Observer-based state feedback for a class of interval Model: Application to multi-Dof micro-positioning system

Mounir Hammouche, Philippe Lutz and Micky Rakotondrabe

Keywords: Robust Observer-Based State Feedback, Interval models, SIVIA (Set Inversion via Interval Analysis), Multi-DOF micropositioning system.

Introduction

During the last decades, micro-positioning systems have gained much attention in microrobotic applications such as micro/nano-assembly, micromanipulation, nanotechnology,... [1,2]. However, the control of micro-positioning systems always presents a very difficult task due to the high sensitivity to the environment at this scale, the characteristics of the used smart actuators (hysteresis, creep, ...), and the cross-couplings effects present between the different axes for multivariable case. In fact, different controllers synthesis approaches have been developed to control such system including real-time adaptive, robust control $(H_{\infty}, \mu - synthesis, ..)$, nonlinear approaches and robust interval-based techniques [1-4]. In this presentation we suggest to model the system uncertainties by a linear and time-invariant interval state-space model which is well adapted to multivariable control systems. Furthermore, we propose a robust Observer-Based State Feedback design using interval techniques to control the micro-positioning systems [4-6]. The proposed control strategy is tested in simulation and validated experimentally using a multi-Dof (degrees of freedom) micro-positioning system.



Figure 1: Observer-based state feedback schema.

Problem formulation

In our work the problem of observer-based state feedback controller with integral compensator is addressed for an interval state-space model with realization ([A], [B], [C]), as shown in fig.1. The objective of the proposed control strategy is to find the set of robust gains matrices [[K] [K_i]] and [L], for the controller and the observer respectively, such that the closed-loop state matrices of the controller and of the observer possess their eigenvalues within two desired subregions: one for the controller $\Omega_{Desired region controller}$ and the other for the observer $\Omega_{Desired region Observer}$, as depicted in fig.2. These two subregions are defined such that the closed-loop of the observer is (i.e. the state estimation), at least, four times faster then the closed-loop of the controller, and such that they provide a guaranteed stability margin and some predefined performances.

Main results

In order to obtain the set of robust gains for the controller and the observer using pole assignment techniques and interval analysis, foremost, we propose to adopt the separation theorem to be able to find the gains of the controller and the observer in separated way and also to reduce the computation complexity [7]. Furthermore we convert the problem of pole assignment to set inversion problem and solve it using the Set Inversion Via Interval Analysis (SIVIA) algorithm [5] with the help of interval eigenvalues computation [8-10]. This recursive



Figure 2: Desired regions for the controller and the observer.

SIVIA-based algorithm approximates with subpaving the set solutions $[[K] \ [K_i]]$ and [L] that satisfy the following inclusions:

$$eig\left[A_{Controller-cl}([A], [B], [C], [[K] [K_i]])\right] \subseteq \Omega_{Desired \, region \, controller} \tag{1}$$

$$eig\left[A_{Observer-cl}([A], [B], [C], [L])\right] \subseteq \Omega_{Desired \, region \, observer} \tag{2}$$

where $A_{controller-cl}$ is the augmented closed-loop matrix for the controller and $A_{Observer-cl}$ is the closed-loop matrix for the observer. These two closed-loop matrices are obtained from the separation theorem.

Finally, the effectiveness of the proposed algorithm is tested in simulation by mean of Monte-Carlo simulation and is illustrated by a real experimentation to control a new multi-Dof micro-positioning system.

References

- M. RAKOTONDRABE, Smart materials-based actuators at the micro/nanoscale, , 2013.
- [2] S. DEVASIA, E. ELEFTHERIOU, AND S. O. R. MOHEIMANI, A survey of control issues in nanopositioning, *TCST*, 2007.
- [3] S. KHADRAOUI, M. RAKOTONDRABE, P. LUTZ, Interval force/position

modeling and control of a microgripper composed of two collaborative piezoelectric actuators and its automation, *IJCAS*, 2014.

- [4] Y. SMAGINA AND I. BREWER, Using interval arithmetic for robust state feedback design, *Systems and Control Letters*, 2002.
- [5] L. JAULIN, Applied interval analysis: with examples in parameter and state estimation, *Springer Science and Business Media*, 2001.
- [6] D. EFIMOV, T. RAISSI., Design of interval observers for uncertain dynamical systems, ARC 2016.
- [7] K. ZHOU, J. C. DOYLE, K. GLOVER, Robust and optimal control, Book vol.40 1996.
- [8] J. ROHN, A handbook of results on interval linear problems, 2005.
- [9] M. HLADIK, Bounds on eigenvalues of real and complex interval matrices , AMC, 2013.
- [10] M. T. HUSSEIN, Assessing 3-d uncertain system stability by using matlab convex hull functions, *IJACSA* 2011.