Analytical Expressions for Parameters of the Dark Resonance in a Vacuum Vapour Cell

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***Abstract*—In spite of great number of papers devoted to theoretical and experimental study of the coherent population trapping phenomenon (CPT), still there is a lack of analytical results for the properties of the CPT-caused dark resonance in some principal cases. We will present the results of our study for single and double Λ schemes of atomic energy levels, which are of great importance for various applications of CPT (atom clocks, magnetometers, etc.). Analytical expressions for width and amplitude of the dark resonances have been obtained, which are valid for wide range of light wave intensities. The results concerns closed as well open Λ schemes. We will also discuss the difference between the two Λ schemes and state the condition when a double scheme can be effectively treated as a single Λ scheme. The analytical approximate formulas obtained have been compared with exact numerical solutions based on the optical Bloch equations as well as experimental.**

Keywords—coherent population trapping; dark resonance; vapour cell; density matrix; quantum metrology

I. Introduction

Coherent population trapping phenomenon (CPT) has been attracting great interest during long time since its discovery in 1976 [1]. It has found numerous applications in laser physics and spectroscopy, nonlinear optics, laser cooling of atoms, quantum metrology and quantum informatics. The basic model underlies the description of CPT is a single Λ scheme of atomic energy levels [2, 3] (Fig. 1a). Also there is a double Λ scheme (Fig. 1b), which is relevant for some applications too, especially for atom clocks.

Dark resonance is the main spectroscopic manifestation of CPT. It can be observed as a narrow dip in absorption (or fluorescence) property of a vapour cell, when it is being irradiated by the resonant light waves. One of the main conditions is that the two-photon (Raman) frequency detuning must be close to zero [2, 3]. The brilliant feature of the dark resonance consists in its potentially small width, which can be much smaller than the natural linewidth γ of optical transition.

In spite of a really huge number of papers in the direction of CPT and its applications, no one paper gives any clear analytical expression for the width and amplitude of the dark resonance in the case of a vacuum vapour cell beyond the perturbation theory limits. Several works contain such kind of analytical expressions only for electromagnetically-induced transparency (EIT) regime, when one of the laser waves is much weaker than the other, i.e. $I_p \ll I_c$ [4, 5]. At the same time, there are a lot of experimental data as well as numerical calculations for the parameters of dark resonances under the CPT regime ($I_p \approx I_c$) in vacuum cells (e.g., see [6-8]). Only analytical expressions valid for wide range of intensities, which can be found in the literature, are some phenomenological approximate formulas to fit experimental curves, which do not follow directly from any theoretical analysis.

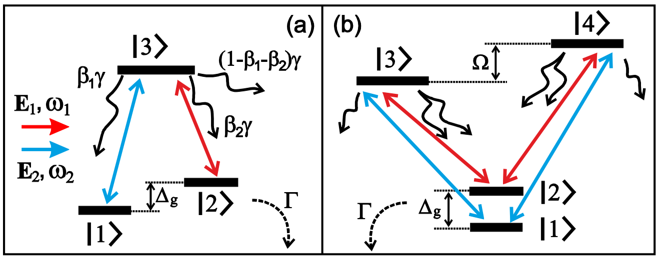

Fig. 1. Single (a) and double (b) Λ schemes of atomic energy levels.

Here, due to the lack of space, we briefly present the results of analytical study of the single Λ scheme only. Analytical expressions for the dark resonance parameters have been derived under several assumptions. In particular, we assume power broadening is the main source of the dark resonance broadening, without taking into account time-of-flight relaxation or residual Doppler broadening (due to difference of absolute values of the wave vectors). The Λ scheme is also assumed to be closed system of levels. These conditions allow us obtaining explicit and relatively simple analytical formulas for the dark resonance parameters, which are valid for wide range of the light wave intensities until Doppler width $k\upsilon_0$ (with υ_0 the most probable thermal velocity in a gas) is much larger than the Rabi frequencies of optical transitions. The latter condition is regularly satisfied in experiments with CPT in a thermal gas. In this abstract we take one-photon frequency

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detuning equal to zero, just for shortness. Our analytical results have been compared with the experimental data obtained with a cesium vapour cell (to be presented during the Conference).

II. Single Λ scheme

Here, for shortness, we focus only on a closed scheme of levels (Γ=0) and assume branching ratios to be equal: $\beta_1=\beta_2=1/2$, as well as the Rabi frequencies: $R_1=R_2=R$ (see Fig. 1a). Then, using the density matrix formalism, we come to the expression for the excited state population:

$$\mathrm{W}_\Lambda = \frac{4R^2\Delta^2}{4\Delta^2 x^2 + 16R^4 + \Delta^2\left(\gamma^2+\Delta^2\right) + 4\Delta^2 R^2} \qquad (1)$$

Here $\Delta=\omega_1-\omega_2-\Delta_g$ is the two-photon (Raman) detuning with Δ_g the frequency difference between ground-state levels, $x=k\upsilon$ is the Doppler frequency shift for a moving atom. Spontaneous relaxation rate of the excited state is equal to γ.

The spectroscopic signal is proportional to $\langle \mathrm{W}_\Lambda \rangle_\upsilon$:

$$\langle \mathrm{W}_\Lambda \rangle_\upsilon = \frac{1}{\sqrt{\pi}\upsilon_0}\int_{-\infty}^{+\infty} \mathrm{W}_\Lambda e^{-\upsilon^2/\upsilon_0^2} d\upsilon \qquad (2)$$

Inserting (1) to (2) one can easily obtain the exact solution expressed through the error function [4]. However, it is not convenient for our theoretical analysis, because in general it cannot provide us with some relatively simple expression for width and amplitude of the resonance. Therefore, the main problem consists in finding the most convenient approximation for (2). Also it would be very useful, if this approximation is valid for wide range of light wave intensities.

Let us write the final approximate formula for the dark resonance width in a vacuum vapour cell:

$$\mathrm{FWHM} \approx \frac{8R^2}{k\upsilon_0\, \xi_1(R)} \qquad (3)$$

with the dimensionless functions of R:

$$\xi_1(R) = \sqrt{\frac{2}{\pi}\left[1+\xi_0(R) - \sqrt{1+2\xi_0(R)}\right]}$$

$$\xi_0(R) = 4\eta(1+\eta)\ , \qquad \eta = \frac{\sqrt{\pi}}{2k\upsilon_0}\sqrt{\gamma^2+12R^2}$$

Fig. 2 shows the expression (3) is quite good approximation for wide range of intensities (compare solid blue and dashed black curves). If condition $R \lesssim \gamma$ is satisfied, expression (3) can be reduced to the following:

$$\mathrm{FWHM}\Big|_{R\lesssim\gamma} \approx \frac{4R^2}{\sqrt{\gamma^2+12R^2}} \qquad (4)$$

One can see that under the weak-field regime the width is linearly proportional to the intensity (as far as $I \sim R^2$). An expression for the resonance amplitude can be obtained analogically, see exp. (5).

It may seem that the closed double Λ scheme is equivalent to the closed single Λ scheme, if the coherence between $|3\rangle$ and $|4\rangle$ upper-state levels is neglected (see Fig. 1b). Indeed, the single Λ scheme does not have this type of coherence (see Fig. 1a). However, it will be shown that only the coherence taken into account makes two closed Λ schemes equivalent (with the effective substitution $R_\Lambda = \sqrt{2}R_{\Lambda-\Lambda}$).

$$\mathrm{A}_0 \approx \frac{4\pi R^2}{(k\upsilon_0)^2\,\xi_0(R)} \qquad (5)$$

The other theoretical and experimental results will be presented and discussed in details during the poster session.

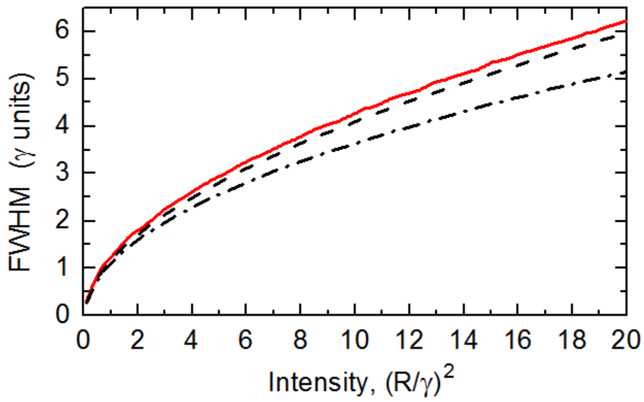

Fig. 2. Width of the dark resonance in a vacuum vapour cell for a single Λ scheme versus the light-wave intensity. Doppler width $k\upsilon_0$ equals to 50γ (typical for the experiments with thermal alkali-metal atoms). Solid line corresponds to the numerical calculations on the basis of the accurate expression (2), while dashed line corresponds to the approximate formula (3). The dashed-dotted line is explained by the simple expression (4), which is valid for the weak and moderate light fields ($R \lesssim \gamma$).

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