate model of $G(s)$. From Figure 6, we have

$$y = \frac{GC}{\left(1 + C\left(G - \hat{G}\right)\right)} r_y + \frac{\left(1 - \hat{G}C\right)}{\left(1 + C\left(G - \hat{G}\right)\right)} e \tag{26}$$

Selecting $C(s) = \frac{1}{\hat{G}(s)}$. Thus, $y = r_y + 0 \times e$, which implies that the steady-state error is always null and the disturbance is always rejected irrespective to the constant $e(t)$, constant reference $r_y(t)$ and any approximate model $\hat{G}(s)$. However, this choice only improves the steady-state regime and ignores transient part. In order to consider a transient part with a desired settling time t_r for the closed-loop, let us choose $C(s) = F(s)\frac{1}{\hat{G}(s)}$ where the filter $F(s) = \frac{1}{1+\frac{t_r}{3}s}$ directly corresponds to the desired closed-loop behavior. Notice that, in that case, if the internal model $\hat{G}(s)$ is exact ($\hat{G}(s) = G(s)$), we have: $y(s) = F(s)r_y(s) + (1 - F(s))\, e(s)$. Consequently, the desired transient part, the zero steady-state error and the disturbance rejection are guaranteed. However, if $\hat{G}(s)$ is not exact, we have

$$y = \frac{\frac{G}{\hat{G}}F}{\left(1 + F\left(\frac{G}{\hat{G}} - 1\right)\right)} r_y + \frac{(1 - F)}{\left(1 + F\left(\frac{G}{\hat{G}} - 1\right)\right)} e.$$

In such case, the steady-state error is still zero and the disturbance still rejected. However, the transient part differs from the desired transient part $F(s)$. This difference increases with if $\hat{G}(s)$ has not been identified properly.

VII. Application to the Piezoelectric Micropositioning Actuator

The boundedness of the compensation error allows designing feedback controllers that can be applied with the proposed rate-dependent compensator to facilitate connecting smart actuators to dynamic plants so as to deliver the desired displacement without hysteresis nonlinearities. The dynamic model $\hat{G}(s)$ was identified from output displacement that was measured after applying a desired step input of $y_r = 40\mu m$ to the compensated system. After application of the ARMAX (Autoregressive–moving-average model with exogenous inputs) system identification technique, the dynamic model $\hat{G}(s)$ was identified as:

$$\hat{G}(s) = \frac{-19.6(s-1.2e5)(s+3e4)(s-2.8e4)(s-1.1e4)(s+1430)}{(s^2+925s+4.6e5)(s^2+55s+1.4e7)(s^2+1.5e4s+4e9)} \frac{(s^2+4.3e4s+3.2e9)}{(s^2+965s+3.9e9)} \tag{27}$$

where $e\theta$ denotes $\times 10^{\theta}$, for instance $1.2e5 = 1.2 \times 10^5$. Figure 7 presents a comparison between both the experimental step response measured from the piezoelectric cantilevered actuator and the step response of the identified model $\hat{G}(s)$.

After estimating the model $\hat{G}(s)$ that can represent the real model $G(s)$ of the compensated system, a feedback behavior $F(s)$ has to be constructed. A transient response

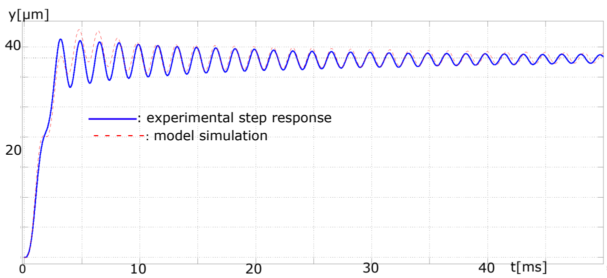

Fig. 7: A comparison between the step response of the piezoelectric cantilevered actuator and the identified linear model $\hat{G}(s)$.

without overshooting and with a settling time of $t_r = 20$ ms was considered as desired specifications for the closed-loop control design. These specifications were selected to obtain a better tracking compared to the response obtained from the initial system $\hat{G}(s)$ (and $G(s)$) in Figure 7. Constructing $F(s)$ based on desired requirements was employed to yield the controller $C(s)$ such that

$$C(s) = F(s)\frac{1}{\hat{G}(s)}. \tag{28}$$

Then, $C(s) = \frac{1}{(1+6.7\times10^{-3}s)\hat{G}(s)}$, where $\hat{G}(s)$ is given by (27).

A step input $r_y = 40$ μm was applied to the closed-loop augmented system. The measured output displacement of the actuator when the reference input $r_y = 40$ μm is applied is shown in Figure 8.

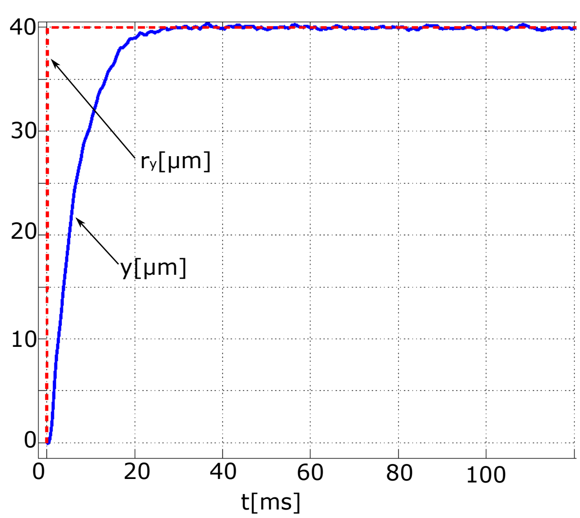

Fig. 8: Experimental step response of the closed-loop with the proposed internal-model controller.

VIII. Conclusions

A model-based feedforward compensator constructed using the rate-dependent Prandtl-Ishlinskii model is applied to compensate for the rate-dependent hysteresis nonlinearities. The proposed model-based feedforward controller can compensate for the rate-dependent hysteresis nonlinearities in micropositioning control when low sampling time is considered. The proposed compensator is applied as a feedforward controller to compensate for the rate-dependent hysteresis nonlinearities of a piezoelectric cantilevered actuator. The results show that the proposed compensator can compensate for rate-dependent hysteresis nonlinearities. A closed-loop control design can be also applied with the proposed compensator to improve the tracking performance of a piezoelectric cantilevered actuator under step and sinusoidal harmonic inputs.

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