Optimal actuator location for electro-active polymer actuated endoscope ⋆

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Abstract: This paper deals with optimal actuator location for a medical endoscope controlled by electro-active polymer (EAP). The inner tube of the endoscope is a flexible structure that can be represented by a Timoshenko beam. Actuators are patches of EAP. There is freedom in the choice of EAP actuators location. In this paper, we first propose a port Hamiltonian model of the endoscope. In order to choose the optimal location for the EAP actuators, we consider the linear quadratic (LQ) performance as the optimal performance objective. At last, some numerical simulation results are given based on the real experimental setup parameters.

Keywords: Medical endoscope; optimal actuator location; linear quadratic optimization; port Hamiltonian system.

1. INTRODUCTION

The theoretical modeling and control of medical endoscopes have been studied since the last century (Anderson et al., 1967). In recent years, technological progresses made possible the use of continuum robots for different applications such as: laser manipulators, catheters and micro-endoscopes (Robert J. Webster and Jones, 2010). Actuated micro-endoscopes have been developed for endonasal skull base surgery in (Chikhaoui et al., 2014) with embedded actuators able to provide additional degrees of freedom to the system. In this paper, the bending of the endoscope is preformed by electro-active polymer (EAP) actuators. One of the most important EAP actuators are Ionic Polymer Composites (IPMC) which have attractive properties such as: low actuation voltage, ease of fabrication, relatively high strain and so on. These properties have been experimentally pointed out in (Shahinpoor and Kim, 2001).

The modeling of medical endoscopes has been considered in (Chikhaoui et al., 2014) by a kinematic approach. The main body of the endoscope is a flexible structure and the IPMC actuators consist in patches of poly-electrolyte gel and metal electrodes plated by a chemical process. The modeling of such kind of system naturally leads to a complex multi physical system which is often governed by partial differential equations (PDEs). The port Hamiltonian framework is a very powerful approach for the modeling and control of mechanical, electro-mechanical and multi physical systems (Duindam et al., 2009). Port Hamiltonian modeling is based on energy exchanges between the different components of the systems. It has been recently extended to distributed parameter systems described by partial differential equations (PDEs). The port Hamiltonian framework is well suited for the modeling of interconnected multiphysical systems and then particularly well adapted for the modeling of medical endoscopes. Moreover, modeling and control of flexible structures by using the port Hamiltonian framework have been widely studied in the last decade (Macchelli and Melchiorri, 2004b,a) and the port Hamiltonian modeling of IPMC soft actuators has been introduced in (Nishida et al., 2011). The actuators being coated outside of the medical endoscope (shown in Fig. 1), this control problem can be regarded as the distributed control of a distributed parameter system and one has to decide the best location of actuators. This naturally leads to the optimal actuator location problem. This problem has been firstly introduced in the context of distributed parameter system in (Slemrod, 1989). The author in (Morris, 2011) proposes to minimize the linear quadratic cost function in order to choose the optimal actuators location. We can also find the other criteria to find the optimal actuators location in the review article (van de Wal and de Jager, 2001).

The organization of this paper is the following. In Section 2, we introduce the port Hamiltonian modeling of the endoscope with its distributed control. The optimal actuator location is considered by minimizing the linear quadratic cost functional in Section 3. In Section 4 is given the discretized model of the endoscope and this model is validated through several simulations. At last, we give the conclusion of this work and some remarks for future works.

2. PORT HAMILTONIAN MODELING OF ENDOSCOPE

A simplified model of a compliant endoscope used for medical examination (Chikhaoui et al., 2014) is presented in Fig 2. The inner tube is actuated by electro-active polymers (EAP) caught on the body of the endoscope. The modeling of EAP can be found in (Nishida et al., 2011). In this paper we do not represent the physical

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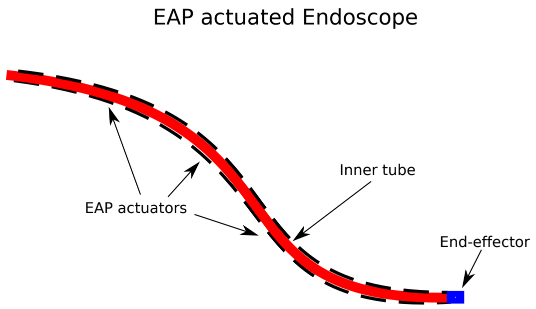

Fig. 1. EAP actuated Endoscope

model of the EAP and consider only the distributed forces and torques applied on the body of the inner tube. The compliant inner tube of the endoscope can be regarded as a flexible beam. One end of this beam is clamped while the other one is free. The actuators and the beam are interconnected through the power conjugated variables. The interconnection relation and causality are indicated also in Fig 2. The compliant inner tube is modeled as an infinite dimensional Timoshenko beam model. In the following subsections we discuss the modeling of this compliant structure and its distributed control.

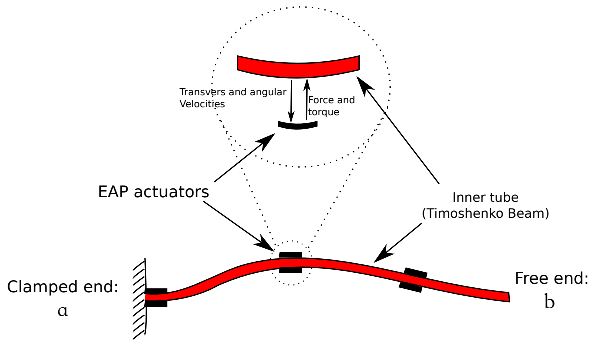

Fig. 2. Simplified EAP actuated Endoscope

2.1 Timoshenko beam

The distributed parameter port Hamiltonian formulation of Timoshenko beam has been represented in(Macchelli and Melchiorri, 2004b; Jacob and Zwart, 2012). This representation has been widely studied for the boundary control problem (Villegas et al., 2009; Ramirez et al., 2014) as well as for the distributed control problem (Macchelli, 2003) of beams. Let consider the port Hamiltonian representation of the Timoshenko beam as follows:

$$\dot{x}(t) = \underbrace{(P_1 \frac{\partial}{\partial z} + P_0)}_{\mathcal{J}} \mathcal{L}x(t) \tag{1}$$

with the operator:

$$\mathcal{L} = \begin{bmatrix} K & 0 & 0 & 0 \\ 0 & \frac{1}{\rho} & 0 & 0 \\ 0 & 0 & EI & 0 \\ 0 & 0 & 0 & \frac{1}{I_\rho} \end{bmatrix}, \tag{2}$$

and the matrices:

$$P_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, P_0 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \tag{3}$$

The state (energy) variables are the shear displacement $x_1 = \frac{\partial w}{\partial z}(z,t) - \phi(z,t)$, the transverse momentum distribution $x_2 = \rho(z)\frac{\partial w}{\partial t}(z,t)$, the angular displacement $x_3 = \frac{\partial \phi}{\partial z}(z,t)$ and the angular momentum distribution $x_4 = I_\rho \frac{\partial \phi}{\partial t}(z,t)$ for $z \in (a,b)$, $t \geq 0$, where $w(z,t)$ is the transverse displacement and $\phi(z,t)$ is the rotation angle of the beam. The coefficients ρ, I_ρ, E, I and K are the mass per unit length, the angular moment of inertia of a cross section, Young's modulus of elasticity, the moment of inertia of a cross section, and the shear modulus respectively, and the state space $X = L_2(a;b;\mathbb{R}^4)$. The energy of the beam is expressed in terms of the energy variables,

$$\begin{aligned} H &= \frac{1}{2}\int_a^b (Kx_1^2 + \frac{1}{\rho}x_2^2 + EIx_3^2 + \frac{1}{I_\rho}x_4^2)dz \\ &= \frac{1}{2}\int_a^b x(z)^T(\mathcal{L}x)(z)dz = \frac{1}{2}\parallel x \parallel_{\mathcal{L}}^2 \end{aligned} \tag{4}$$

The medical endoscope is clamped at one end while the other end is free. The endoscope is actuated in its domain by the use of EAP patches but does not have any control at the boundary. Thus, the boundary conditions of the endoscope are $Kx_1(b,t) = EIx_3(b,t) = 0 \ \forall t \geq 0$ and $\frac{1}{\rho}x_2(a,t) = \frac{1}{I_\rho}x_4(a,t) = 0 \ \forall t \geq 0$, The domain of the operator $\mathcal{J}$ is

$$D(\mathcal{J}) = \left\{ x \in H_1(0,1;\mathbb{R}^n) \left| \begin{array}{l} x_2(a,t) = 0 \\ x_4(a,t) = 0 \\ x_1(b,t) = 0 \\ x_3(b,t) = 0 \end{array} \right., \forall t \geq 0 \right\} \subset X \tag{5}$$

The operator $\mathcal{J} = P_1\frac{\partial}{\partial z} + P_0$ defined by the matrices $P_1 = P_1^T$ and $P_0 = -P_0^T$ is a first order skew adjoint differential operator acting on the state space X with the boundary condition (5). We also consider the material of the endoscope is uniform, *i.e.* ρ, I_ρ, E, I and K are constant. Hence the operator $\mathcal{L}$ is self-adjoint and coercive.

2.2 Distributed control of Timoshenko beam

As previously mentioned, the endoscope is controlled by the EAP actuators caught on its body. In this section, we discuss the distributed control of the inner tube body (Timoshenko beam).

Assume the EAP actuators can provide uniform torques. We place the EAP actuators on the different small intervals $I_i = [a_i, b_i]$ of the beam (on the spatial domain $[a,b]$). The torque given by each EAP can be written as $b_i(z)u_i(t)$ with $b_i(z) = 1$ if $z \in I_{bi}$ and $b_i(z) = 0$ elsewhere.

The torque given by each EAP is $b_i(z)u_i(t)$ on the $i-$th on the small interval $I_i = [a_i, b_i]$ of the spatial space $[a,b]$ *i.e.* $b_i(z) = 1$ if $z \in I_{bi}$ and $b_i(z) = 0$ elsewhere. Thus the input operator and input are:

$$\mathcal{B}(z)u(t) = \sum_i \begin{bmatrix} 0 \\ 0 \\ 0 \\ b_i(z) \end{bmatrix} u_i(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ b(z) \end{bmatrix} u(t) \tag{6}$$

where $\mathcal{B} : \mathbb{C}^i \mapsto X$, $b(z) = [b_1(z), \cdots, b_i(z), \cdots]$ and $u(z) = [u_1(z), \cdots, u_i(z), \cdots]^T$.

Example 1. Consider that three EAP actuators are placed on the three small intervals of the beam $I_1 = [0, 0.1c]$, $I_2 = [0, 4c, 0.5c]$ and $I_1 = [0.9c, 1c]$ with $c = \frac{b-a}{10}$. The three inputs given by the three actuators are $u_1(t)$, $u_2(t)$ and $u_3(t)$. Thus the distributed control is given by

$$\mathcal{B}u = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ b_1(z) & b_2(z) & b_3(z) \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} \quad (7)$$

where

$$b_i(z) \begin{cases} = 1 \text{ if } z \in I_i \\ = 0 \text{ if } z \notin I_i \end{cases} \quad i \in \{1, 2, 3\}. \quad (8)$$

The output is power conjugated to the input, *i.e.*

$$y = \mathcal{B}^* \mathcal{L} x(t) \quad (9)$$

The input-output model of the endoscope can be described by the following port Hamiltonian formulation :

$$\begin{aligned} \dot{x}(t) &= \mathcal{J}\mathcal{L}x(t) + \mathcal{B}u(t) \\ y(t) &= \mathcal{B}^*\mathcal{L}x(t) \end{aligned} \quad (10)$$

The derivative of the Hamiltonian can be easily computed by using the total energy of the system (4) and the system (10):

$$\frac{\partial H}{\partial t} = y^T u. \quad (11)$$

3. LINEAR QUADRATIC OPTIMAL LOCATION

In this section, we discuss the optimal placement of the actuator which minimizes a quadratic performance. Before analyzing the optimal location problem, let recall the linear-quadratic optimal problem (Curtain and Zwart, 1995). The linear-quadratic optimal controller design is to find a control $u(t)$ such that the cost functional:

$$J_{co}(u, x_0) = \int_0^\infty \left(\langle x(t), Qx(t) \rangle + \langle u(t), Ru(t) \rangle \right) dt \quad (12)$$

is minimized. $x(t) \in X$ is the state variable defined in (1). The state and control weighting operators $Q : X \mapsto X$ and $R : \mathcal{U} \mapsto \mathcal{U}$ are bounded, symmetric and positive definite.

Definition 2. The system (10) with cost functional (12) is optimizable if for every $x_0 \in X$, there exists $u \in L_2([0, \infty); \mathcal{U})$ such that the cost is finite.

Definition 3. The pair $(Q^{1/2}, \mathcal{J}\mathcal{L})$ is detectable if there exists $F : \mathcal{Y} \mapsto X$ such that $\mathcal{J}\mathcal{L} - \mathcal{F}\mathcal{C}$ generates an exponentially stable semigroup.

Theorem 4. (Curtain and Zwart, 1995) The system (10) with cost functional (12) is optimizable and detectable, then the cost function has a minimum for every $x_0 \in X$. Furthermore, there exists a self-adjoint non-negative operator $P : X \mapsto X$ such that

$$\min_{u \in L_2([0,\infty];\mathcal{U})} J_{co}(u, x_0) = \langle x_0, Px_0 \rangle \quad (13)$$

The operator P is the non-negative unique solution of Riccati equation:

$$\left((\mathcal{J}\mathcal{L})^* P + P\mathcal{J}\mathcal{L} - P\mathcal{B}^T R \mathcal{B} P + Q \right) x = 0 \quad (14)$$

with $x \in D(\mathcal{L})$. Defining $K = R^{-1}\mathcal{B}^* P$, the optimal control is $u = -Kx(t)$ and $\mathcal{J}\mathcal{L} - BK$ generates an exponentially stable semigroup.

Definition 5. The pair $(\mathcal{J}\mathcal{L}, \mathcal{B})$ is stabilizable if there exists $K : \mathcal{U} \mapsto X$ such that $\mathcal{J}\mathcal{L} - \mathcal{B}K$ generates an exponentially stable semigroup.

Now we consider there are m actuators and their locations can be varied over the compact set Ω. We parametrize the location by r. The input operator is denoted as $\mathcal{B}(r)$ which depends on the parameter r. This parameter r is a vector of length m with components on Ω so r is varied on the space denoted by Ω^m. Hence for each r we have an optimal control problem (12) which we denote by $J^r_{co}(u, x_0)$ which its corresponding optimal cost $\langle x_0, P(r)x_0 \rangle$.

Normally, the initial condition x_0 is not fixed. In this paper, we will consider that the optimal location will minimize the cost function with the worst choice of the initial condition (Curtain and Zwart, 1995, Lemma A.3.70), *i.e.* to choose r in order to minimize

$$\begin{aligned} \max_{\substack{x_0 \in X \\ \|x_0\| = 1}} \min_{u \in L_2([0,\infty];\mathcal{U})} J^r_{co}(u, x_0) &= \max_{\substack{x_0 \in X \\ \|x_0\| = 1}} \langle x_0, Px_0 \rangle \\ &= \|P(r)\|. \end{aligned} \quad (15)$$

We denote the performance on location r is $\mu(r) = \|P(r)\|$ and the optimal performance is

$$\hat{\mu} = \inf_{r \in \Omega^m} \|P(r)\|. \quad (16)$$

Theorem 6. (Morris, 2011) Let $\mathcal{B}(r) : \mathcal{U} \mapsto X$, $r \in \Omega^m$,be a family input operator such that for any $r_0 \in \Omega^m$

$$\lim_{r \to r_0} \|\mathcal{B}(r) - \mathcal{B}(r_0)\| = 0. \quad (17)$$

Assume that $(\mathcal{J}\mathcal{L}, \mathcal{B}(r))$ are all exponentially stabilizable and that $(Q^{1/2}, \mathcal{J}\mathcal{L})$ is detectable where $Q^{1/2} : X \mapsto \mathcal{Y}$ is compact. If $\mathcal{U}$ and $\mathcal{Y}$ are finite dimensional, then there exists an optimal actuator location $\hat{r}$ such that

$$\|P(\hat{r})\|_1 = \inf_{r \in \Omega^m} \|P(r)\|_1 = \hat{\mu} \quad (18)$$

The Theorem 6 shows that we can find the optimal actuator locations if the input operators $\mathcal{B}(r)$ and the operator $Q^{1/2}$ are compact, the Riccati equations have the unique non-negative solutions, then the optimal location with the performance $\hat{\mu} = \inf_{r \in \Omega^m} \|P(r)\|$ exists. This result is proven following the Theorem 3.1 (Curtain and Sasane, 2001).

4. COMPUTATION OPTIMAL LOCATIONS AND SIMULATION RESULTS

In this section, we will discuss how to choose the optimal locations of the EAP actuator to control the beam position.

We first focus on the one actuator case. In this part, we will discuss two different situation. First one, we will consider optimal actuator location when we measure the power conjugate output of the port Hamiltonian system. Secondly, we will consider the optimal actuator location when we fix the output measure, in this case, we will measure the displacement of the middle of the beam as output.

The system (10) can be presented as:

$$\dot{x}(t) = \mathcal{J}\mathcal{L}x(t) + \mathcal{B}(r)u(t) \quad (19)$$

The input operator $\mathcal{B}(r)$ depends on the actuator location r. We denote Δ the length of the actuator. Thus the input operator can be written as:

$$\mathcal{B}(r) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ b_r(z) \end{bmatrix} \text{ with } b_r(z) = \begin{cases} 1, \ |r-z| < \frac{1}{\Delta} \\ 0, \ |r-z| > \frac{1}{\Delta} \end{cases}. \quad (20)$$

The power conjugated output are

$$y(t) = \mathcal{B}^*(r)\mathcal{L}x(t), \quad (21)$$

which also depends on the actuator location r. Consider the state weighting operator $Q = \mathcal{L}\mathcal{B}(r)\mathcal{B}^*(r)\mathcal{L}$ and the input weighting operator $R = I$, the cost functional (12) becomes:

$$J_{co}(u, x_0) = \int_0^\infty (\langle y(t), y(t)\rangle + \langle u(t), u(t)\rangle)\, dt. \quad (22)$$

The optimal objective is minimize the norm of the response over times. The Riccati equation associated with this optimal problem is

$$((\mathcal{J}\mathcal{L})^* P + P\mathcal{J}\mathcal{L} - P\mathcal{B}(r)\mathcal{B}^*(r)P + \mathcal{L}\mathcal{B}(r)\mathcal{B}^*(r)\mathcal{L})\, x = 0 \quad (23)$$

with $x \in D(\mathcal{L})$. Since the operator Riccati equation (23) can not be solved in practice, we need an approximation of the system (19) to compute the control law. We will discuss the discretization of the system (10) in the next paragraph.

We use the mixed-finite element discretization method proposed in (Golo et al., 2004). The idea of this method is to approximate flows and efforts with differential forms related to their physical (geometrical) natural. In the case of the Timoshenko beam, defined on a one-dimensional spatial domain, The efforts (torque) correspond to the zero forms (functions) and the flows (angular velocities) correspond to the one forms respectively. This spatial discretization method has been used on the different physical models, the reader can read (Hamroun et al., 2009; Baaiu et al., 2009) for more details and particularly in the Timoshenko beam case can be find in (Macchelli et al., 2009). The explicit finite dimensional port Hamiltonian discretization of the Timoshenko beam is given as follow:

$$\dot{x}_d = J_d \frac{\partial H_d}{\partial x_d} + B_d(r)u \quad (24)$$

where $J_d = -J_d^T \in \mathbb{R}^{4N}$ with N infinitesimal subsections for the discretization, $H_d = \frac{1}{2}x_d^T L_d x_d$ is the Hamiltonian function with L_d the approximation matrix of operator $\mathcal{L}$. The following matrices presents the discretized structure operator of the infinite dimensional model :

$$J_d = \underbrace{\begin{bmatrix} 0 & M & 0 & 0 \\ M^T & 0 & 0 & 0 \\ 0 & 0 & 0 & M \\ 0 & 0 & M^T & 0 \end{bmatrix}}_{\tilde{P}_1} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & -\Phi \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \Phi^T & 0 & 0 & 0 \end{bmatrix}}_{\tilde{P}_2} \quad (25)$$

where the sub-matrices are:

$$M = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & 0 & -1 \end{bmatrix} \text{ with } M \in \mathbb{R}^{N\times N} \quad (26)$$

$$\Phi = diag(\beta, \cdots, \beta) \text{ with } \Phi \in \mathbb{R}^{N\times N} \quad (27)$$

where β is the distance of the infinitesimal section. The matrix $B_d(r)$ is the approximation of the input operator $\mathcal{B}(r)$:

$$B_d(r) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ b_r \end{bmatrix} \in \mathbb{R}^{4N} \quad (28)$$

where the vector $b_r \in \mathbb{R}^N$ depends on the actuator location r. By using the approximation (24) of the system (19), we can get an approximate solution P_d to the infinite dimensional Riccati equation with solving the following finite dimensional Riccati equation:

$$\begin{aligned} (J_d L_d)^T P_d + P_d J_d L_d - P_d B_d(r) B_d^T(r) P_d \\ + L_d B_d(r) B_d^T(r) L_d = 0. \end{aligned} \quad (29)$$

We consider now the numerical simulation of optimal actuator location. The parameters used for the numerical simulations are given in Tab. 1. These are the real parameters of the experimental setup in department AS2M of Institute FEMTO-ST (shown in Fig. 3).

Parameters of beam and actuator	Value (unit)
Length L	30 cm
Width b	2 cm
Thickness h	2 mm
Young's modulus E	0.2 GPa
Mass density ρ	920 kg/m^2
Actuator length Δ	3 cm

Table 1. Parameters of the beam

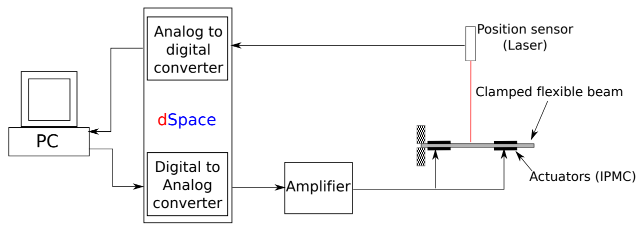

Fig. 3. Clamped flexible beam experimental setup

We illustrate the optimal actuator location for system (19) with the LQ cost function (22). We take the length of the actuator ten times shorter than the beam *i.e.* $\Delta = \frac{b-a}{10}$. The optimal actuator locations are computed by the approximations (24) with the different numbers of infinitesimal subsection (N). We take N from 10 to 200. The optimal actuator locations are shown in Fig 4. In this simulation result, we find that for the power conjugated input-output case, *i. e.* $Y = \mathcal{B}^*(r)\mathcal{L}x$, the optimal actuator location is on the clamped side of the beam.

In the Fig. 5, we show the LQ-performance $\|P\|$ for the different actuator locations. This variation of the LQ norm $\|P(r)\|$ has been computed when $N = 60$. The actuator locations are evaluated by each finite element of discretization from the clamped side to the free side of the beam. The actuator location which minimizes LQ-norm $\|P(r)\|$ is on the clamped side.

Now we consider the second case. The measurement is the displacement of the middle of the beam. Where is the optimal actuator location which minimize the LQ

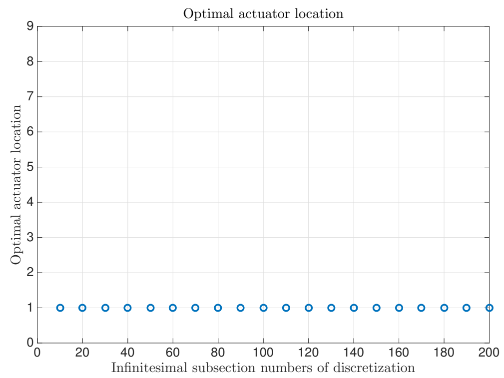

Fig. 4. Optimal location

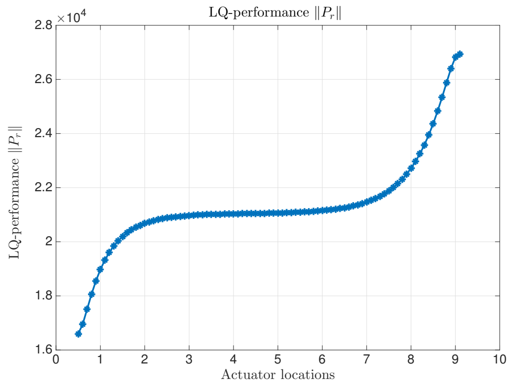

Fig. 5. Variation of LQ performance $\|P\|$ with respect to actuator location

performance with this measurement? In this situation, the output of the system is

$$y_2(t) = \mathcal{B}^* \mathcal{L} x(t), \tag{30}$$

with

$$\mathcal{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ b(z) \end{bmatrix} \quad \text{with} \quad b(z) = \begin{cases} 1, & z \in I \\ 0, & z \notin I \end{cases} \tag{31}$$

where I is a small interval $I = [0.4c, 0.5c]$, $c = \frac{b-a}{10}$.

The cost functional of LQ problem can be written as following:

$$J_{co}(u, x_0) = \int_0^\infty \left(\langle y_2(t), y_2(t) \rangle + \langle u(t), u(t) \rangle \right) dt. \tag{32}$$

with the state weighting operator $Q = \mathcal{L}\mathcal{B}\mathcal{B}^*\mathcal{L}$. Then the optimal objective becomes to minimize the norm of the response at the fixed interval $I = [0.4c, 0.5c]$ over time.

We illustrate the optimal actuator location of above LQ optimal problem by Fig. 6. We can see now the optimal actuator location is not the same shown in Fig. 4. Because we change the LQ cost functional. We obtain the optimal actuator location is as same as interval we measure output, *i.e.* $I = [0.4c, 0.5c]$. In the Fig. 7, we show the LQ-performance $\|P\|$ for the different actuator locations. This variation of the LQ norm $\|P(r)\|$ has been computed when $N = 60$.

This simulation result shows that in order to minimize the the norm of the response at the fixed interval $I = [0.4c, 0.5c]$ over time, we have to place the actuator in the same interval. The input and output are collocated in this case. After several simulations (which are not shown in this paper), the optimal actuator locations are always collocated at the output measurements.

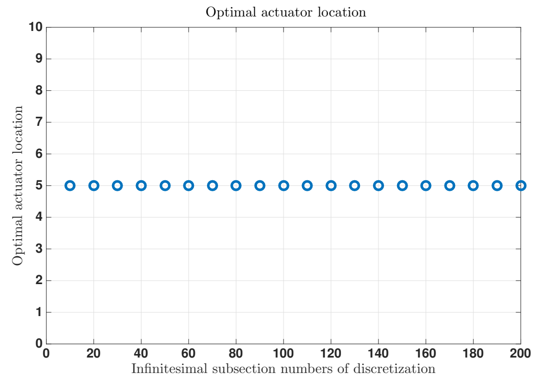

Fig. 6. Optimal location

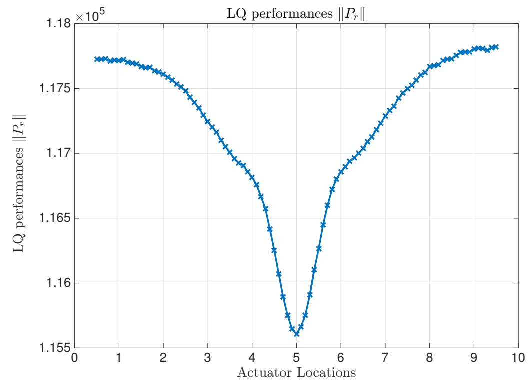

Fig. 7. Variation of LQ performance $\|P\|$ with respect to actuator location

Now we consider two actuators placement case. We suppose that the measurements are the displacement at the intervals $I = [0.2c, 0.3c]$ and $I = [0.5c, 0.6c]$. The Fig. 8 shows that the optimal actuator locations are also around these two intervals. This variation of the LQ norm $\|P(r)\|$ has been computed when $N = 100$. In this figure, we want to underline that the anti-diagonal elements from the bottom left to the top right are no meaning from two actuator point of view. Because these elements show the norms $\|P_r\|$ when the two actuators are superposed. They don't have the real physical meaning in practical application. This simulation shows the similar result as the one actuator case.

5. CONCLUSION AND FUTURE WORK

In this paper, the port Hamiltonian framework has been used to model and reduce one class of bio-medical endoscope. This medical endoscope is controlled by the distributed torques provided by EAP actuators. We formulate the endoscope with its distributed control as an abstract system by port Hamiltonian approach. Then we have considered a LQ optimal actuator location problem for this system. This optimal problem consists in the minimization

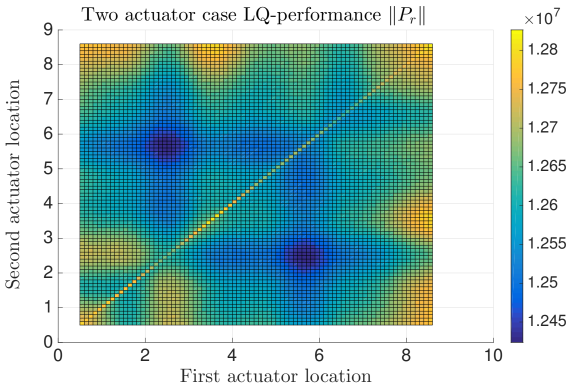

Fig. 8. Actuators locations

of the LQ cost functions which are related to the actuator locations. This method has been illustrated by the numerical simulations. The parameters of a real experimental setup have been used in the simulation.

The ongoing work is to implant this method on the experimental benchmark in order to compare with the numerical simulation results. Since the EAP can also be used as the deformation sensor, we will consider both optimal sensor/actuator locations in the future work.

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