A complete framework for the design of the viscoelastic insulators

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Abstract

This work aims to provide a toolbox for the design of optical image stabilizer. Both mechanical and electro-mechanical devices can be used to stabilize imaging devices. The mechanical devices are constituted with viscoelastic materials that provide damping and flexibility in order to isolate the optical devices from vibrations and shocks. The purpose of this paper is to give a framework for taking into account viscoelastic materials behavior in different simulations, complex eigenvalue analysis, frequency response or time response. To achieve this goal, the generalized Maxwell model is associated to an original state-space formulation. To reduce CPU cost, it is associated with model order reduction. As these materials are very sensitive to temperature variation, the model has been adapted to take easily into account different temperature fields over the domain. Many results are presented and compared to experimental data in order to validate the algorithms. Besides, in a context where the temperature and thus the performance of the insulators may vary in a unknown way an optimal robust choice of the design is studied to ensure the efficiency of the systems.

Keywords : viscoelasticity, damping, complex eigenvalue analysis, robustness, info-gap

1 Introduction

Viscoelastic materials are commonly recognized as efficient passive damping systems, and are widely used in many applications such as automotive, marine or aerospace. The design of structures exhibiting such a behavior is complexified by many aspects as the need to choose between different material models not valid for any application, the difficulties in identifying the parameters for these models, and besides all the varying properties with frequency, temperature, pre-load, aging above all. Furthermore, the finite element models used for industrial applications are large-sized models which lead to crippling calculation times. The wide use of viscoelastic structures as insulators makes essential the development of efficient and robust tools of design, for modal analyses for instance, in a context of severe, thermal, uncertainty, in particular. The work presented in this paper aims at proposing a global framework in this purpose.
The paper is organized as follows. The considered application is detailed in the first section, as well as the modeling of the viscoelastic materials whose behavior is assumed to vary with frequency and temperature. The strategy developed to solve the eigenvalue problem is described in the second section, and obtained results are compared to experimental data obtained on the considered structure. Finally, a robustness analysis is presented in the third section as an efficient tool to quantify the impact of the temperature uncertainties on damping.

2 Modeling

2.1 Case study

A structure integrating viscoelastic components is used as starting point to the led works. It consists in a damper introduced in an optical image stabilizer fixed under fighter jets, where viscoelastic elements are introduced to achieve a sufficient damping level in all the directions of space during application phases (Fig.1-A). The damper is made of a steel frame and viscoelastic elements are added by melt-injection. One part of this damper is directly linked to the suspended mass, and the other is attached to the main frame. In our case, the vibrations transmitted to the optics come from this structure. The discretization of the model is made with the Finite Element Method (FEM) using ten nodes tetrahedron elements. The nominal model contains 83037 nodes, and 249111 Degrees Of Freedom (DOF). The mesh between the viscoelastic and the steel part is assumed to be compatible.

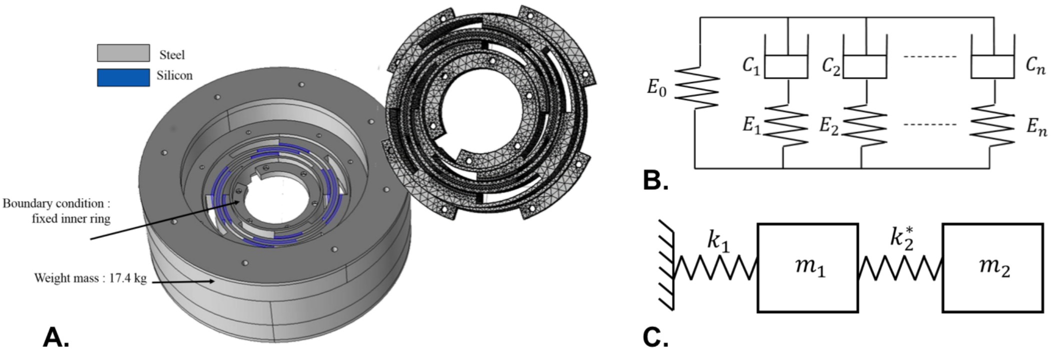

Fig. 1: A. : Design of a damper made of a steel frame and silicon viscoelastic elements, finite element modeling with tetrahedron elements ; B. : Rheological representation of the Generalized Maxwell Model (GMM) as a series of spring and viscous elements ; C. : 2-DOFs modal model of the device

2.2 Rheological behavior of the viscoelastic components

The basic assumption of linear viscoelasticty is the existence of a relaxation function such that the stress is obtained as a convolution with the strain history [7, 10]. Using a Fourier transform, one obtains an equivalent representation where the material is now characterized by a complex modulus E^* which depends on the frequency ω and on the temperature T. It can also depend on initial stresses σ_0. In many papers, it is divided into a real part, the storage modulus $E^{'}$, and an imaginary part $E^{''}$, the loss modulus:

$$\hat{\sigma}(\omega) = E^*(\omega, T, \sigma_0)\varepsilon(\omega) = [E^{'}(\omega, T, \sigma_0) + j.E^{''}(\omega, T, \sigma_0)]\hat{\varepsilon}(\omega) \tag{1}$$

In order to perform accurate simulations of the damper behavior, the rheological model for the viscoelastic elements has to be realistic on a large frequency bandwidth, and has to allow to take into account the variations of behavior with the temperature. The Generalized Maxwell Model (GMM) [12] is chosen as this model has both a time and frequency representation and is thus adapted for many different excitations, it can be adapted for studies on large bandwidth, and thermal dependency can be integrated in the model parameters. The complex dynamic modulus can thus be written as follows:

$$E^* = E_\infty + \sum_{i=1}^{ncell} \frac{E_i \tau_i j\omega}{1 + \tau_i j\omega} = E_\infty \left(1 + \sum_{i=1}^{ncell} \frac{\alpha_i \tau_i j\omega}{1 + \tau_i j\omega}\right) \tag{2}$$

where E_∞ is the long term Young modulus, E_i the dynamic modulus of the i^{th} cell and τ_i its time constant. Increase the number of cells ($ncell$) allows the model to be consistent on a larger frequency range in return of an increase in the number of parameters. The second form is more adapted to the dynamic simulation. It introduces the frequency stiffening α_i as the ratio E_i/E_∞.

It has to be noted that the GMM model can be enhanced to take into account a possible dependence to the strain amplitude (known at Payne effect) by the fretting elements addition. It thus allows to envisage later simulations in a nonlinear context.Fig. 1-B shows the rheological representation of the GMM.

3 Simulation and validation

3.1 Numerical simulation : Complex Eigenvalue Analysis (CEA)

The eigenvalue analysis of a viscoelastic problem is not classic due to the frequency dependency of the parameters. When considering the GMM to model the viscoelastic components, and assuming the rest of the structure as elastic, the complex eigenvalue problem to solve can be written as,

$$-\omega^2[M]\hat{U} + [K_{elastic}]\hat{U} + \sum_{i=1}^{nbmat} [K_{visco}^i] \left(\sum_{k=1}^{ncell} \frac{\alpha_k^i \tau_k^i j\omega}{1+\tau_k^i j\omega} \hat{U}^i \right) = 0 \tag{3}$$

$[M]$ and $[K_{elastic}]$ denote the mass and elastic stiffness matrices. $nbmat$ is the number of different viscoelastic materials, $ncell$ is the number of Maxwell cells in each GMM. α_{ik} and τ_{ik} are respectively the viscoelastic constant and the relaxation time for the k^{th} cell of the model associated to the i^{th} viscoelastic material. K^i_{visco} is a stiffness matrix based on the stiffness matrix of the i^{th} viscoelastic subdomain completed with null terms to fit the size of the finite element model. This equation is not in a classical form to be solved. The system is thus transformed to obtain a generalized eigenvalue problem by a state space formulation [6, 8]. Fig.2-left presents the steps of the proposed approach.

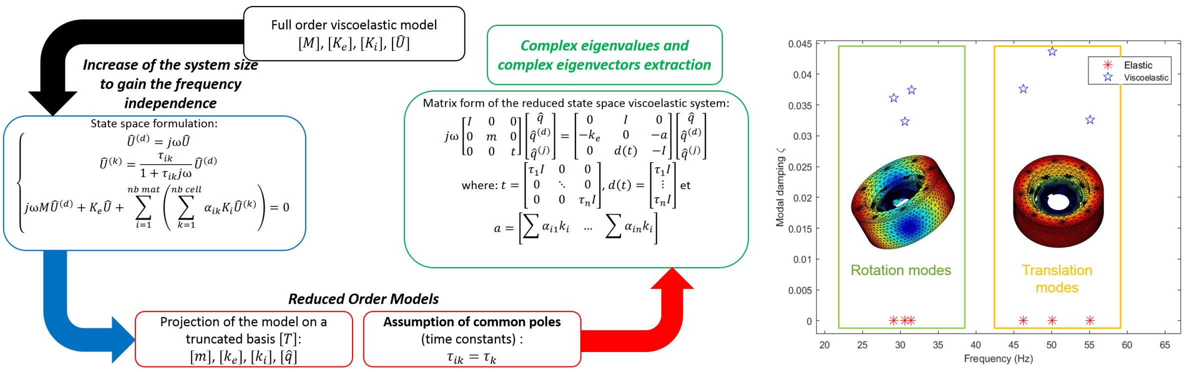

Fig. 2: LEFT : State space formulation with reduction of the model approach ; RIGHT : Complex Eigenvalue Analysis: Elastic (red asterisk) versus viscoelastic (blue star) system

The main issue with this expression is the size of the system. Indeed, if there are $nbcell$ terms in each GMM, the size of the state space formulation is $(ncell + nbmat + 2)N_{DOF}$ where N_{DOF} is the number of DOF in the initial finite element model. In order to reduce the size, the relaxation times are constrained to be common for all the viscoelastic materials. The identification of the GMM parameters under this assumption is still realistic [5]. As the proposed damper only contains one viscoelastic material, equations can be reduced to,

$$-\omega^2[M]\hat{U} + [K_{elastic}]\hat{U} + [K_{visco}] \sum_{k=1}^{ncell} \frac{\alpha_k \tau_k j\omega}{1+\tau_k j\omega} \hat{U} = 0 \tag{4}$$

A second action to improve the computation cost is to reduce directly the size of the system. The operators $[M]$, $[K_{elastic}]$ and $[K_{visco}]$ are projected on a truncated multi-model basis [2, 11]. In a dynamic problem with viscoelastic behavior, the multi-model approach allows us to build a projection basis that is representative of the viscoelastic material stiffening according to the rise of the frequency. To create this basis, a "low frequency" basis $[\Phi_{BF}]$ made of the p first normal modes is extracted by solving:

$$\left(-\omega_p^2[M] + [K_{elastic}] + [K_{visco(\omega=0)}]\right)\phi_p = 0 \tag{5}$$

These normal modes verify the two orthogonality conditions:

$$[\Phi_{BF}]^T[M][\Phi_{BF}] = [I] \tag{6}$$

$$[\Phi_{BF}]^T[K_{elastic+visco(\omega=0)}][\Phi_{BF}] = [\Omega^2] \tag{7}$$

Another basis $[\Phi_{HF}]$ also composed of the p first modes at "high frequency" is stocked:

$$\left(-\omega_p^2[M]+[K_{elastic}]+Re([K_{visco}]\sum_{k=1}^{ncell}\frac{\alpha_k\tau_k j\omega_{HF}}{1+\tau_k j\omega_{HF}})\right)\phi_p=0 \tag{8}$$

where ω_{HF} is the pulsation associated to the maximum frequency of the studied domain. Then, a modified Gram-Schmidt ortho-normalization is used to concatenate these two basis in one multi-model basis $[T]$. The dimension of this operator is $N_{DOF}*2p$.
Due to these two assumptions, the final formulation size is $2p(2+nbcell)$. The computation cost is hence more affordable and the extraction of the complex eigenvalues ω_i and the complex eigenvectors ϕ_i is possible. In our case, the eigenfrequency and the modal damping associated are determined by:

$$f_{pi} = |(f_i)/2\pi| \tag{9}$$
$$\zeta_i = -Im(f_i)/f_{pi} \tag{10}$$

3.2 Validation

Experimental tests on the proposed damper have allowed to identify the first resonance frequencies around 30 Hz for the three rotations and around 50 Hz for the three translations. A sweep sinus of 0.5 G magnitude over the frequency range $[10;1000]$ Hz is used to excite the damper. The Complex Eigenvalue Analysis (CEA) is achieved and a diagram frequency-modal damping is plotted (Fig.2-right).

A comparison between the viscoelastic (blue star) and the purely elastic (red asterisk) system is made. All the elastic identified frequencies lie on the horizontal axis as no damping is introduced in the system. The CEA allows to take into account the dissipative behavior of the viscoelastic elements, and to compute the modal damping for each mode contribution. It can be noticed a slight shift of the eigenfrequencies between the two problems. Such a representation is useful to quantify the effects of the system variables (such as temperature) on the localization of the poles as showed further is the paper. In order to validate this analysis, the Frequency Response Function (FRF) of the full order model is calculated for a sweep sinus excitation on X direction. The resonance frequency and the associated modal damping (f_0, ζ) are compared for the X translation mode. From, the FRF, we get ($f_0 = 47\ Hz$, $\zeta = 0.043$) whereas from, the CEA, we get ($f_0 = 49\ Hz$, $\zeta = 0.044$). Thus, the CEA appears to correctly predict the modal behavior of the system.

4 Robustness analysis under temperature uncertainties

4.1 Context of the study

The mechanical device presented in this paper is subjected to large derivation on temperature that can consist in sudden variations between $-20°C$ and $50°C$. Due to the position of the damper in the final system, it is hard to accurately know the surrounding temperature of the viscoelastic parts and, as told previously, the damping behavior of viscoelastic materials depends on the temperature. The effective temperature in which materials are subjected in the real applications may vary, and the profile of variation is an unknown considered as a lack-of-knowledge. In such a thermal environment it would thus be interesting to be able to guarantee a certain damping level, and maybe selecting a viscoelastic material less absorbing but more robust in temperature, than a material more absorbing but less robust. In this context, the aim of this last part is to study the robustness of a given viscoelastic material when the temperature field is uncertain.
For sake of simplicity, without loss of generality, the approach has been developed on a viscously damped two degree of freedom spring-mass system (Fig.1-C). This model is dimensioned to be almost representative of the two first modes of the studied device.

The spring between the two masses contains the viscoelastic behavior and the parameters k_1, k_2, m_1 and m_2 are set to obtain the two first resonance frequencies of the damper. The GMM is also reduced to a Zener model according to the smaller frequency interval observed. Hence, the form of the model is:

$$\left(-\omega^2[M]+[K_{elastic}]+\frac{j\alpha(T)\omega\tau(T)}{1+j\omega\tau(T)}[K_{visco}(T)]\right)\hat{U}=0 \tag{11}$$

Moreover, the time constant τ is assumed to be constant and no longer depending of the temperature field. It becomes a user choice to make the Zener model efficient on the studied frequency range. Hence, in this section, only the long-term modulus E_{infty} (linked to $[K_{visco}]$) and the α coefficient are functions of the temperature. An identification of these parameters based on experimental curves extracted at different temperature can be considered. However, in practice, it is really hard to know and control this variable during the characterization tests. In order to avoid this difficulty, a chosen temperature dependence is introduced. The viscoelastic material used in this work has a glass transition temperature around $50°C$. Thus in order to highlight the importance of robustness study, the temperature range of the study is $[45°C;55°C]$. Fig. 3 shows the impact of the α variation according to the temperature on the eigenfrequencies and the modal damping of the two-DOF system. A expected, it appears that the modal damping of the modes changes with temperature, and this effect is more important on the first mode on which the robustness study is going to be applied.

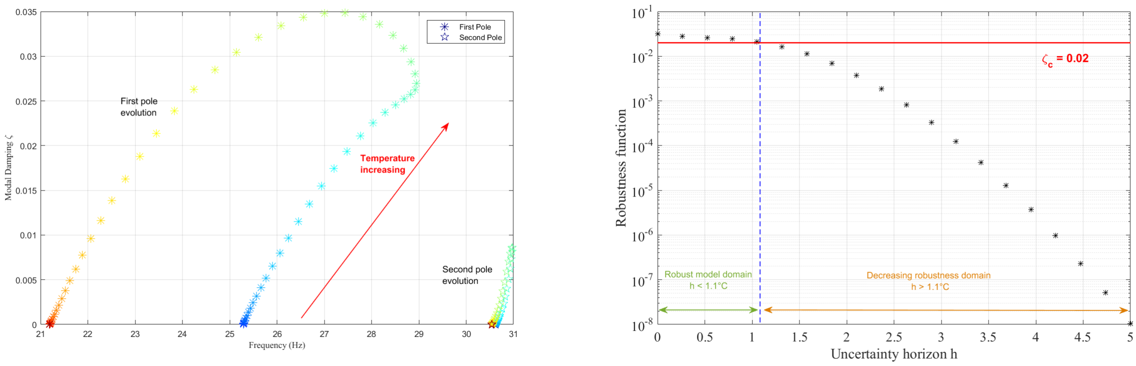

Fig. 3: LEFT : Impact of the α variation according to the temperature on the eigenfrequencies and the modal damping of the two-DOF system ; RIGHT : Info-gap robustness curve for the modal damping of the first mode

4.2 Info-gap robustness analysis: introduction and application

The approach is based on the info-gap decision theory [1, 3, 4, 9]. This theory is based on three elements: an *uncertainty model*, a *system model* and a *performance criterion*. The response of interest in the study is the modal damping of the first mode. Thus, the system model is the model used to compute this damping, given all the input parameters (dimensions, materials, $\cdots$). Equation 9 is used for that purpose, based on eigenfrequencies computed using the CEA on the complex eigenvalue problem,

$$\left(-\omega_p^2[M] + [K_{elastic}] + \frac{j\alpha(T)\omega_p\tau}{1+j\omega_p\tau}[K_{visco}(T)]\right)\phi_p = 0 \tag{12}$$

The info-gap model for uncertainty on temperature T is formulated under the following form:

$$U(h) = \left\{T : |T - \tilde{T}| \leq h\right\}, h \geq 0 \tag{13}$$

where h is the horizon on uncertainty, varying in the study between 0 and $5°C$, and $\tilde{T}$ is the nominal temperature fixed at the glass transition value. Hence, the interval covered by the temperature is between $45°C$ and $55°C$. The main goal is to determine how the modal damping fluctuates with the temperature deviation. In practice, this approach can be used to know when the modal damping of the considered mode ζ_1 will be smaller than a critical value ζ_c regarding the uncertainty on temperature. Hence, the performance requirement can be expressed as inequality:

$$\zeta_1(T) \geq \zeta_c \tag{14}$$

The robustness is the largest horizon of uncertainty h that can be tolerated on the temperature while sastifying the performance requirement:

$$\hat{h}(\zeta_c) = max\left\{h : \left[min_{T\in U(h)}\zeta_1(T)\right] \geq \zeta_c\right\}, \zeta_c \geq 0 \tag{15}$$

Thanks to the CEA and the info-gap formulation, it is possible to extract the minimum modal damping of the considered mode and observe the impact of the temperature deviation on this variable. Figure 3 - right presents the results of the info-gap analysis for the damping of the first mode. It clearly appears that for an horizon of uncertainty smaller than $1.1°C$, the ensured modal damping is greater than the performance requirement. In other words, the dynamic model is assumed to be robust for a temperature variation of $\pm 1.1°C$ around the nominal temperature. Out of this range, the robustness decreases with the growth of the horizon of uncertainty.

5 Conclusion

A framework is proposed in this paper to design the dynamic behavior of structures including viscoelastic elements. Such an approach is not easy to carry out due to the dependency of viscoelastic materials to many factors as the frequency or the temperature, and due to large computation times when working with finite element models. To show and validate the proposed methodology, this one is applied to a specific damper including viscoelastic elements. To take into account the viscoelastic behavior, the GMM is used due to its flexibility and its easy numerical implementation. A complex eigenvalue analysis is applied and numerical results are compared to given experimental data. The CEA has been driven under several assertions: the system poles are common for all of the viscoelastic material and the operators can be projected on a multi-model basis. These assumptions lead to an affordable computation cost. This approach has thus been validated, and has shown its efficiency to predict the system eigenfrequencies and the associated modal damping.
As well-known in viscoelasticity, the temperature is a leading parameter in the dynamic behavior and the temperature field introduces uncertainties due to its complexity. An info-gap robustness analysis has been led to quantify the impact of these uncertainties on the modal damping. It appears that the dynamic model well manages the small deviation of temperature but the robustness quickly decreases when the horizon of uncertainty growth. The method appears to be validate for a simplified model of viscoelastic dynamic system.
Ongoing work deals with the simulation of the dynamic response of structures with viscoelastic elements in case of multiple kinds of excitations, and taking into account non-linearities. Moreover, it will be interesting to apply the robustness analysis to the real dynamic system with adequate assumptions on temperature dependence.

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