Modelling and control of a class of lumped beam with distributed control

Andrea Mattioni [**] **Yongxin Wu** [*] **Hector Ramirez** [*]
Yann Le Gorrec [*] **Alessandro Macchelli** [**]

[*] *FEMTO-ST, Univ. Bourgogne Franche-Comté, CNRS, Besançon, France. 24 rue Savary, F-25000 Besançon, France. (e-mail: yongxin.wu@femto-st.fr; hector.ramirez@femto-st.fr; yann.le.gorrec@ens2m.fr).*

[**] *Department of Electrical, Electronic and Information Engineering "Guglielmo Marconi" (DEI), University of Bologna, viale del Risorgimento 2, 40136 Bologna, Italy. (e-mail: andrea.mattioni@studio.unibo.it; alessandro.macchelli@unibo.it).*

Abstract: A simple lumped port-Hamiltonian model for an actuated flexible beam is proposed. The flexible beam is modelled as a n-DOF actuated beam, and the port-Hamiltonian model is constructed by a systematic interconnection of the links of the beam. The proposed model is then instrumental to derive a stabilizing controller using interconnection and damping assignment - passivity based control considering an underactuated scenario. The work has been developed motivated by the practical application to a medical endoscope with distributed actuation by electro-active polymers. The lumped parameter model offers the possibility of having input/output ports in every joint between successive links, this permits to easily model the action of the actuators as an input force applied to a specific joint.

Keywords: Port-Hamiltonian system, IDA-PBC, medical endoscope, actuated beam

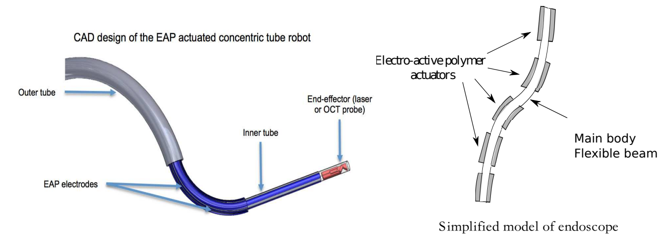

Fig. 1. Medical endoscope and its simplified model

1. INTRODUCTION

Port-Hamiltonian systems (PHS) (Maschke and van der Schaft, 1992, 1994; van der Schaft, 2000) have proven to be powerful for the modelling and control of complex physical control systems (Duindam et al., 2009), such as multi-physical (Falaize and Hélie, 2017; Doria-Cerezo et al., 2010), non-linear (Ramirez et al., 2016), described by partial differential equations (Macchelli et al., 2009; Jacob and Zwart, 2012; Ramirez et al., 2014) or with irreversible thermodynamic behaviour (Ramirez et al., 2013). Modelling by the PHS approach is based on the characterization of energy exchanges between components of a system. The framework permits in a quite straightforward and elegant manner to interconnect the different parts of a system through energy exchange ports, hence it is well suited for the modelling of the medical endoscope interconnected with the ionic polymer metal composites (IPMC) actuators. On the other hand, the PHS approach is well adapted for the application of powerful passivity based control tools with clear physical interpretation, such as energy shaping and control by interconnection and damping assignment (IDA-PBC) (Ortega et al., 2001, 2002; Macchelli et al., 2017).

In this paper we use the PHS formalism to model a class of n-degree of freedom (DOF) actuated beam. The class of actuated beam proposed in this work is a simple but realistic approximation of an actuated medical endoscope (Chikhaoui et al., 2014). The principle of the actuated medical endoscope (Fig. 1) is that the bending of the main body is achieved with IPMC actuators, which are a class of electro-active polymer actuators. The main body of the endoscope is a flexible structure and in this work we shall consider a lumped model of interconnected links to represent it. An IDA-PBC for the PHS model of the endoscope is proposed, assuming an underactuated scenario, and numerical simulation results provided.

This paper is organized as follows. Section 2 presents the PH model of the endoscope. A position controller is proposed in Section 3 by using the IDA-PBC. Simulation results are shown in Section 4. Finally, Section 5 gives some final remarks and perspectives of future work.

[⋆] The authors gratefully acknowledge the support of the ANR-DFG (French-German) project INFIDHEM with Reference Code ANR-16-CE92-0028. The first and second authors acknowledge the support of the ENSMM BQR project with Reference Code BQR ENSMM N^{o}06.2017

2. PORT HAMILTONIAN MODEL OF A N-DOF FLEXIBLE BEAM

The beam is modelled as n different elements connected through n joints. Between every joint there is an angular spring and a damper as shown in Figure 2. We shall assume a planar model, and so all the links are allowed to move only in the x-y plane.

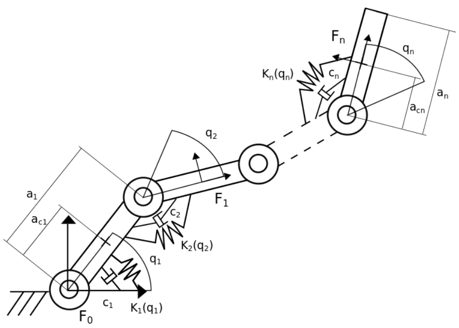

Fig. 2. Lumped parameters Beam

The parameters of the n-degree of freedom mechanism, with $i = 1, 2, ..., n$, are:

- q_i the i-th joint angular displacement;
- m_i the i-th link's mass;
- I_i the moment of inertia about the axis passing through the Center of Mass (CoM) of the i-th link;
- a_i length of the i-th link;
- a_{Ci} distance between the i-th Joint and the CoM of the i-th link;
- τ_i applied torque on the i-th joint;
- K_i stiffness of the i-th joint;
- $c_i(\dot{q}_i)$ viscous non-linear damping at the i-th joint;
- P_i, E_i Potential and Kinetic energy of the i-th link.
- F_0 is the inertial frame.
- F_i is the reference frame attached to the CoM and with axis parallel to principal axis of inertia of the i-th link .

2.1 The Hamiltonian function

In this subsection we derive the Hamiltonian function of the system with respect to the coordinate frame in which the system has been set up. The Hamiltonian corresponds to the total physical energy of the system which is the sum of the kinetic and potential energy. The kinetic energy of the i-th link has the form

$$E_i = \frac{1}{2} m_i v_{Ci}^T v_{Ci} + \frac{1}{2} w_i^T R_i \tilde{I}_i R_i^T w_i,$$

where v_{Ci} is the speed of the center of mass (CoM) of the i-th link, w_i is the angular speed of the i-th link with respect to F_0, $\tilde{I}_i$ is the inertia matrix of the i-th link with respect to F_i and R_i is the Rotational matrix between F_i and F_0. The goal is to express the kinetic energy of every link only with respect to the derivatives of the angular displacements. Thanks to the rigidity of the links, it is possible to relate both the speed of the CoM and the angular speeds to the derivative of the angular displacement of every joint. The relation that links angular displacements derivative to angular speeds is trivial

$$w_i = \dot{q}_1 + \dot{q}_2 + ... + \dot{q}_i.$$

Then, this relation can be expressed through the use of the so called angular Jacobian,

$$w_i = J_w^i \dot{q},$$

where q is the vector containing all the angular displacement and $\dot{q}$ is the one containing all the derived angular displacement. In this case it can be seen that the angular velocity Jacobian does not depend on the angular displacements. This is not the case for the Jacobian related to the velocities of the center of mass. The velocity Jacobian of the i-th link can be found differentiating with respect to time the position of the i-th center of mass in the F_0 frame,

$$q_{Ci} = \begin{bmatrix} x_{Ci} \\ y_{Ci} \end{bmatrix} = \begin{bmatrix} f_{xi}(q) \\ f_{yi}(q) \end{bmatrix} = f_i(q),$$

where,

$$f_{xi}(q) = \sum_{k=1}^{i-1} a_k \cos(\sum_{j=1}^{k} q_j) + a_{Ci} \cos(\sum_{k=1}^{i} q_k),$$

$$f_{yi}(q) = \sum_{k=1}^{i-1} a_k \sin(\sum_{j=1}^{k} q_j) + a_{Ci} \sin(\sum_{k=1}^{i} q_k).$$

Differentiating q_{Ci} with respect to time, we obtain $\dot{q}_{Ci} = v_{Ci} = \frac{df_i(q)}{dq}\dot{q}$, hence the velocity Jacobian is

$$J_v^i = \frac{df_i(q)}{dq}.$$

Now it is possible to express the kinetic energy of every link with respect to the derivative of the displacement vector

$$E_i = \frac{1}{2} \dot{q}^T (m_i J_v^{iT}(q) J_v^i(q) + J_w^{iT}(q) R_i \tilde{I}_i R_i^T J_w^i(q)) \dot{q}.$$

The total kinetic energy of the beam is then

$$E = \frac{1}{2} \dot{q}^T M(q) \dot{q},$$

where $M(q)$ is the mass matrix of the system, given by

$$M(q) = \sum_{i=1}^{n} [m_i J_v^{iT}(q) J_v^i(q) + J_w^{iT}(q) R_i I_i R_i^T J_w^i(q)].$$

The mass matrix allows to relate the generalized speed with momentum of the mechanical system

$$p = M(q)\dot{q},$$

where $p = [p_1 \; p_2 \; \cdots \; p_n]^T$. The kinetic energy expressed as a function of the momentum is then

$$E(q, p) = p^T M^{-1}(q) p.$$

In our framework we are supposing that the work plane is parallel to the ground, therefore we ignore the effect of the gravity on the dynamic of the system. Then, the potential energy is only due to the springs deformation. To find the potential energy we first define the stiffness matrix of the system

$$K = \operatorname{diag}[K_1, \; K_2, \; \cdots \; , \; K_n].$$

The constitutive relation between elastic torques and springs deformation is given by $\tau_e = Kq$, hence the total potential energy can be trivially expressed as

$$P(q) = \frac{1}{2} q^T K q.$$

Finally, define the Hamiltonian as the total energy of the system, i.e., the sum of the total kinetic and total potential energy of the system

$$H(q, p) = E(q, p) + P(q) = p^T M^{-1}(q) p + \frac{1}{2} q^T K q. \quad (1)$$

2.2 The input matrix

In this case study the beam is force actuated. Hence, it is necessary to find the input matrix which maps the normal forces at every joint into torques at each joint. Define by $u \in \mathbb{R}^m$ the vector of magnitudes of the input forces applied to the beam, where m is the number of actuated joints. Define an arbitrary point of the lumped beam as δ_i. The velocity of point δ_i is mapped to the joint's velocity using the Jacobian matrix as follow

$$\dot{\delta}_i = J_v^{\delta_i} \dot{q},$$

where $J_v^{\delta_i}$ is the Jacobian matrix related toan arbitrary point δ_i. By conservation of energy, we have that $F^T \dot{\delta}_i = \tau^T \dot{q}$, where F is the vector containing the x and y components of every applied force

$$F = \left[F_{1x} \ F_{1y} \ \cdots \ F_{mx} \ F_{my}\right]^T.$$

The velocity Jacobian matrix is constructed as the composition of all the Jacobian of every force application point

$$J_v = \left[J_v^{\delta_1} \ J_v^{\delta_2} \ \cdots \ J_v^{\delta_m}\right]^T,$$

and it is direct to show that

$$\tau = J_v^T F(q),$$

where τ is the vector of torque applied to the n different joints. Since the direction of every application force is assumed to be always perpendicular to the joint itself, the F_{ix} and F_{iy} components of every force can be written with respect to the force magnitude and the joint configuration. For this reason we define

$$f_{xi}^F(q) = -\sin\Big(\sum_{k=1}^{l_{app}(i)} q_i\Big), \qquad f_{yi}^F(q) = +\cos\Big(\sum_{k=1}^{l_{app}(i)} q_i\Big)$$

where $l_{app}(i)$ gives the link in which is applied the i-th force. Then one has that

$$F_{xi}(q) = -|F_i(t)| f_{xi}^F(q),$$
$$F_{yi}(q) = +|F_i(t)| f_{yi}^F(q).$$

Since the vector of input is composed by the magnitude of every applied force $u = |F| = \left[|F_1(t)| \ \cdots \ |F_m(t)|\right]^T$, the vector of applied forces can be written as

$$F(q) = L_F(q) u$$

where,

$$L_F(q) = \mathrm{diag}\left[\begin{bmatrix} F_{x1}(q) \\ F_{y1}(q) \end{bmatrix}, \begin{bmatrix} F_{x2}(q) \\ F_{y2}(q) \end{bmatrix}, \cdots, \begin{bmatrix} F_{xm}(q) \\ F_{ym}(q) \end{bmatrix}\right].$$

Finally, the input matrix of the system is

$$g(x) = \begin{bmatrix} 0_{n\times m} \\ J_v^T(q) L_F(q) \end{bmatrix}.$$

The upper part of the $g(x)$ matrix is null because the torques don't affect the first n equations which correspond to the displacement dynamics.

2.3 The port-Hamiltonian model

Define as state vector $x = [q \ p]^\top$, then it is straightforward to define the following port-Hamiltonian representation of the system

$$\begin{aligned} \dot{x} &= (J - R)\frac{\partial H(x)}{\partial x} + g(x) u \\ y &= g(x)^T \frac{\partial H(x)}{\partial x} \end{aligned} \tag{2}$$

with

$$J = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix}, \qquad R = \begin{bmatrix} 0 & 0 \\ 0 & C_n \end{bmatrix},$$

with $C_n = \mathrm{diag}\left[c_1(\dot{q}_1), \, c_2(\dot{q}_2), \, \cdots, \, c_n(\dot{q}_n)\right]$, a positive diagonal matrix containing the viscous friction coefficients of the dampers between every joint. The structural matrix $J = -J^\top$ represent how the energy is internally exchanged within the system while the damping matrix $R = R^\top > 0$ captures the internal dissipation of the system. It is direct to verify that the system is passive, indeed noticing that $H > 0$ and $H(0) = 0$, and taking the time derivative of the Hamiltonian

$$\dot{H} = -\frac{\partial H}{\partial x}^\top R \frac{\partial H}{\partial x} + y^\top u \leq y^\top u.$$

For further details and considerations concerning stability and stabilization properties the reader is referred for instance to van der Schaft (2000); Duindam et al. (2009)

3. IDA-PBC DESIGN

In this section we design a controller for the system which allows to change configurations in an stable fashion. To this end we employ interconnection and damping assignement passivity based control (IDA-PBC) Ortega et al. (2001, 2002). The main idea is to match the open-loop dynamic of the system with the one defined by a target system and solve the resulting PDE. Define an asymptotically stable PHS target system

$$\dot{x} = (J_d - R_d)\frac{\partial H_d}{\partial x} \tag{3}$$

with H_d a positive definite Hamiltonian function with strict minimum at the desired equilibrium. Then the feedback law

$$\beta(x) = g^\top \left(g g^\top\right)^{-1}\left((J_d - R_d)\frac{\partial H_d}{\partial x} - (J - R)\frac{\partial H}{\partial x}\right)$$

asymptotically stabilizes the closed-loop system provided that the matching condition

$$g^\perp (J_d - R_d)\frac{\partial H_d}{\partial x} = g^\perp (J - R)\frac{\partial H}{\partial x}, \tag{4}$$

with $g^\perp$ a full rank left annihilator of g, i..e, $g^\perp g = 0$, is satisfied. The closed-loop dynamic will then behave as (3). We shall consider that the beam is of degree $n = 3$, i.e.,

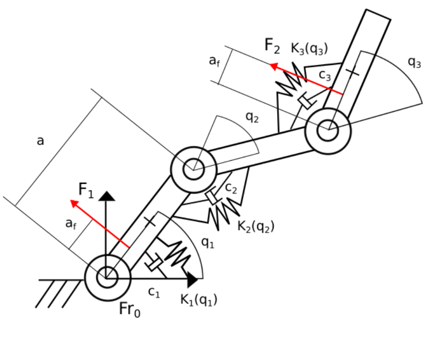

Fig. 3. Three elements force actuated beam

that it contains three links, and that two of the links are actuated. The inputs the force magnitudes applied to the first and third links at a distance of a_f from respectively the first and third joint. The direction of the forces is supposed to be always perpendicular to the respective link as can be seen in the Figure (3). It is important to underline that the springs and the dampers are considered as torsional, i.e., they act punctually at each joint. The input matrix is in this case

$$g(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ a_f & a_f + acos(q_3) + acos(q_2 + q_3) \\ 0 & a_f + acos(q_3) \\ 0 & a_f \end{bmatrix}$$

Lets assume the desired Hamiltonian H_d in the form

$$H_d(x) = H(x) + H_a(x).$$

In order to guarantee that it is strictly positive with a strict minimum at the desired equilibrium x^* it should satisfy

$$\frac{\partial H_d}{\partial x}(x^*) = 0, \tag{5}$$

$$\frac{\partial^2 H_d}{\partial x^2}(x^*) > 0. \tag{6}$$

We shall not modify the closed-loop interconnection and damping matrices, hence $J_d = J$ and $R_d = R$. The matching condition then becomes

$$g^{\perp}(J - R)\frac{\partial H_a}{\partial x} = 0$$

A possible annihilator of g is

$$g^{\perp}(x) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -a_f & a_f + acos(q_3) \end{bmatrix}$$

which leads to the following matching equations

$$\begin{aligned} \frac{\partial H_a}{\partial p_1} = \frac{\partial H_a}{\partial p_2} = \frac{\partial H_a}{\partial p_3} &= 0 \\ a_f \frac{\partial H_a}{\partial q_2} - (a_f + a\cos(q_3))\frac{\partial H_a}{\partial q_3} &= 0 \end{aligned} \tag{7}$$

The first line of (7) implies that H_a cannot depend on the momentum variables, while the last is a partial differential equation imposes a relation between the displacement variables q_2 and q_3. Since H_a cannot depend on momentum variables, the only part of the total energy that we can modify is the one depending only on displacement variables, *i.e.* the potential energy. From (1), we define the desired Hamiltonian as the composition of the desired potential energy and the desired kinetic energy

$$H_d(q, p) = E_d(q, p) + P_d(q)$$

From the first equation of (7) we obtain

$$E_d(q, p) = E(q, p).$$

Since the only part of the energy that can be modified is $P_d(q)$, (5) and (6) become

$$\frac{\partial P_d}{\partial q}(q^*) = 0, \tag{8}$$

$$\frac{\partial^2 P_d}{\partial q^2}(q^*) > 0. \tag{9}$$

The general solution of (7) is

$$H_a = H_a\left(q_1, -q_2 - \frac{2a_f \tanh^{-1}\left(\frac{(a-a_f)\tan(\frac{q_3}{2})}{\sqrt{a^2 - a_f^2}}\right)}{\sqrt{a^2 - a_f^2}}\right) \tag{10}$$

A simple form of (10) leads to the following solution

$$\begin{aligned} \frac{\partial H_a}{\partial q_2} &= \kappa q_2^* \\ \frac{\partial H_a}{\partial q_3} &= \frac{a_f}{a_f + a\cos(q_3)}\kappa q_2^* \end{aligned} \tag{11}$$

The part of H_a which depends on q_1, i.e., $H_a(q_1)$, can be freely chosen, since there is no condition on $\frac{\partial H_a}{\partial q_1}$. Then, $H_a(q_1, q_2, q_3)$ can be found by direct integration of (11). Once H_a has been chosen it remains to check that P_d is a Lyapunov function for the closed-loop system. This is achieved verifying that (8) and (9) are satisfied. The forced equilibrium position, and the corresponding steady state input, is computed from the dynamic model of the system (2). Since the system has two inputs we shall impose q_1^* and q_2^*, such that q_3^* is a function of q_1^* and q_2^*. Choosing H_a such that $\frac{\partial P_d}{\partial q_1} = K_1(q_1 - q_1*)$, and making $\kappa = -K_2$ we obtain

$$\frac{\partial P_d}{\partial q} = \begin{bmatrix} K_1(q_1 - q_1^*) \\ K_2(q_2 - q_2^*) \\ K_3 q_3 - \dfrac{a_f K_2 q_2^*}{a_f + acos(q_3)} \end{bmatrix}$$

It is straightforward to verify (8),

$$\frac{\partial P_d}{\partial q}(q^*) = 0 \quad \forall q_1^*, q_2^* \in [0, 2\pi].$$

On the other hand, condition (9) leads to

$$\frac{\partial^2 P_d}{\partial q^2}(q^*) = \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & \dfrac{\partial^2 P_d}{\partial q_3^2}(q^*) \end{bmatrix}$$

where

$$\frac{\partial^2 P_d}{\partial q_3^2}(q^*) = K_3 - K_2\frac{a_f q_2^* asin(q_3)}{(a_f + acos(q_3))^2}$$

Numerically, using the parameters in Table 1, we obtain that

$$\frac{\partial^2 P_d}{\partial q_3^2}|_{x^*} > 0 \quad \forall q_1^* \in \mathbb{R}, q_2^* \in [-0.75\pi, 0.75\pi].$$

The stable configuration of q_3^* is computed by q_1^* and q_2^*. Hence, we obtain that the stable configuration space that can be reached with the proposed controller is given by

$$\begin{bmatrix} q_1^* \\ q_2^* \\ q_3^* \end{bmatrix} \in \begin{bmatrix} -\pi, \pi \\ -0.75\pi, 0.75\pi \\ -0.33\pi, 0.33\pi \end{bmatrix}.$$

The controlled beam with proposed method cannot move over all the $x - y$ plane. However, from a practical point of view, this stable configuration space is large enough for the clamped medical endoscope beam.

4. SIMULATION RESULTS

In this section, simulation results are shown using a beam modelled with 3 elements. The parameters used in the simulation are resumed in Table 1.

4.1 Open-loop response

The free oscillations of the beam starting from an initial condition $q_0 = [+\frac{\pi}{8},\ -\frac{\pi}{8},\ +\frac{\pi}{6}]^\top$ and $p_0 = [0,\ 0,\ 0]^\top$ are

$m_1 = m_2 = m_3 = 0,0017 \quad [kg]$
$I_1 = I_2 = I_3 = 1,3801x10-6 \quad [kg*m^2]$
$a_1 = a_2 = a_3 = 0,1 \quad [m]$
$a_{C1} = a_{C2} = a_{C3} = 0,05 \quad [m]$
$K_i = 10 \quad i = 1,2,3 \quad [\frac{N}{m}]$
$c_i(\dot{q}_i) = 0.05|\tan^{-1}(\dot{q}_i * 4)| + 0.03 \quad i = 1,2,3 \quad [Pa*s]$

Table 1. Actuated beam parameters

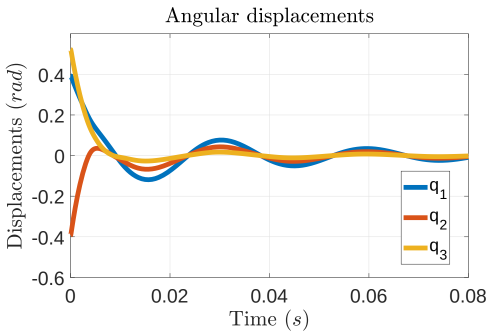

Fig. 4. Free Beam oscillations

illustrated in Figure 4. The beam goes back to its natural equilibrium position $q = [0,\ 0,\ 0]^T$ with external forces equal to zero. We can observe that the beam is badly damped and the presence of large oscillations.

4.2 Closed-loop response

In this sub-section the IDA-PBC feedback law is illustrated. In a first instance a controller that doesn't modify the closed-loop stiffness of the first spring is employed, while in a second instance a controller which modifies this stiffness is used.

Unmodified stiffness The controller is defined setting

$$H_a = +\frac{1}{2}K_1 q_1^{*2} - q_1^* q_1 K_1 + \frac{1}{2}K_2 q_2^{*2} - q_2^* q_2 K_2 - \frac{2a_f q_2^* K_2 \tanh^{-1}\left(\frac{(a-a_f)\tan(\frac{q_3}{2})}{\sqrt{a^2-a_f^2}}\right)}{\sqrt{a^2-a_f^2}} \tag{12}$$

where one has the possibility of choosing q_1^* and q_2^*, begin q_3^* a function of the other equilibria. This leads to

$$\frac{\partial H_a}{\partial x} = \begin{bmatrix} -K_1 q_1^* \\ -K_2 q_2^* \\ -\frac{a_f}{a_f + acos(q_3)} K_2 q_2^* \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The time responses of the controlled three element beam with different initial positions are presented. In the first simulation we set the natural equilibrium position as the initial condition, *i.e.* all the angular displacements equal to zero and the final equilibrium position set as $q^* = [q_1^*\ q_2^*\ q_3^*] = [0.5\ 0.52\ 0.15]$. The initial position and the final position are shown in Fig. 5 and Fig. 6 respectively. The time responses of the angular displacement of every joint is shown in the Fig. 7. We observe that the response

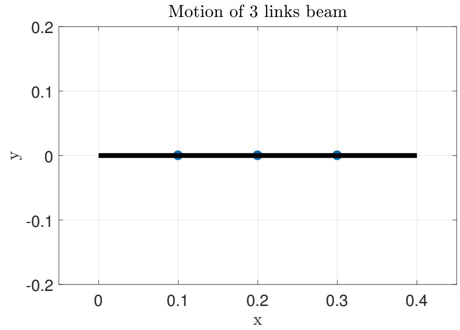

Fig. 5. Initial position with $q_0 = [0, 0, 0]^\top$

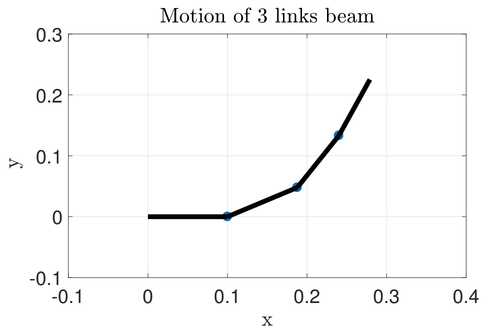

Fig. 6. Final position

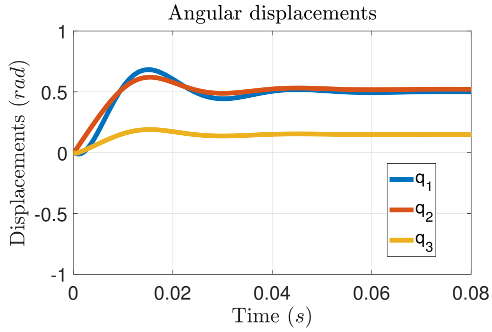

Fig. 7. Beam time response, zero initial position.

time is faster than the for the open-loop system and with less oscillations. The angle of every joint converges to the desired equilibrium position as expected.

In the second simulation, we consider a "non-natural" initial condition, $q_0 = [-0.4\ -0.8\ 0.7]$, shown in Fig. 8. The desired equilibrium position is always the same as in Fig. 6. The time responses of the angular displacement

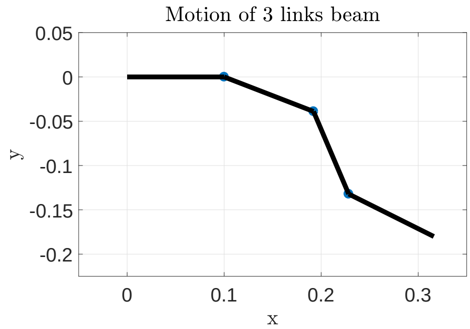

Fig. 8. Initial position with $q_0 = [-0.4, -0.8, 0.7]^\top$

of every joint is shown in the Fig. 9. In this case we can

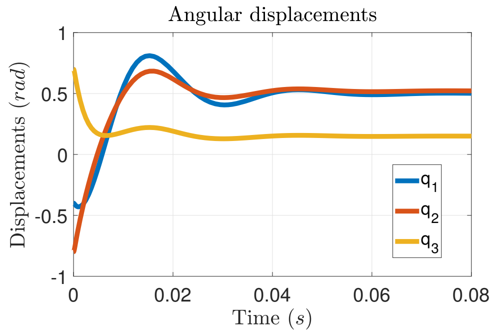

Fig. 9. Beam time response with worse initial condition

appreciate from Fig. 9 that every joint reach the desired equilibrium position. Moreover, compare to the previous simulation, the response time is the same, however, one can see that the overshoot of the response is larger. That is because in this case, the equilibrium position is further away to the initial position than in the previous case.

Modified Stiffness Now we define the controller Hamiltonian such that

$$H_a = -\frac{1}{2}K_1 q_1^2 + \frac{1}{2}K_1'(q_1 - q_1^*)^2 + \frac{1}{2}K_2 q_2^{*2} - q_2^* q_2 K_2 - \frac{2a_f q_2^* K_2 \tanh^{-1}\left(\frac{(a-a_f)\tan(\frac{q_3}{2})}{\sqrt{a^2-a_f^2}}\right)}{\sqrt{a^2 - a_f^2}}$$

where the stiffness of the first link in the closed-loop system has been modified to

$$K_1' = 50 \quad [\frac{N}{m}].$$

This lead to

$$\frac{\partial H_a}{\partial x} = \begin{bmatrix} -K_1 q_1 + K_1'(q_1 - q_1^*) \\ -K_2 q_2^* \\ -\dfrac{a_f}{a_f + a\cos(q_3)} K_2 q_2^* \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The initial and final configurations are the same as before. The time response of the angular displacement can be seen in Fig. 10. Comparing Fig. 9 with Fig.10, it is possible to notice that the response time has been significantly decreased in this case. This is due to the augmentation of the stiffness of the first joint.

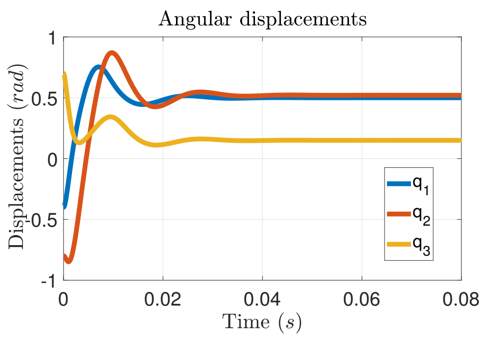

Fig. 10. Beam time response with modified stiffness.

5. CONCLUSION

The PHS framework has been used to model and control an actuated medical endoscope system. The endoscope is modelled as a lumped parameter flexible beam. A position control, that uses forces as inputs, has been proposed for this model using the IDA-PBC. The proposed model and control law have been illustrated by means of numerical simulations. The closed-loop system behaves in a satisfactory manner using the quite simple approach proposed in this work. One of the interesting aspect of the stability analysis is that it is possible to characterize the stable configuration space a priori. Ongoing work deals with potential non-linearity of the stiffness of the beam, the robustness of the proposed control and the experimental implementation of the controller.

REFERENCES

Chikhaoui, M.T., Rabenorosoa, K., and Andreff, N. (2014). *Kinematic Modeling of an EAP Actuated Continuum Robot for Active Micro-endoscopy*, 457–465. Springer International Publishing, Cham.

Doria-Cerezo, A., Batlle, C., and Espinosa-Perez, G. (2010). Passivity-based control of a wound-rotor synchronous motor. *IET Control Theory Applications*, 4(10), 2049–2057.

Duindam, V., Macchelli, A., Stramigioli, S., and Bruyninckx, H.e. (2009). *Modeling and Control of Complex Physical Systems - The Port-Hamiltonian Approach*. Springer. ISBN 978-3-642-03195-3.

Falaize, A. and Hélie, T. (2017). Passive simulation of the nonlinear port-Hamiltonian modeling of a rhodes piano. *Journal of Sound and Vibration*, 390(Supplement C), 289 – 309.

Jacob, B. and Zwart, H. (2012). *Linear Port-Hamiltonian Systems on Infinite-dimensional Spaces*, volume 223 of *Operator Theory: Advances and Applications*. Birkhäuser, Basel, Switzerland.

Macchelli, A., Le Gorrec, Y., Ramirez, H., and Zwart, H. (2017). On the synthesis of boundary control laws for distributed port-Hamiltonian systems. *IEEE Transactions on Automatic Control*, 62(4), 1700–1713.

Macchelli, A., Melchiorri, C., and Stramigioli, S. (2009). Port-based modeling and simulation of mechanical systems with rigid and flexible links. *IEEE Transactions on Robotics*, 25(5), 1016–1029.

Maschke, B. and van der Schaft, A. (1992). Port controlled Hamiltonian systems: modeling origins and system theoretic properties. In *Proceedings of the 3rd IFAC Symposium on Nonlinear Control Systems, NOLCOS'92*, 282–288. Bordeaux, France.

Maschke, B. and van der Schaft, A. (1994). On the Hamiltonian Formulation of Nonholonomic Mechanical Systems. *Reports Mathematical Physics*, 34, 225–233.

Ortega, R., van der Schaft, A., B., M., and Escobar, G. (2002). Interconnection and damping assignment: passivity-based control of port-controlled Hamiltonian systems. *Automatica*, 38(4), 585–596.

Ortega, R., van der Schaft, A., Mareels, I., and Maschke, B. (2001). Putting energy back in control. *IEEE Control Systems Magazine*, 21(2), 18– 32.

Ramirez, H., Le Gorrec, Y., Macchelli, A., and Zwart, H. (2014). Exponential stabilization of boundary controlled port-Hamiltonian systems with dynamic feedback. *IEEE Transactions on Automatic Control*, 59(10), 2849–2855.

Ramirez, H., Maschke, B., and Sbarbaro, D. (2013). Irreversible port-Hamiltonian systems: A general formulation of irreversible processes with application to the CSTR. *Chemical Engineering Science*, 89(0), 223 – 234.

Ramirez, H., Le Gorrec, Y., Maschke, B., and Couenne, F. (2016). On the passivity based control of irreversible processes: A port-Hamiltonian approach. *Automatica*, 64, 105 – 111.

van der Schaft, A.J. (2000). *L2-Gain and Passivity Techniques in Nonlinear Control*.