

Identification of Precision Motion Systems with Prandtl-Ishlinskii Hysteresis Nonlinearities

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Abstract—In this paper, we introduce an algorithm to identify the nonlinear dynamics of a class of precision motion systems, which is modeled as a Hammerstein system, that is, a cascade of a Prandtl-Ishlinskii nonlinearity with a linear system. The hysteresis nonlinearity, the linear system, and the intermediate signal between them are assumed to be unknown. The first stage in the algorithm is to identify the linear plant from measurements of the input and the output of the Hammerstein system. Then, the unknown intermediate signal is reconstructed using the output and the identified model of the linear system. Finally, the Prandtl-Ishlinskii nonlinearity is estimated using the input and the reconstructed intermediate signal. We show that under some conditions, the identified model of the linear plant and the estimated Prandtl-Ishlinskii nonlinearity are correct up to an unknown scalar factor.

I. INTRODUCTION

Smart material-based actuators, such as piezoceramic and magnetostrictive actuators, are considered an attractive choice for applications where fast response with high resolution is desired. These actuators are used in motion control applications to deliver fast output displacement in the micro/nano level in response to an external voltage or current inputs. Scanning in atomic force microscopy, surface finishing, micro-machining devices, and manipulating objects in micro-environments are examples of relevant applications that integrate smart actuators [1]–[4]. The advantages of smart material-based actuators, however, come with the hysteresis nonlinearities, which affect the performance and cause positioning errors [5], [6].

The dynamics of smart material-based actuators can be characterized by a Hammerstein system, that is, a cascade of a hysteresis nonlinearity and a linear dynamic system, see for example [7], [8]. Different models have been used to model hysteresis nonlinearities in smart material-based actuators. These models include the Preisach model, Bouc-Wen model, Prandtl-Ishlinskii model, Maxwell-Slip model, and Krasnoselskii-Pokrovskii model [9].

The Prandtl-Ishlinskii model has been recently used in different studies to model hysteresis nonlinearities because

it can be constructed with a fewer number of parameters than other hysteresis models, and the inverse model can be obtained analytically. In this study, we consider a Hammerstein system with a Prandtl-Ishlinskii model to characterize the dynamics of smart-material based actuators. This Hammerstein system has been used to model different smart material-based actuators, see for example [7], [10], [11]. In [7], [10], the Preisach model has been used to model the hysteresis nonlinearities of a magnetostrictive actuator. In [11], the Prandtl-Ishlinskii model has been used to model the dynamics of the piezo-cantilever beam over different excitation frequencies.

Identification of Hammerstein systems has been well-studied in the literature [12]–[18]. However, most studies consider Hammerstein systems with memoryless nonlinearities. Identification of Hammerstein systems in the presence of hysteresis-backlash and hysteresis-relay nonlinearities was studied in [19]. However, hysteresis-backlash and hysteresis-relay nonlinearities cannot describe hysteresis nonlinearities that appear in smart material-based actuators. In [5], pseudo random binary sequences were used to identify the linear dynamic part only of the piezoceramic actuator.

In this paper, we consider the problem of identifying Hammerstein systems with hysteresis nonlinearities. We assume that only the input and the output of the Hammerstein system are known, where the intermediate signal of the Hammerstein system is inaccessible. The first stage in the algorithm is to identify the linear plant from measurements of the input and the output of the Hammerstein system. Then, the unknown intermediate signal in the Hammerstein system is reconstructed using the output and the identified model of the linear part of the Hammerstein system. Finally, the hysteresis nonlinearity is estimated using the input and the reconstructed intermediate signal. The approach presented in this paper can be applied to Hammerstein systems with either static or dynamic nonlinearities.

II. THE HAMMERSTEIN SYSTEM WITH THE PRANDTL-ISHLINSKII MODEL

This section presents the Hammerstein system that characterizes the dynamics of a class of smart material-based actuators such as piezoceramic and magnetostrictive actuators.

A. The Hammerstein system

Consider the discrete-time SISO Hammerstein system shown in Figure 1, where u is the input, $\mathcal{P} : \mathbb{R} \rightarrow \mathbb{R}$ is the

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Prandtl-Ishlinskii hysteresis nonlinearity, v is the intermediate signal, and y_0 is the output of the asymptotically stable, SISO, linear, time-invariant, discrete-time system G . We assume that G has no poles on the unit circle. Hammerstein systems with hysteresis nonlinearities have been used in different studies to model the dynamics of smart material-based actuators, see for example [5], [7], [10].

B. The hysteresis model

The Prandtl-Ishlinskii hysteresis model has been used to model nonlinearities in the output displacement of piezoceramic and magnetostrictive actuators [6], [11]. This model is constructed based on a linear combination of play hysteresis operators. For all $k \geq 0$, the output v of the Prandtl-Ishlinskii model is represented by [20]

$$v(k) = \mathcal{P}[u](k) \triangleq \sum_{i=1}^n p_i \Gamma_{r_i}[u](k), \quad (1)$$

where n is the number of play operators, p_1, \dots, p_n are positive weights, r_1, \dots, r_n are positive constants represent thresholds, and $\Gamma_{r_i}[u](k)$ is the output of the i th play operator at time step k , where

$$\Gamma_{r_i}[u](k) = \max\{u(k) - r_i, \min\{u(k) + r_i, \Gamma_{r_i}[u](k-1)\}\}, \quad (2)$$

which is also equivalent to

$$\Gamma_{r_i}[u](k) = \begin{cases} u(k) + r_i, & u(k) < u(k-1) \text{ and } u(k) + r_i < \Gamma_{r_i}[u](k-1), \\ u(k) - r_i, & u(k) > u(k-1) \text{ and } u(k) - r_i > \Gamma_{r_i}[u](k-1), \\ \Gamma_{r_i}[u](k-1), & \text{otherwise,} \end{cases} \quad (3)$$

where for $k < 0$, we consider $u(k) = 0$ and $\Gamma_{r_i}[u](k) = 0$.

Note that, (3) can be written as

$$\Gamma_{r_i}[u](k) = u(k) + q_i[u](k), \quad (4)$$

where

$$q_i[u](k) \triangleq \begin{cases} r_i, & u(k) < u(k-1) \text{ and } u(k) + r_i < \Gamma_{r_i}[u](k-1), \\ -r_i, & u(k) > u(k-1) \text{ and } u(k) - r_i > \Gamma_{r_i}[u](k-1), \\ \Gamma_{r_i}[u](k-1) - u(k), & \text{otherwise.} \end{cases} \quad (5)$$

Therefore, using (4), (1) becomes

$$v(k) = \alpha u(k) + \sum_{i=1}^n p_i q_i[u](k), \quad (6)$$

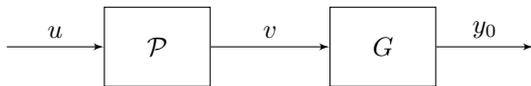


Fig. 1. A SISO Hammerstein System, where u is the input, \mathcal{P} is the Prandtl-Ishlinskii hysteresis nonlinearity, v is the unknown intermediate signal, and y_0 is the output of the linear, time-invariant, discrete-time system G .

where $\alpha \triangleq \sum_{i=1}^n p_i$ is a positive number. For all $j \geq 0$, let H_j denote the j th Markov (impulse response) parameter of G . Then, y_0 can be written as [21]

$$y_0(k) = \sum_{j=0}^k H_j v(k-j). \quad (7)$$

Moreover, using (6), (7) can be written as

$$\begin{aligned} y_0(k) &= \sum_{j=0}^k H_j (\alpha u(k-j) + R_j(k)) \\ &= \alpha \sum_{j=0}^k H_j u(k-j) + \sum_{j=0}^k H_j R_j(k), \end{aligned} \quad (8)$$

where $R_j(k) \triangleq \sum_{i=1}^n p_i q_i[u](k-j)$.

III. IDENTIFICATION OF THE LINEAR PART OF THE HAMMERSTEIN SYSTEM

Consider the FIR model of G given by [21]

$$G_\mu(\mathbf{q}) \triangleq \sum_{i=0}^{\mu} H_i \mathbf{q}^{-i}, \quad (9)$$

where $\mu \geq 0$ is the order of \hat{G}_μ , \mathbf{q}^{-1} is the backward shift operator, and for all $i = 0, \dots, \mu$, H_i is the i th Markov parameter of G .

For all $k \geq 0$, (7) can be written as

$$y_0(k) = y_{0,\mu}(k) + e_\mu(k), \quad (10)$$

where

$$y_{0,\mu}(k) \triangleq \sum_{j=0}^{\min\{\mu, k\}} H_j v(k-j) \quad (11)$$

is the output of the FIR model (9) of G . The FIR model output error at time k defined by

$$e_\mu(k) \triangleq y_0(k) - y_{0,\mu}(k) \quad (12)$$

is the difference between the output of G and the FIR model output at time k . Since G has no poles on the unit circle, then we can find finite μ such that e_μ is negligible [21]. Taking the limit of (11) as μ tends to infinity, and using (7) yields, for all $k \geq 0$,

$$\lim_{\mu \rightarrow \infty} y_{0,\mu}(k) = \sum_{j=0}^k H_j v(k-j) = y_0(k). \quad (13)$$

Therefore, for all $k \geq 0$,

$$\lim_{\mu \rightarrow \infty} e_\mu(k) = y_0(k) - \lim_{\mu \rightarrow \infty} y_{0,\mu}(k) = 0. \quad (14)$$

Consider the identification problem shown in Figure 2, where u is a deterministic signal that is persistently exciting of a sufficient order, w is a realization of a zero-mean, stationary, white random process \mathcal{W} with Gaussian probability density function $\mathcal{N}(0, 1)$, and the intermediate signal v is unknown.

Note that (10) can be expressed as

$$y_0(k) = \theta_\mu \phi_v(k) + e_\mu(k), \quad (15)$$

where

$$\begin{aligned} \theta_\mu &\triangleq [H_0 \quad \cdots \quad H_\mu], \\ \phi_v(k) &\triangleq [v(k) \quad \cdots \quad v(k-\mu)]^T. \end{aligned}$$

Moreover, for all $k \geq 0$

$$y(k) = \theta_\mu \phi_v(k) + w(k) + e_\mu(k). \quad (16)$$

The least squares estimate $\hat{\theta}_{\mu,\ell}$ of θ_μ is given by

$$\hat{\theta}_{\mu,\ell} = \arg \min_{\bar{\theta}_\mu} \|\Psi_{y,\ell} - \bar{\theta}_\mu \Phi_{\mu,\ell}\|_F, \quad (17)$$

where $\bar{\theta}_\mu \in \mathbb{R}^{1 \times \mu}$,

$$\begin{aligned} \Psi_{y,\ell} &\triangleq [y(\mu) \quad \cdots \quad y(\ell)], \\ \Phi_{\mu,\ell} &\triangleq [\phi_\mu(\mu) \quad \cdots \quad \phi_\mu(\ell)], \\ \phi_\mu(k) &\triangleq [u(k) \quad \cdots \quad u(k-\mu)]^T, \end{aligned}$$

and ℓ is the number of samples.

The eigensystem realization algorithm (ERA), which is based on the Ho-Kalman realization theory, can be used to construct a transfer function estimate \hat{G} of G from the estimated Markov parameters $\hat{\theta}_{\mu,\ell}$ [22], [23].

IV. CONSISTENCY ANALYSIS

It follows from (17) that the least squares estimate $\hat{\theta}_{\mu,\ell}$ of θ_μ satisfies

$$\Psi_{y,\ell} \Phi_{\mu,\ell}^T = \hat{\theta}_{\mu,\ell} \Phi_{\mu,\ell} \Phi_{\mu,\ell}^T. \quad (18)$$

Moreover, it follows from (16) that

$$\Psi_{y,\ell} = \theta_\mu \Phi_{v,\ell} + \Psi_{w,\ell} + \Psi_{e_\mu,\ell}, \quad (19)$$

where

$$\Phi_{v,\ell} \triangleq [\phi_v(\mu) \quad \cdots \quad \phi_v(\ell)], \quad (20)$$

$$\Psi_{w,\ell} \triangleq [w(\mu) \quad \cdots \quad w(\ell)], \quad (21)$$

$$\Psi_{e_\mu,\ell} \triangleq [e_\mu(\mu) \quad \cdots \quad e_\mu(\ell)]. \quad (22)$$

Using (19), (18) becomes

$$(\theta_\mu \Phi_{v,\ell} + \Psi_{w,\ell} + \Psi_{e_\mu,\ell}) \Phi_{\mu,\ell}^T = \hat{\theta}_{\mu,\ell} \Phi_{\mu,\ell} \Phi_{\mu,\ell}^T. \quad (23)$$

Using (6), note that for all $k \geq 0$,

$$\phi_v(k) = \alpha \phi_u(k) + \phi_r(k), \quad (24)$$

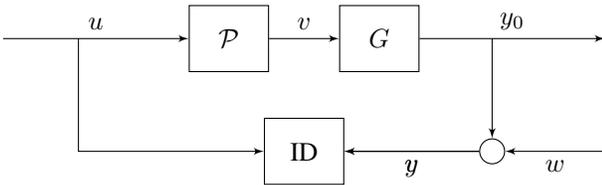


Fig. 2. Identification of the Hammerstein system consisting of the hysteresis nonlinearity \mathcal{P} and the linear system G , where u and y are used to identify G , and w is the output sensor noise. The intermediate signal v is unknown.

where

$$\phi_r(k) \triangleq [\sum_{i=1}^n p_i q_i [u](k) \quad \cdots \quad \sum_{i=1}^n p_i q_i [u](k-\mu)]^T. \quad (25)$$

Therefore, using (24) we can write

$$\Phi_{v,\ell} = \alpha \Phi_{\mu,\ell} + \Phi_{r,\ell}, \quad (26)$$

where

$$\Phi_{r,\ell} \triangleq [\phi_r(\mu) \quad \cdots \quad \phi_r(\ell)]. \quad (27)$$

Then, using (26), (23) can be written as

$$\begin{aligned} \alpha \theta_\mu \Phi_{\mu,\ell} \Phi_{\mu,\ell}^T + \theta_\mu \Phi_{r,\ell} \Phi_{\mu,\ell}^T + \Psi_{w,\ell} \Phi_{\mu,\ell}^T + \Psi_{e_\mu,\ell} \Phi_{\mu,\ell}^T \\ = \hat{\theta}_{\mu,\ell} \Phi_{\mu,\ell} \Phi_{\mu,\ell}^T. \end{aligned} \quad (28)$$

Dividing (28) by ℓ and taking the limit as ℓ tends to infinity yields

$$\begin{aligned} \alpha \theta_\mu \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \Phi_{\mu,\ell} \Phi_{\mu,\ell}^T + \theta_\mu \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \Phi_{r,\ell} \Phi_{\mu,\ell}^T + \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \Psi_{w,\ell} \Phi_{\mu,\ell}^T \\ + \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \Psi_{e_\mu,\ell} \Phi_{\mu,\ell}^T = \lim_{\ell \rightarrow \infty} \hat{\theta}_{\mu,\ell} \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \Phi_{\mu,\ell} \Phi_{\mu,\ell}^T. \end{aligned} \quad (29)$$

Since w is a realization of a white, zero-mean random processes and u is deterministic, then

$$\lim_{\ell \rightarrow \infty} \frac{1}{\ell} \Psi_{w,\ell} \Phi_{\mu,\ell}^T = 0_{1 \times \mu}.$$

Therefore, (29) becomes

$$\begin{aligned} \alpha \theta_\mu \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \Phi_{\mu,\ell} \Phi_{\mu,\ell}^T + \theta_\mu \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \Phi_{r,\ell} \Phi_{\mu,\ell}^T \\ + \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \Psi_{e_\mu,\ell} \Phi_{\mu,\ell}^T = \lim_{\ell \rightarrow \infty} \hat{\theta}_{\mu,\ell} \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \Phi_{\mu,\ell} \Phi_{\mu,\ell}^T. \end{aligned} \quad (30)$$

Since u is a persistently exciting signal of sufficient order, then

$$Q \triangleq \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \Phi_{\mu,\ell} \Phi_{\mu,\ell}^T$$

has full rank. Therefore, multiplying (30) by Q^{-1} from the right yields

$$\alpha \theta_\mu + \theta_\mu R Q^{-1} + \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \Psi_{e_\mu,\ell} \Phi_{\mu,\ell}^T Q^{-1} = \lim_{\ell \rightarrow \infty} \hat{\theta}_{\mu,\ell}, \quad (31)$$

where

$$R \triangleq \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \Phi_{r,\ell} \Phi_{\mu,\ell}^T.$$

Note that

$$\begin{aligned} R &= \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \Phi_{r,\ell} \Phi_{\mu,\ell}^T \\ &= \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \begin{bmatrix} r(\mu) & \cdots & r(\ell) \\ \vdots & \ddots & \vdots \\ r(0) & \cdots & r(\ell-\mu) \end{bmatrix} \begin{bmatrix} u(\mu) & \cdots & u(0) \\ \vdots & \ddots & \vdots \\ u(\ell) & \cdots & u(\ell-\mu) \end{bmatrix} \\ &= \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \begin{bmatrix} \sum_{j=\mu}^{\ell} r(i)u(i) & \cdots & \sum_{j=\mu}^{\ell} r(i)u(i-\mu) \\ \vdots & \ddots & \vdots \\ \sum_{j=\mu}^{\ell} r(i-\mu)u(i) & \cdots & \sum_{j=\mu}^{\ell} r(i)u(i) \end{bmatrix}. \end{aligned} \quad (32)$$

Moreover, note that

$$\begin{aligned}
Q &= \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \Phi_{\mu, \ell} \Phi_{\mu, \ell}^T \\
&= \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \begin{bmatrix} u(\mu) & \cdots & u(\ell) \\ \vdots & \ddots & \vdots \\ u(0) & \cdots & u(\ell - \mu) \end{bmatrix} \begin{bmatrix} u(\mu) & \cdots & u(0) \\ \vdots & \cdots & \vdots \\ u(\ell) & \cdots & u(\ell - \mu) \end{bmatrix} \\
&= \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \begin{bmatrix} \sum_{j=\mu}^{\ell} u(i)^2 & \cdots & \sum_{j=\mu}^{\ell} u(i)u(i-\mu) \\ \vdots & \ddots & \vdots \\ \sum_{j=\mu}^{\ell} u(i)u(i-\mu) & \cdots & \sum_{j=\mu}^{\ell} u(i)^2 \end{bmatrix}. \tag{33}
\end{aligned}$$

Note from (32) and (33) that if the entries of R are much smaller than the entries of Q , then RQ^{-1} can be neglected. Therefore we choose the amplitude of the input signal u to be as large as possible. Assuming that RQ^{-1} can be neglected, (31) becomes

$$\alpha\theta_{\mu} + \lim_{\ell \rightarrow \infty} \frac{1}{\ell} \Psi_{e_{\mu}, \ell} \Phi_{\mu, \ell}^T Q^{-1} \approx \lim_{\ell \rightarrow \infty} \hat{\theta}_{\mu, \ell}. \tag{34}$$

Note from (14) and (22) that, as μ increases, the entries of $\Psi_{e_{\mu}, \ell}$ become smaller. Therefore, we choose μ to be large enough such that

$$\lim_{\ell \rightarrow \infty} \frac{1}{\ell} \Psi_{e_{\mu}, \ell, \mu} \Phi_{\mu, \ell}^T \approx 0_{1 \times (\mu+1)}. \tag{35}$$

Therefore, (34) becomes

$$\lim_{\ell \rightarrow \infty} \hat{\theta}_{\mu, \ell} \approx \alpha\theta_{\mu}. \tag{36}$$

It follows from (36) that $\hat{\theta}_{\mu, \ell}$ is approximately a semiconsistent estimate of θ_{μ} , that is, $\lim_{\ell \rightarrow \infty} \hat{\theta}_{\mu, \ell}$ is a correct estimate of θ_{μ} up to an unknown scalar factor.

V. IDENTIFICATION OF THE HYSTERESIS NONLINEARITY

Identification of the hysteresis nonlinearity is performed by first estimating the unknown intermediate signal v , and then using the input u and the estimated intermediate signal to construct an estimate of the hysteresis nonlinearity.

Note that, if we use y as an input to the transfer function $G^{-1} = 1/G$, then the output of G^{-1} is the unknown intermediate signal v . Assuming that \hat{G} is an estimate of the transfer function G , then using y as an input to the transfer function $\hat{G}^{-1} = 1/\hat{G}$, the output of \hat{G}^{-1} is an estimate \hat{v} of the unknown intermediate signal v . However, if \hat{G} is strictly proper, then \hat{G}^{-1} is improper, that is, noncausal. Moreover, if \hat{G} has a nonminimum-phase zero, that is, a zero that is outside the closed unit disk, then \hat{G}^{-1} is unstable. In order to simulate \hat{G}^{-1} with y as an input, we need to circumvent these two problems.

Noncausal FIR models have been used to obtain asymptotically stable approximations of unstable and noncausal systems [21], [24]. A noncausal FIR model of a transfer function G is a truncation of the Laurent expansion of G in an annulus that contains the unit circle [21].

A. Asymptotically Stable Inversion of G

Let $\mathbb{A}(\rho_1, \rho_2) \triangleq \{z \in \mathbb{C} : |z| > \rho_1 \text{ and } |z| < \rho_2\}$ denote an open annulus in the complex plane centered at the origin with inner radius ρ_1 and outer radius ρ_2 , where $\rho_1 < 1 < \rho_2$. Then, assuming that \hat{G}^{-1} is analytic in $\mathbb{A}(\rho_1, \rho_2)$, the Laurent expansion of \hat{G}^{-1} in $\mathbb{A}(\rho_1, \rho_2)$ can be written as

$$\hat{G}^{-1}(z) = \sum_{i=-\infty}^{\infty} \hat{h}_i z^{-i}, \tag{37}$$

where \hat{h}_i is the i th coefficient of the Laurent expansion of \hat{G}^{-1} in $\mathbb{A}(\rho_1, \rho_2)$. Truncating the sum in (37) yields the truncated model

$$\hat{G}_{\text{inv}, r, d}(\mathbf{q}) \triangleq \sum_{i=-d}^r \hat{h}_i \mathbf{q}^{-i}, \tag{38}$$

where r is the order of the causal part of $\hat{G}_{\text{inv}, r, d}$ and d is the order of the noncausal part of $\hat{G}_{\text{inv}, r, d}$. Note that all poles of $\hat{G}_{\text{inv}, r, d}$ are located at zero, and thus $\hat{G}_{\text{inv}, r, d}$ is an asymptotically stable approximation of \hat{G}^{-1} . Since \hat{G}^{-1} is analytic in $\mathbb{A}(\rho_1, \rho_2)$, then we can find finite r and d such that $\|\hat{G}^{-1} - \hat{G}_{\text{inv}, r, d}\|$ is negligible [21, Theorem 4.1], [25].

Using y as an input to $\hat{G}_{\text{inv}, r, d}$, and using (38), yields, for all $k \geq r$,

$$\hat{v}(k) = \hat{G}_{\text{inv}, r, d}(\mathbf{q})y(k) = \sum_{i=-d}^r \hat{h}_i y(k-i). \tag{39}$$

Note from (39) that computing $\hat{v}(k)$ requires knowledge of $y(k+d), \dots, y(k-r)$, which makes $\hat{G}_{\text{inv}, r, d}$ a noncausal model.

B. Identification of the Hysteresis Nonlinearity

To identify the hysteresis nonlinearity, we apply a pure sinusoidal input signal u to the Hammerstein system. Then, we use the output of the Hammerstein system due to the sinusoidal signal u along with $\hat{G}_{\text{inv}, r, d}$ obtained from the previous subsection to construct an estimate \hat{v} of the intermediate signal v . Then, we plot \hat{v} versus the sinusoidal input u to obtain a nonparametric model of the hysteresis nonlinearity. If the hysteresis nonlinearity is rate independent, then the estimated hysteresis nonlinearity is independent of the frequency of the single sinusoidal input.

VI. A NUMERICAL EXAMPLE

Consider the hysteresis nonlinearity described by (1) with $r_1 = 0$, $r_2 = 0.2$, $r_3 = 0.3$, $r_4 = 0.4$, $r_5 = 0.5$, and $p_1 = 0.1$, $p_2 = 0.2$, $p_3 = 0.3$, $p_4 = 0.4$, $p_5 = 0.6$, and the linear plant

$$G(\mathbf{q}) = \frac{(\mathbf{q} - 0.4)(\mathbf{q} - 0.5)}{(\mathbf{q} - 0.3)(\mathbf{q} + 0.1)}. \tag{40}$$

For all $k \geq 0$, let $u_0(k) = 500 \sin(k)$. Moreover, for all $k \geq 0$, let $u(k)$ be obtained by saturating $u_0(k)$ between -100 and 100 , that is,

$$u(k) = \begin{cases} -100, & u_0(k) \leq -100, \\ u_0(k), & -100 < u_0(k) < 100, \\ 100, & u_0(k) \geq 100. \end{cases} \tag{41}$$

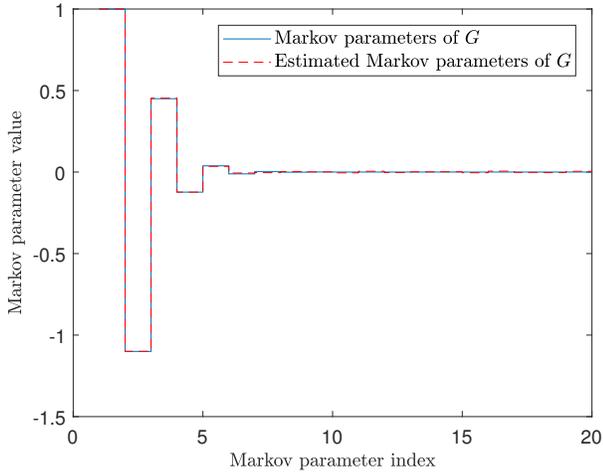


Fig. 3. Markov parameters of G and the estimated Markov parameters of G after scaling, that is, dividing all the estimated Markov parameters by $\alpha = \sum_{i=1}^5 p_i = 1.6$.

To identify G , we use $\ell = 10,000$ samples of u and y with least squares and an FIR model with order $\mu = 20$. Figure 3 shows the Markov parameters of G and the estimated Markov parameters of G obtained using (17) after scaling, that is, dividing all the estimated Markov parameters by $\alpha = \sum_{i=1}^5 p_i = 1.6$. Then, we construct an IIR model \hat{G} of G using ERA and the estimated Markov parameters of G after scaling. Figure 4 shows the Bode plots of G and \hat{G} , which are very close to each other. Next, we find a noncausal FIR approximation $\hat{G}_{inv,r,d}$ of \hat{G}^{-1} by truncating the Laurent expansion of \hat{G}^{-1} in the annulus that contains the unit circle with $r = d = 25$. Applying y as an input to the noncausal FIR approximation $\hat{G}_{inv,r,d}$ of \hat{G}^{-1} yields an estimate \hat{v} of the intermediate signal v . Figure 5 shows the intermediate signal v , the estimated intermediate signal \hat{v} , and the difference between them.

Next, we use $u(k) = \sin(T_s k)$ where $k \geq 0$ as an input to the Hammerstein system, and we obtain the output y of the Hammerstein system due to u . We use y and the noncausal FIR approximation $\hat{G}_{inv,r,d}$ of \hat{G}^{-1} obtained above to obtain an estimate \hat{v} of the unknown intermediate signal v . Figure 6 shows the intermediate signal v and the estimated intermediate signal \hat{v} of v . Finally, Figure 7 shows the hysteresis loop obtained by plotting v versus u and the estimated hysteresis loop obtained by plotting \hat{v} versus u , which are very close to each other.

VII. CONCLUSIONS

This paper considered the problem of identification of the nonlinear dynamics of a class of precision motion systems. The precision motion system was modeled as a Hammerstein system, that is, a cascade of a nonlinearity and a linear plant, where a Prandtl-Ishlinskii hysteresis nonlinearity was considered. We used least squares with an FIR model to identify the linear part of the Hammerstein system using measurements of the input and the output of the Hammerstein

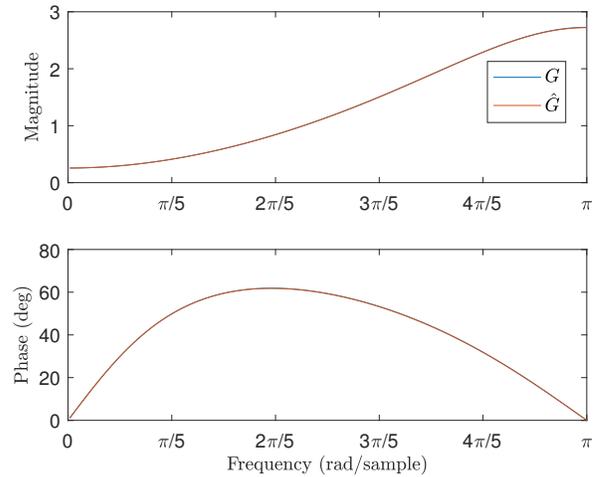


Fig. 4. Bode plot of G and the identified model \hat{G} obtained using ERA and the estimated Markov parameters of G after scaling.

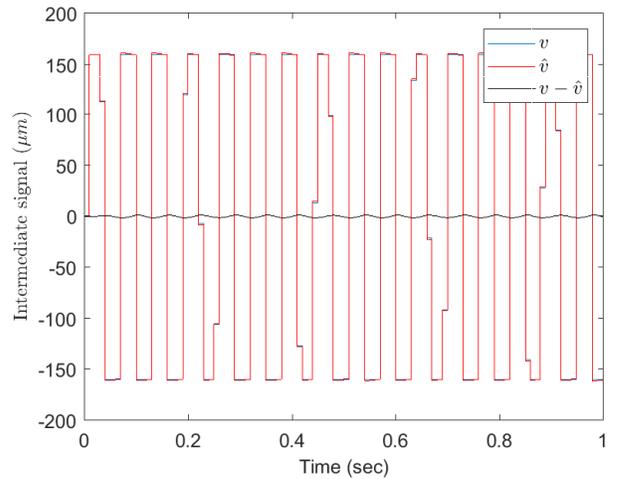


Fig. 5. The intermediate signal v and the estimated intermediate signal \hat{v} obtained by applying y as an input to the noncausal FIR approximation $\hat{G}_{inv,r,d}$ of \hat{G}^{-1} , where u is as given in (41).

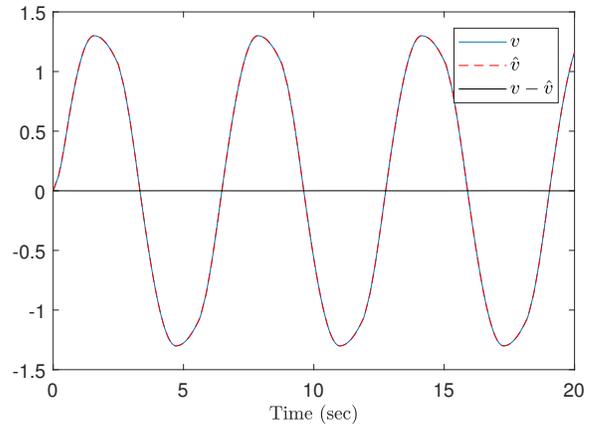


Fig. 6. The intermediate signal v and the estimated intermediate signal \hat{v} obtained by applying y as an input to the noncausal FIR approximation $\hat{G}_{inv,r,d}$ of \hat{G}^{-1} , where for all $k \geq 0$, $u(k) = \sin(T_s k)$.

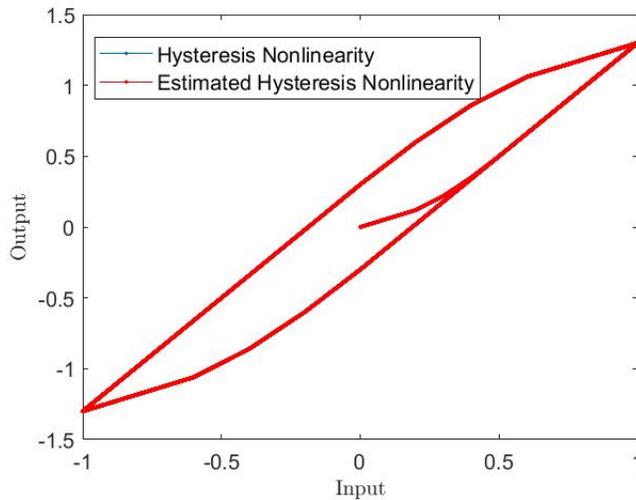


Fig. 7. The hysteresis loop obtained by plotting v versus u and the estimated hysteresis loop obtained by plotting \hat{v} versus u .

system, where the intermediate signal was assumed to be unknown. The ERA algorithm was used to construct a transfer function model from the estimated FIR model. A noncausal FIR model was used to obtain an asymptotically stable approximation of the inverse of the estimated linear part of the Hammerstein system. To obtain an estimate of the hysteresis nonlinearity, a pure sinusoidal signal was used as an input to the Hammerstein system. The output of the Hammerstein system due to the sinusoidal input was used along with the noncausal FIR approximation of the inverse of the estimated linear part of the Hammerstein system to obtain an estimate of the unknown intermediate signal. Finally, the estimated hysteresis nonlinearity was obtained by plotting the estimated intermediate signal versus the sinusoidal input.

VIII. FUTURE RESEARCH

Future research will consider studying the accuracy of different model structures and identification methods to identify the linear part of the model. Moreover, future research will focus on using unknown input reconstruction methods, such as [26], [27] to reconstruct the intermediate signal using the output measurements and the identified linear part of the Hammerstein system. The algorithm will be applied to the piezoelectric cantilever actuator presented in [11].

REFERENCES

- [1] M. Ragazzon, M. Ruppert, D. Harcombe, A. Fleming, and J. Gravdahl, "Lyapunov estimator for high-speed demodulation in dynamic mode atomic force microscopy," *IEEE Transactions on Control Systems Technology*, vol. 26, no. 2, pp. 765–772, 2017.
- [2] M. Al Janaideh, M. Rakotondrabe, and X. Tan, "Hysteresis in smart mechatronic systems: Modeling, identification, and control," *IEEE/ASME Transactions on Mechatronics*, vol. 21, pp. 1–3, 2016.
- [3] M. Al Janaideh, "About the output of the inverse compensation of the prandtl-ishlinskii model," *American Control Conference*, pp. 247–252, 2013.
- [4] J. Zhang, D. Torres, J. Ebel, N. Sepulveda, and X. Tan, "A composite hysteresis model in self-sensing feedback control of fully integrated VO₂ microactuators," *IEEE/ASME Transactions on Mechatronics*, vol. 21, pp. 2405–2417, 2016.
- [5] M. Butcher, A. Giustiniani, and A. Masi, "On the identification of Hammerstein systems in the presence of an input hysteretic nonlinearity with nonlocal memory: Piezoelectric actuators an experimental case study," *Physica B*, vol. 486, pp. 101–105, 2016.
- [6] M. Al Janaideh and O. Aljanaideh, "Further results on open-loop compensation of rate-dependent hysteresis in a magnetostrictive actuator with the Prandtl-Ishlinskii model," *Mechanical Systems and Signal Processing*, vol. 104, pp. 835–850, 2018.
- [7] X. Tan and J. Baras, "Modeling and control of hysteresis in magnetostrictive actuators," *Automatica*, vol. 40, pp. 1469–1480, 2004.
- [8] M. Al Janaideh, M. Rakotondrabe, and O. Aljanaideh, "Further results on hysteresis compensation of smart micropositioning systems with the inverse Prandtl-Ishlinskii compensator," *IEEE Transactions on Control Systems Technology*, vol. 24, no. 2, pp. 428–439, 2016.
- [9] H. Vahid, T. Tegoeh, and T. NhoDo, "A survey on hysteresis modeling, identification and control," *Mechanical Systems and Signal Processing*, vol. 49, no. 1-2, pp. 209–233, 2014.
- [10] D. Davino, C. Natale, S. Pirozzi, and C. Visone, "Phenomenological dynamic model of a magnetostrictive actuator," *Physica B*, vol. 343, pp. 112–116, 2004.
- [11] M. Al Janaideh, M. Rakotondrabe, I. Al-Darabseh, and O. Aljanaideh, "Internal model-based feedback control design for inversion-free feedforward rate-dependent hysteresis compensation of piezoelectric cantilever actuator," *Control Engineering Practice*, vol. 72, pp. 29–41, 2018.
- [12] J. Schoukens, J. Nemeth, P. Crama, Y. Rolain, and R. Pintelon, "Fast approximate identification of nonlinear systems," *Automatica*, vol. 39, pp. 1267–1274, 2003.
- [13] T. H. Van Pelt and D. S. Bernstein, "Non-linear system identification using Hammerstein and non-linear feedback models with piecewise linear static maps," *International Journal of Control*, vol. 74, no. 18, pp. 1807–1823, 2001.
- [14] B. Ninness and S. Gibson, "Quantifying the accuracy of Hammerstein model estimation," *Automatica*, vol. 38, pp. 2037–2051, 2002.
- [15] E.-W. Bai, "Identification of linear systems with hard input nonlinearities of known structure," *Automatica*, vol. 38, no. 5, pp. 853–860, 2002.
- [16] F. Ding, X. P. Liu, and G. Liu, "Identification methods for Hammerstein nonlinear systems," *Dig. Sig. Proc.*, vol. 21, no. 2, pp. 215–238, 2011.
- [17] K. F. Aljanaideh and D. S. Bernstein, "Nonparametric identification of Hammerstein systems using orthogonal basis functions as ersatz nonlinearities," in *ASME Dynamic Systems and Control Conference*, p. V003T35A004, 2013.
- [18] B. J. Coffey, K. Aljanaideh, and D. S. Bernstein, "Identification of Hammerstein systems using input amplitude multiplexing," in *American Control Conference*, pp. 3930–3935, 2013.
- [19] F. Giri, Y. Rochdi, F. Chaoui, and A. Brouri, "Identification of Hammerstein systems in presence of hysteresis-backlash and hysteresis-relay nonlinearities," *Automatica*, vol. 44, pp. 767–775, 2008.
- [20] M. Al Janaideh and P. Krejčí, "Inverse rate-dependent prandtl-ishlinskii model for feedforward compensation of hysteresis in a piezomicropositioning actuator," *IEEE/ASME Transactions on Mechatronics*, vol. 18, pp. 1498–1507, 2013.
- [21] K. F. Aljanaideh and D. S. Bernstein, "Closed-loop identification of unstable systems using noncausal FIR models," *International Journal of Control*, vol. 90, pp. 168–185, 2017.
- [22] J.-N. Juang, *Applied system identification*. Prentice Hall, 1994.
- [23] B. Ho and R. Kalman, "Efficient construction of linear state variable models from input/output functions," *Regelungstechnik*, vol. 14, pp. 545–548, 1966.
- [24] K. F. Aljanaideh and D. S. Bernstein, "Experimental application of time-domain transmissibility identification to fault detection and localization in acoustic systems," *Journal of Vibration and Acoustics*, vol. 140, no. 2, p. 021017, 2018.
- [25] D. S. Bernstein, K. F. Aljanaideh, and A. E. Frazho, "Laurent series and l^p sequences," *American Mathematical Monthly*, vol. 123, no. 4, pp. 398–398, 2016.
- [26] S. Gillijns and B. De Moor, "Unbiased minimum-variance input and state estimation for linear discrete-time systems," *Automatica*, vol. 43, no. 1, pp. 111–116, 2007.
- [27] H. J. Palanthandalam-Madapusi and D. S. Bernstein, "Unbiased minimum-variance filtering for input reconstruction," in *American Control Conference*, pp. 5712–5717, 2007.