Two-Dimensional Exact Analytical Method for Steady-State Heat Transfer Prediction in Rotating Electrical Machines

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Actually, the heat transfer prediction and temperature distribution in electrical machines are realized by using finite-element method (Fem) and/or thermal equivalent circuit (TEC). The analytical resolution of partial differential equations (PDEs) representing the temperature distribution in electrical machines doesn’t exist. In this paper, a two-dimensional (2-D) exact analytical calculation of steady-state temperature distribution in rotating electrical machines by solving the heat equation in homogenous and non-homogenous regions by using the Fourier’s series and the separation of variables method is presented. It is based on the new subdomain technique where the solution depends on both directions (r, θ) and able to model the different materials of the machine with different thermal conductivities. The heat sources are volumic power losses due to hysteresis, eddy-current and Joule losses in all the regions of machine. A simplified method is used to determine the power losses in permanent-magnet (PM) motors. The main studied problem is conductive with convection in radial and axial directions (r, z) inside the magnet (one region) of a PM machine.

The most used method in thermal modeling of electrical machines is TEC which is able to predict the temperature distribution in steady-state and/or transient. It is based on the representation of the machine materials by different thermal resistances representing the thermal conduction, convection and radiation. This method is fast and can be used to determine the necessary insulation characteristics and normal temperature functioning of the machine [6]-[8]. Although, there are many different lumped circuits that can be adopted for representing the heat transfer in an electrical machine. The thermal designers chose generally a TEC with low number of nodes and thermal resistances especially in transient calculation. The numerical methods are also frequently used to analyze the machine’s temperatures and heat transfer due to power losses (i.e., hysteresis losses, eddy-current losses and Joule losses). The thermal calculation by using Fem can be done alone using power losses as heat sources or coupled directly to electromagnetic analysis [3] and [9]-[11]. The Fem and computational fluid dynamics (CFD) take into account the variation of materials conductivities with temperature and permit to model convective problems accurately [12]-[13]. Recently, there are some references that used a hybrid method. The stator and rotor TEC is coupled to an exact analytical solution only in the air-gap [14]-[15].

In this paper, we present a new 2-D exact analytical prediction of steady-state temperature and heat flux in rotating electrical machines considering a conductive heat transfer inside the machine and air-gap and convective heat transfer outside the machine and inside rotor. It is based on the 2-D exact subdomain technique in polar coordinates developed by Dubas and Boughrara (2017) [16] and applied to radial-flux electrical machines for electromagnetic performances calculation [17]. In this paper, this novel scientific contribution allows to take into account the different materials conductivities (i.e., stator/rotor iron, air, air-gap, PM, slots coils, cage bars). Of course, the conventional subdomain technique used by many authors in electromagnetic performances prediction cannot be applied to thermal modeling even if the heat equations in the different regions of the machine are similar to magnetic differential equations. The conventional subdomain technique considers the ferromagnetic iron having infinity permeability [18]-[21] and cannot take into account the different thermal conductivities of the machine. This is why at this time there is not any prediction of heat transfer in electrical machines using the subdomain technique. In [17], the authors confirmed also that the approach applied to electrical machines is less time consuming (12 sec) in comparison with Fem (20 sec). The developed analytical method is used to determine heat transfer...
in surface-mounted PM machine (SPM), inset PM machine (IPM), induction machine (IM) with cage rotor, and spoke-type PM machine (Spoke). Moreover, it is valid for most electrical machines.

The heat transfer by convection in the air-gap of IPM is also investigated by using the developed analytical method. However, it cannot be validated with the used Fem code [22] and necessitate a Fem with CFD [23][25]. This is why we have limited the investigation of air-gap convection to IPM only. The next step in the analytical thermal modeling of rotating machines would be the introduction of the conductivity variation with the temperature. This variation is similar to the $B(H)$ curve in the iron for the saturation effect and can be introduced in the same way [26]. To determine the heat sources, a simple method is used to determine power losses for studied SPM and IPM [27]. Heat sources of IM and Spoke machines are considered similar to those of IPM.

II. STUDIED MACHINES AND ANALYTICAL TEMPERATURE CALCULATION

A. Problem Description, Assumptions and PDEs

The main heat transfer studied in this paper is conductive with volumique power sources representing the power losses in each region. Heat PDEs are the Laplace’s equation in the air-gap and the Poisson’s equations in the other regions. The model is adopted with the following assumptions:

- The materials of the machines are considered isotropic having constant thermal conductivities without any variation with temperature;
- The stator and rotor slots have radial sides;
- The heat sources are uniform and constant in each region (or subdomain);
- The radiation outside stator and inside the rotor is ignored;
- The interfaces between regions are considered perfect.

Four electrical machines have been studied in this paper [see Fig. 1], viz., i) a surface-mounted PM machine, ii) an inset PM machine, iii) an induction machine with cage rotor, and iv) a spoke-type PM machine. The analyzed IM has 6 stator slots and 4 bars [see Fig. 1(c)].

The machines are subdivided by 7 regions. Region I represents the air-gap, Region IIj the PMs in the SPM, IPM or Spoke and cage bars in the IM, Region III the stator yoke, Region IVi the stator slots, Region V the rotor yoke, Region VIj the air between PMs in the SPM and rotor teeth in the IPM, IM and Spoke, and Region VIIi the stator teeth. It is interesting to note that Region V does not exist in Spoke machine.

In steady-state, the PDEs representing the temperature distribution in each region are given

- in Region I by

$$\lambda_r \Delta T I (r, \theta) = 0$$

where $\lambda_r$ is the air-gap thermal conductivity (in $W/mK$).
• in Regions IIj by
\[ \Delta TII_j (r, \theta) = -\frac{Pm_j}{\lambda_m} \] \hspace{1cm} (2)
where \( \lambda_m \) is the PMs or cage bars thermal conductivity, and \( Pm_j \) the power loss density of the \( j^{th} \) Region II (in W/m³).

• in Region III by
\[ \Delta TIII (r, \theta) = -\frac{P_r}{\lambda_s} \] \hspace{1cm} (3)
where \( \lambda_s \) is the stator yoke thermal conductivity, and \( P_r \) the power loss density in the stator iron. This power loss is considered uniform and constant in the whole stator iron.

• in Regions IVj by
\[ \Delta TVI_j (r, \theta) = -\frac{Psl_j}{\lambda_s} \] \hspace{1cm} (4)
where \( \lambda_s \) is the stator slots thermal conductivity, and \( Psl_j \) the Joule losses considered constant in each stator slot.

• in Region V by
\[ \Delta TV (r, \theta) = -\frac{P_r}{\lambda_s} \] \hspace{1cm} (5)
where \( \lambda_s \) is the rotor yoke thermal conductivity, and \( P_r \) the power loss density in the rotor iron.

• in Regions VIj by
\[ \Delta TVII_j (r, \theta) = -\frac{P_{dr_j}}{\lambda_d} \] \hspace{1cm} (6)
where \( \lambda_d \) is the air between PMs or rotor teeth thermal conductivity, and \( P_{dr_j} \) the power loss density of the \( j^{th} \) Region VI. This power loss is null in the case of the SPM where the region represents the air space between PMs.

• in Regions VIIj by
\[ \Delta TVIII_j (r, \theta) = -\frac{P_{dr_j}}{\lambda_d} \] \hspace{1cm} (7)
where \( \lambda_d \) is the stator teeth thermal conductivity, and \( P_{dr_j} \) the power loss density in the stator teeth.

It is noticed that \( \lambda_s = \lambda_{sm} = \lambda_{ar} \) and \( \lambda_u = \lambda_{du} = \lambda_r \) for ferromagnetic material.

Using \( q = -\lambda \cdot \nabla T \), the heat flux density components in W/m² in polar coordinates are defined as
\[ q_r = -\lambda \frac{\partial T(r, \theta)}{\partial r} \] \hspace{1cm} (8)
\[ q_\theta = -\lambda \frac{\partial T(r, \theta)}{\partial \theta} \] \hspace{1cm} (9)
where \( \lambda \) is the thermal conductivity.

### B. Temperature Solution in each Subdomain

The steady-state heat transfer in the four machines is studied by using the 2-D exact subdomain technique developed recently in [16]-[17] with non-homogenous BCs. The subdomains connection is performed directly in both directions (i.e., \( r \) and \( \theta \)-edges ICs) considering the different conductivities of machines. The general solutions of Laplace’s and Poisson’s equations in non-homogenous BCs [see Fig. 2(a)] are deduced by applying the principle of superposition [see Fig. 2(b)] and by using the Fourier’s series as well as the separation of variables method.

The solution of Laplace’s equation (1) in the air-gap, where the BCs are homogenous, is given by well-known general form in \( 2\pi \) periodic subdomain:
\[ TI(r, \theta) = A_1 + A_2 \ln(r) \] \hspace{1cm} (10)
\[\ldots + \sum_{n=1}^{\infty} \left[ A_1 \left( \frac{r}{R_e} \right)^n + A_2 \left( \frac{r}{R_m} \right)^n \right] \sin(n\theta) \]
\[\ldots + \sum_{n=1}^{\infty} \left[ A_3 \left( \frac{r}{R_e} \right)^n + A_4 \left( \frac{r}{R_m} \right)^n \right] \cos(n\theta) \]
where \( R_e \) and \( R_m \) are respectively the external and internal radius of the air gap region. \( nn \) is the number of harmonics in the air gap, stator yoke and the rotor.

The solution of Poisson’s equation (2) in Region IIj, where the BCs are non-homogenous, is given by
\[ TII_j (r, \theta) = B_{1,j} + B_{2,j} \ln(r) - \frac{Pm_j}{4\pi a} r^2 \] \hspace{1cm} (11)
\[\ldots + \sum_{n=1}^{\infty} \left[ B_{1,j,n} \left( \frac{r}{R_e} \right)^{n_m} + B_{2,j,n} \left( \frac{r}{R_m} \right)^{n_m} \right] \cos(fra_{j}(\theta - \theta a_1)) \]
\[\ldots + \sum_{n=1}^{\infty} \left[ B_{3,j,n} \right] \sin(fra_{j}(\theta - \theta a_2)) \]
\[\ldots + \sum_{n=1}^{\infty} \left[ B_{4,j,n} \right] \sin(fra_{j}(\theta - \theta a_2)) \]
\[ f_{n_m} = k_{m_n} \; g_{m_n} = \ln \left( \frac{R_e}{R_m} \right) ; \; fra_{j} = \frac{m \pi}{a} ; \; \theta a_j = g_j \frac{a}{2} ; \]
\[ \theta a_{2j} = g_j + \frac{a}{2}; \ R_e, a \text{ and } g_j \text{ are respectively the internal radius, the opening width and angle of symmetry axis of PM; } \]

\[ kk \text{ and } mm \text{ are the number of harmonics in the PM region.} \]

The stator yoke represented by Region III has homogenous BCs and the solution of (3) is

\[ TIII(r, \theta) = A6_0 + A5_0 \ln(r) - \frac{P_d}{4\lambda_s} r^2 \]

\[ + \sum_{n=1}^{\infty} \left( A5_n \left( \frac{r}{R_{ext}} \right)^n + A6_n \left( \frac{r}{R_{ext}} \right)^n \right) \cos(n\theta) + \]

\[ + \sum_{n=1}^{\infty} \left( A7_n \left( \frac{r}{R_{ext}} \right)^n + A8_n \left( \frac{r}{R_{ext}} \right)^n \right) \sin(n\theta) \]  

(12)

where \( R_{ext} \) is the external radius of the machine.

The \( Q \) stator slots represented by Region IVi with non-homogenous BCs, the solution of (4) is given by

\[ TVI(r, \theta) = C1_{i0} + C2_{i0} \ln(r) - \frac{P_{ds}}{4\lambda_{si}} r^2 \]

\[ + \sum_{n=1}^{\infty} \left[ C1_{in} \left( \frac{r}{R_{i}} \right)^n + C2_{in} \left( \frac{r}{R_{i}} \right)^n \right] \cos(fsc_{in}(\theta - \theta c1)) + \]

\[ + \sum_{n=1}^{\infty} \left[ C3_{in} \frac{sh(fsc_{in}(\theta - \theta c1))}{sh(fsc_{in})} \right] \sin\left( fsc_{in} \ln\left( \frac{r}{R_{i}} \right) \right) \]  

(13)

where \( fsc_{1i} = \frac{k1\pi}{ff}; \ ff = \ln\left( \frac{r_{a}}{R_{i}} \right); \ fsc_{max} = \frac{m1\pi}{c}; \ \theta c1 = \alpha_i - \frac{c}{2}; \)

\[ \theta c2 = \alpha_i + \frac{c}{2}; \ r_a, c \text{ and } \alpha_i \text{ are respectively the external radius of rotor tooth opening width and the angle of symmetry axis of rotor tooth.} \]

The rotor yoke represented by Region V has homogenous BCs and the solution of (5) is

\[ TV(r, \theta) = A10_0 + A9_0 \ln(r) - \frac{P_d}{4\lambda_s} r^2 \]

\[ + \sum_{n=1}^{\infty} \left[ A9_n \left( \frac{r}{R} \right)^n + A10_n \left( \frac{r}{R} \right)^n \right] \cos(n\theta) + \]

\[ + \sum_{n=1}^{\infty} \left[ A11_n \left( \frac{r}{R} \right)^n + A12_n \left( \frac{r}{R} \right)^n \right] \sin(n\theta) \]  

(14)

where \( R \) is the radius of rotor shaft.

In Region VIj with non-homogenous BCs, the solution of (6) is given by

\[ TVI_j(r, \theta) = B6_{j,0} + B5_{j,0} \ln(r) - \frac{Pdr}{4\lambda_{si}} r^2 \]

\[ + \sum_{n=1}^{\infty} \left( B5_{j,n} \left( \frac{r}{R_m} \right)^n + B6_{j,n} \left( \frac{r}{R_m} \right)^n \right) \cos(frb_{n}(\theta - \theta b1)) + \]

\[ + \sum_{n=1}^{\infty} \left[ B7_{j,n} sh(fr_{j}(\theta - \theta b1)) \right] \frac{sh(fr_{b})}{sh(fr_{b})} \left( \frac{r}{R_m} \right) \]  

\[ + \sum_{n=1}^{\infty} \left[ B8_{j,n} sh(fr_{j}(\theta - \theta b2)) \right] \frac{sh(fr_{b})}{sh(fr_{b})} \left( \frac{r}{R_m} \right) \]  

(15)

where \( frb_{n} = \frac{m\pi}{b}; \ \theta b1 = \beta_j - \frac{b}{2}; \ \theta b2 = \beta_j + \frac{b}{2}; \ b \) and \( \beta_j \) are respectively the rotor tooth opening width and the angle of symmetry axis of rotor tooth.

In region VII representing the stator teeth with \( Pds \) power losses, we have

\[ TVII(r, \theta) = C5_{i,0} + C6_{i,0} \ln(r) - \frac{P_{ds}}{4\lambda_{si} a} r^2 \]

\[ + \sum_{n=1}^{\infty} \left( C5_{in} \left( \frac{r}{r_{a}} \right)^n + C6_{in} \left( \frac{r}{r_{a}} \right)^n \right) \cos(fsd_{in}(\theta - \theta d1)) + \]

\[ + \sum_{n=1}^{\infty} \left[ C7_{in} \frac{sh(fsd_{in}(\theta - \theta d1))}{sh(fsd_{in})} \right] \sin\left( fsd_{in} \ln\left( \frac{r}{r_{a}} \right) \right) \]  

(16)

where \( fsd_{1i} = \frac{k1\pi}{ff}; \ ff = \ln\left( \frac{r_{a}}{r_{a}} \right); \ fsd_{max} = \frac{m1\pi}{c}; \ \theta d1 = \gamma_i - \frac{d}{2}; \)

\[ \theta d2 = \gamma_i + \frac{d}{2}; \ d \text{ and } \gamma_i \text{ are respectively the stator tooth opening width and the angle of symmetry axis of stator tooth.} \]

C. Interface Conditions in the \( \theta \)- and \( r \)-Directions

To determine the unknown coefficients of temperature in each subdomain, there are 18 ICs, viz., 14 ICs in the \( \theta \)-direction and 4 ICs in the \( r \)-direction.

When considering heat transfer inside the machine by conduction and outside the machine by convection, the ICs are given as follows:

- in the \( \theta \)-direction
\[ qV_i (R, \theta) = -h_i \left( TV(R, \theta) - T_{int} \right) \]  

(17)

where \( T_{int} \) and \( h_i \) are respectively the temperature and the coefficient of convection of air in the rotor shaft.

\[ TV(R, \theta) = TII_j(R, \theta) \]  

(18)

\[ TV(R, \theta) = TVI_j(R, \theta) \]  

(19)

\[ qV_i (R, \theta) = \begin{cases} qII_{r, i} (R, \theta) & \text{for } \theta \in \left[ g_j + \frac{a}{2}, g_j - \frac{a}{2} \right] \\ qVI_{r, i} (R, \theta) & \text{for } \theta \in \left[ \beta_j + \frac{b}{2}, \beta_j - \frac{b}{2} \right] \end{cases} \]  

(20)

\[ TI(R, \theta) = TII_j (R, \theta) \]  

(21)

\[ TI(R, \theta) = TVI_j (R, \theta) \]  

(22)
The heat transfer in the $\theta$- and $r$-directions by convection between air regions and solid regions of the machine can be introduced in the analytical model in the same way. This situation can be found in the SPM with air space between PMs.

Outside the stator and inside the rotor, the heat transfer by radiation can occur and can be taken into account by modifying (17) and (30) respectively by

$$q_{v_\alpha} (R_\alpha, \theta) = -h_{c} (T_{I\alpha} (R_\alpha, \theta) - T_{stg})$$

$$q_{v_\alpha} (R_\alpha, \theta) = h_{c} (T_{I\alpha} (R_\alpha, \theta) - T_{stg})$$

where $h_{c}$ is the coefficient of convection in the air gap (in $W/m^2K$).

In the case of SPM with PMs mounted at the surface of rotor using glue. In this case, IC (18) is modified as

$$q_{v_\alpha} (R_\alpha, \theta) = \frac{1}{R_{c}} (T_{I\alpha} (R_\alpha, \theta) - T_{stg})$$

where $R_{c}$ is the contact resistance between PMs and the rotor yoke due to glue (in $m^2K/W$).

The development of ICs permits to obtain a system of equations whose unknowns are the coefficients of Fourier’s series solution in each subdomain. The resolution of this system gives the distribution of temperature and heat flux in the entire machine. In this section, we develop only the ICs of the conduction problem [i.e., (17) to (34b)] with convective heat transfer to ambient air and at the rotor.

The development of ICs for the introduction of convection in the air-gap [i.e., (35) to (38)], the radiation outside the stator and inside the rotor [i.e., (39) and (40)], the contact resistance between subdomains for non-perfect contact [i.e., (41)], and convective BC in the rotor for SPM machine [i.e., (42) and (43)] can be done easily in the same way as in the conduction problem.

In the $\theta$-direction, the IC (17), which represents the interface of heat transfer by convection inside the rotor, gives 3 equations

$$\lambda_{c} \left( \frac{p_{R} R_{c}}{2\lambda_{c}} + A_{0} \right) = h_{c} \left( \frac{p_{R} R_{c}}{2\lambda_{c}} + A_{0} \ln (R) + A_{10} - T_{stg} \right)$$

$$\frac{\lambda_{c}}{R_{c}} (A_{0} - A_{10}) = h_{c} (A_{0} + A_{10})$$

where $A_{0}$ is the surface of the rotor and $A_{10}$ is the surface of the stator.
\[
\frac{nA}{R_1} \left( A1 - A1_2 \right) = h_1 \left( A1_1 + A1_2 \right) \tag{46}
\]

From IC (18), which represents the perfect contact between PMs and the rotor yoke, we obtain 2 equations

\[
B_{1,0} + B_{2,0} \ln(R_1) = \frac{PmR_1^2}{4\lambda_m} = \frac{1}{a} \int TV(R_1, \theta) d\theta \tag{47}
\]

\[
B_{1,m} \left( \frac{R_m}{R_1} \right) + B_{2,m} = \frac{1}{a} \int TV(R_1, \theta) \cos \left( \frac{m\pi}{a} \left( \theta - \frac{\beta}{2} \right) \right) d\theta \tag{48}
\]

From IC (19), which represents the perfect contact between rotor yoke and rotor teeth (or air space in the SPM), we obtained 2 equations

\[
B_{6,0} + B_{5,0} \ln(R_m) = \frac{PdR_m^2}{4\lambda_m} = \frac{1}{b} \int TV(R_m, \theta) d\theta \tag{49}
\]

\[
B_{6,m} \left( \frac{R_m}{R_1} \right) + B_{5,m} = \frac{1}{b} \int TV(R_1, \theta) \cos \left( \frac{m\pi}{b} \left( \theta - \frac{\beta}{2} \right) \right) d\theta \tag{50}
\]

The heat flux transfer in IC (20) between Regions V, IIj and VIj permit to obtain 3 equations as

\[
-\lambda_1 \left( \frac{P \cdot R_1}{2\pi R_1} + A9_{-a} \right) = \frac{1}{2\pi} \sum_{j=1}^{2} \int qU_{ij}(R_1, \theta) d\theta \tag{51}
\]

\[
\cdots + \frac{1}{2\pi} \sum_{j=1}^{2} \int qV_{ij}(R_1, \theta) d\theta \tag{52}
\]

\[
\text{where } p \text{ is the number of poles pairs.}
\]

\[
-\lambda_p \frac{A9}{R_1^2} \left( \frac{R_m}{R_1} \right)^{\alpha} \left( \frac{R_m}{R_1} \right)^{\alpha} + \frac{2}{\alpha} \sum_{j=1}^{2} \int qU_{ij}(R_1, \theta) \cos(n\theta) d\theta \tag{53}
\]

\[
\cdots + \frac{1}{2\pi} \sum_{j=1}^{2} \int qV_{ij}(R_1, \theta) \cos(n\theta) d\theta \tag{54}
\]

\[
\text{The IC (24) between air-gap and stator slots gives}
\]

\[
C_{1,0} + C_{2,0} \ln(R_1) = \frac{PsL_1^2}{4\lambda_a} = \frac{1}{c} \int TV(R_1, \theta) d\theta \tag{55}
\]

\[
\text{The IC (25) between air-gap and stator teeth gives}
\]

\[
C_{5,0} + C_6 \ln(R_1) = \frac{PdR_1^2}{4\lambda_a} = \frac{1}{d} \int TV(R_1, \theta) d\theta \tag{56}
\]
The heat flux continuity in IC (26) permits to get 3 equations
\[ -\frac{\lambda_{2s}}{R_{ext}} + \frac{\lambda_{1s}}{R_{st}} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = \frac{1}{\pi} \sum_{i=1}^{\int_{a_{i}} \int_{t_{i}}} \int qIV_{s}(R, \theta) d\theta + \frac{1}{\pi} \sum_{i=1}^{\int_{a_{i}} \int_{t_{i}}} \int qVII_{s}(R, \theta) d\theta \]
(65)
\[ -\frac{\lambda_{n}}{R_{r}} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = \frac{1}{\pi} \sum_{i=1}^{\int_{a_{i}} \int_{t_{i}}} \int qIV_{s}(R, \theta) \cos(n\theta) d\theta \]
\[ -\frac{\lambda_{n}}{R_{r}} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = \frac{1}{\pi} \sum_{i=1}^{\int_{a_{i}} \int_{t_{i}}} \int qVII_{s}(R, \theta) \sin(n\theta) d\theta \]
(66)
\[ -\frac{\lambda_{n}}{R_{r}} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = \frac{1}{\pi} \sum_{i=1}^{\int_{a_{i}} \int_{t_{i}}} \int qIV_{s}(R, \theta) \cos(n\theta) d\theta \]
\[ -\frac{\lambda_{n}}{R_{r}} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = \frac{1}{\pi} \sum_{i=1}^{\int_{a_{i}} \int_{t_{i}}} \int qVII_{s}(R, \theta) \sin(n\theta) d\theta \]
(67)

The IC (27) between temperatures of stator yoke and stator slots gives
\[ C_{1,0} + C_{2,0} \ln(r_{i}) - \frac{P_{st}}{4 \lambda_{0}} = \frac{1}{c} \int \frac{T_{III}(r, \theta)}{r} d\theta \]
(68)
\[ C_{2,0} \left( \frac{r_{c}}{R_{ext}} \right)^{\eta_{0}} + C_{1,0} = \frac{2}{c} \int \frac{T_{III}(r, \theta)}{r} \cos(\frac{m_{0} \pi}{c} \left( \theta - a_{i} - \frac{\pi}{2} \right)) d\theta \]
(69)

The IC (28) between temperatures of stator yoke and stator teeth gives
\[ C_{5,0} + C_{6,0} \ln(r_{i}) - \frac{P_{st}}{4 \lambda_{0}} = \frac{1}{d} \int \frac{T_{III}(r, \theta)}{r} d\theta \]
(70)
\[ C_{6,0} \left( \frac{r_{c}}{R_{ext}} \right)^{\eta_{0}} + C_{5,0} = \frac{2}{d} \int \frac{T_{III}(r, \theta)}{r} \cos(\frac{m_{0} \pi}{d} \left( \theta - a_{i} - \frac{\pi}{2} \right)) d\theta \]
(71)

In conduction problems, the heat flux continuity between stator yoke, stator slots and stator teeth is represented by IC (29) giving
\[ \frac{p_{st}}{2 \lambda_{0}} + \frac{4 \lambda_{0}}{R_{st}} = \frac{1}{\pi} \sum_{i=1}^{\int_{a_{i}} \int_{t_{i}}} \int qIV_{s}(r, \theta) d\theta + \frac{1}{\pi} \sum_{i=1}^{\int_{a_{i}} \int_{t_{i}}} \int qVII_{s}(r, \theta) d\theta \]
(72)

The IC (32a) between temperatures of PM (or cage bar in the IM) and rotor teeth (or air between PMs in the SPM) gives
\[ -\left( B_{8,j_{k},+} + B_{3,j_{k},+} \right) = \frac{2}{g_{k}} \left( -B_{5,j_{k},0} + B_{2,j_{k},0} \right) \int 2 + \left( B_{1,j_{k},0} - B_{6,j_{k},0} \right) \int 1 + \left( -\frac{P_{m}}{4 \lambda_{m}} + \frac{P_{dr}}{4 \lambda_{r}} \right) \int 7 \]
\[ + \sum_{n=1}^{\int_{a_{n}} \int_{t_{n}}} \left( -B_{5,j_{n},m} \int 3 - B_{6,j_{n},m} \int 4 + \cos(m \pi) \left( B_{1,j_{n},m} \int 5 + B_{2,j_{n},m} \int 6 \right) \right) \]
(81)
Development of IC (32b) for the first PM (or cage rotor bar) and rotor teeth (or air between PMs in the SPM) gives

\[
\begin{align*}
B_{7,2p,k} + B_{4,1,k} &= \frac{2}{gg} \left( -B_{5,2p,0} + B_{2,1,0} \right) \text{int } 2 + \left( B_{1,0} - B_{6,2p,0} \right) \text{int } 1 + \left( \frac{Pm_{i+1}}{4\lambda_{m}} + \frac{Pdr_{j}}{4\lambda_{j}} \right) \text{int } 7 \\
&\quad \cdots + \frac{2}{gg} \sum_{n=1}^{m} \left( -B_{5,2p,n} \text{int } 3 + B_{6,2p,n} \text{int } 4 \right) \cos \left( \frac{m\pi}{2} \right) + B_{1,n} \text{int } 5 + B_{2,1,n} \text{int } 6 \\
\end{align*}
\]

(82)

For the other \(2p-1\) PM, we obtain

\[
\begin{align*}
B_{7,2p,k} + B_{4,1,k+1} &= \frac{2}{gg} \left( -B_{5,2p,0} + B_{2,1,0} \right) \text{int } 2 + \left( B_{1,0} - B_{6,2p,0} \right) \text{int } 1 + \left( \frac{Pm_{j+1}}{4\lambda_{m}} + \frac{Pdr_{j}}{4\lambda_{j}} \right) \text{int } 7 \\
&\quad \cdots + \frac{2}{gg} \sum_{n=1}^{m} \left( B_{5,2p,n} \text{int } 3 + B_{6,2p,n} \text{int } 4 \right) \cos \left( \frac{m\pi}{2} \right) + B_{1,n} \text{int } 5 + B_{2,1,n} \text{int } 6 \\
\end{align*}
\]

(83)

where

\[
\begin{align*}
\text{int } 1 &= \frac{1}{h_{r}} \int \frac{k_{r}}{gg} \ln \left( \frac{r}{R_{s}} \right) \text{d}r, \\
\text{int } 2 &= \frac{1}{h_{r}} \int \frac{k_{r}}{gg} \ln \left( \frac{r}{R_{s}} \right) \text{d}r, \\
\text{int } 3 &= \frac{1}{h_{r}} \int \frac{k_{r}}{gg} \ln \left( \frac{r}{R_{s}} \right) \left( \frac{r}{R_{s}} \right) \text{d}r, \\
\text{int } 4 &= \frac{1}{h_{r}} \int \frac{k_{r}}{gg} \ln \left( \frac{r}{R_{s}} \right) \left( \frac{r}{R_{s}} \right)^{m_{g}} \text{d}r, \\
\text{int } 5 &= \frac{1}{h_{r}} \int \frac{k_{r}}{gg} \ln \left( \frac{r}{R_{s}} \right) \left( \frac{r}{R_{s}} \right)^{m_{g}} \text{d}r, \\
\text{int } 6 &= \frac{1}{h_{r}} \int \frac{k_{r}}{gg} \ln \left( \frac{r}{R_{s}} \right) \left( \frac{r}{R_{s}} \right)^{m_{g}} \text{d}r, \\
\end{align*}
\]

For the stator slots and stator teeth, the development of IC (34a) gives

\[
\begin{align*}
C_{3,i,1} + C_{8,i,1} &= \frac{2}{ff} \left( -C_{2,i,0} + C_{6,i,0} \right) \text{int } 2s + \left( C_{5,i,0} - C_{1,i,0} \right) \text{int } 1s + \left( -\frac{Pd_{i+1}}{4\lambda_{d}} + \frac{Ps_{i+1}}{4\lambda_{s}} \right) \text{int } 7s \\
&\quad \cdots + \frac{2}{ff} \sum_{n=1}^{m_{i}} \left( C_{5,i,n} \text{int } 3s + C_{6,i,n} \text{int } 4s - \cos \left( \frac{m\pi}{2} \right) \left( C_{1,i,n} \text{int } 5s + C_{2,i,n} \text{int } 6s \right) \\
\end{align*}
\]

(84)

Development of IC (34b) for the first stator slot and the last stator tooth permits to get

\[
\begin{align*}
-C_{4,i,1} - C_{7,Q,i,1} &= \frac{2}{ff} \left( -C_{2,i,0} + C_{6,Q,i,0} \right) \text{int } 2s + \left( C_{5,Q,i,0} - C_{1,i,0} \right) \text{int } 1s + \left( -\frac{Pd_{Q,i+1}}{4\lambda_{d}} + \frac{Ps_{Q,i+1}}{4\lambda_{s}} \right) \text{int } 7s \\
&\quad \cdots + \frac{2}{ff} \sum_{n=1}^{m_{i}} \left( C_{5,Q,i,n} \text{int } 3s + C_{6,Q,i,n} \text{int } 4s \cos \left( \frac{m\pi}{2} \right) - C_{1,i,n} \text{int } 5s - C_{2,i,n} \text{int } 6s \right) \\
\end{align*}
\]

(85)

For the other stator slots and stator teeth (with \(i\) vary from 1 to \(Q-1\)), we have

\[
\begin{align*}
-C_{7,i,1} - C_{4,i+1,1} &= \frac{2}{ff} \left( -C_{2,i+1,0} + C_{6,Q,i,0} \right) \text{int } 2s + \left( -C_{1,i+1,0} + C_{5,i,0} \right) \text{int } 1s + \left( -\frac{Pd_{i+1}}{4\lambda_{d}} + \frac{Ps_{i+1}}{4\lambda_{s}} \right) \text{int } 7s \\
&\quad \cdots + \frac{2}{ff} \sum_{n=1}^{m_{i}} \left( C_{5,i,n} \text{int } 3s + C_{6,i,n} \text{int } 4s \cos \left( \frac{m\pi}{2} \right) - C_{1,i+1,n} \text{int } 5s - C_{2,i+1,n} \text{int } 6s \right) \\
\end{align*}
\]

(86)

The integrals in the stator slots and teeth in the \(r\)-direction \(\text{int } 1s \sim \text{int } 7s\) between \(R_{s}\) and \(r_{s}\) are similar to above integrals at the rotor subdomains between \(R_{s}\) and \(R_{r}\). In the \(\theta\)-direction, the integrals are simple and easy to calculate.

The heat flux continuity between stator slots and stator teeth is represented by IC (33a) giving

\[
\begin{align*}
\frac{k_{1}\lambda_{sl}}{sh} \left( C_{3,i,1}ch \left( \frac{k_{1}r_{c}}{ff} \right) + \cdots + C_{4,i,1} \right) &= \frac{k_{1}\lambda_{sl}}{sh} \left( C_{8,i,1}ch \left( \frac{k_{1}r_{d}}{ff} \right) + \cdots + C_{7,i,1} \right) \\
\end{align*}
\]

(87)

The IC (33b) gives \(Q_{s}\) equations. For the first stator slot, we have

\[
\begin{align*}
\frac{k_{1}\lambda_{sl}}{sh} \left( C_{4,i,1}ch \left( \frac{k_{1}r_{c}}{ff} \right) + \cdots + C_{3,i,1} \right) &= \frac{k_{1}\lambda_{sl}}{sh} \left( C_{7,Q,i,1}ch \left( \frac{k_{1}r_{d}}{ff} \right) + \cdots + C_{8,Q,i,1} \right) \\
\end{align*}
\]

(88)

For the other \(Q_{s}-1\) slots (with \(i\) vary from 2 to \(Q_{s}\), we obtain

\[
\begin{align*}
\frac{k_{1}\lambda_{sl}}{sh} \left( C_{4,i,1}ch \left( \frac{k_{1}r_{c}}{ff} \right) + \cdots + C_{2,i,1} \right) &= \frac{k_{1}\lambda_{sl}}{sh} \left( C_{7,i+1,1}ch \left( \frac{k_{1}r_{d}}{ff} \right) + \cdots + C_{8,i+1,1} \right) \\
\end{align*}
\]

(89)
III. TEMPERATURE AND HEAT FLUX RESULTS

The parameters and dimensions of the studied electrical machines are given in Table I. The machines have a simple distributed 4 poles winding. The power losses of SPM and IPM at 500 rpm used as sources for the thermal model are calculated using simple formulas presented in [27]. The $B(H)$ curve of stator and rotor iron for the electromagnetic power losses determination by Fem is given in Fig. 3. The hysteresis and eddy current coefficients and lamination factor used in losses calculation are also given in Table I. For the IM with cage rotor, 6 stator slots and 4 rotor bars are considered to show the validity of the analytical model to predict temperature and heat flux in this type of machines. The heat sources for the IM and Spoke machine are considered the same as in the IPM. The PM losses is considered equal to loss in cage rotor bar. The thermal conductivities, convection coefficients, ambient temperatures and power losses at 500 rpm used in the thermal model are listed in Table II. The variation of power losses in the SPM and IPM with speed are shown in Figs. 4 – 5. The harmonics number of analytical model is $m$, $mn$, $kk$, $mm$ and $kk1$ are 200, 50, 50, 40, and 40 respectively. This harmonics number permits to obtain a very good accuracy in comparison with Fem and a reasonable computing time. However, the analytical program is not optimized to compare computational time with Fem. The average number of elements and nodes of the Fem calculation [22] are respectively 109,168 and 55,484.

![B(H) curve of stator and rotor iron.](image)

**TABLE I**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{lm}$</td>
<td>Remanence flux density of PMs</td>
<td>1.3 T</td>
</tr>
<tr>
<td>$\mu_{lm}$</td>
<td>Relative permeability of PMs</td>
<td>1.0277</td>
</tr>
<tr>
<td>$N_c$</td>
<td>Number of conductors per stator slot</td>
<td>23</td>
</tr>
<tr>
<td>$I_m$</td>
<td>Peak phase current</td>
<td>7 A</td>
</tr>
<tr>
<td>$Q_c$</td>
<td>Number of stator slots</td>
<td>6</td>
</tr>
<tr>
<td>$c$</td>
<td>Stator slot-opening</td>
<td>30 deg.</td>
</tr>
<tr>
<td>$a$</td>
<td>PM-opening</td>
<td>40 deg.</td>
</tr>
<tr>
<td>$p$</td>
<td>Number of pole pairs</td>
<td>2</td>
</tr>
<tr>
<td>$R_{ext}$</td>
<td>Radius of the external stator surface</td>
<td>110 mm</td>
</tr>
<tr>
<td>$r_s$</td>
<td>Outer radius of stator slot</td>
<td>97 mm</td>
</tr>
<tr>
<td>$R_s$</td>
<td>Radius of the stator inner surface</td>
<td>80.5 mm</td>
</tr>
<tr>
<td>$R_{rot}$</td>
<td>Radius of the rotor outer surface at the PM</td>
<td>79.3 mm</td>
</tr>
<tr>
<td>$R_i$</td>
<td>Radius of the rotor inner surface at the PM</td>
<td>73 mm</td>
</tr>
<tr>
<td>$R_{sh}$</td>
<td>Radius of the rotor shaft</td>
<td>40 mm</td>
</tr>
<tr>
<td>$\sigma_{lm}$</td>
<td>Electrical conductivity of PMs</td>
<td>0.556E6 S/m</td>
</tr>
<tr>
<td>$\sigma_w$</td>
<td>Electrical conductivity of stator coil wires</td>
<td>58E6 S/m</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>Electrical conductivity of M-19 Steel</td>
<td>1.9E6 S/m</td>
</tr>
<tr>
<td>$c_s$</td>
<td>Lamination stacking factor</td>
<td>0.95</td>
</tr>
<tr>
<td>$c_h$</td>
<td>Hysteresis coefficient</td>
<td>143</td>
</tr>
<tr>
<td>$c_e$</td>
<td>Eddy-current coefficient</td>
<td>0.530</td>
</tr>
<tr>
<td>$g$</td>
<td>Air-gap length</td>
<td>0.8 mm</td>
</tr>
<tr>
<td>$L_s$</td>
<td>Axial length</td>
<td>40 mm</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Mechanical speed</td>
<td>500 rpm</td>
</tr>
</tbody>
</table>

![Figure 3: B(H) curve of stator and rotor iron.](image)

**TABLE II**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_e$</td>
<td>Thermal conductivity of air-gap</td>
<td>0.03 W/(m °K)</td>
</tr>
<tr>
<td>$\lambda_a$</td>
<td>Thermal conductivity of air</td>
<td>0.03 W/(m °K)</td>
</tr>
<tr>
<td>$\lambda_{lm}$</td>
<td>Thermal conductivity of PMs</td>
<td>9 W/(m °K)</td>
</tr>
<tr>
<td>$\lambda_s$</td>
<td>Thermal conductivity of stator iron</td>
<td>55 W/(m °K)</td>
</tr>
<tr>
<td>$\lambda_r$</td>
<td>Thermal conductivity of rotor iron</td>
<td>55 W/(m °K)</td>
</tr>
<tr>
<td>$\lambda_t$</td>
<td>Thermal conductivity of cage rotor bars</td>
<td>394 W/(m °K)</td>
</tr>
<tr>
<td>$\lambda_d$</td>
<td>Thermal conductivity of stator slot coil</td>
<td>1.73 W/(m °K)</td>
</tr>
<tr>
<td>$p_s$</td>
<td>Stator losses in the SPM</td>
<td>4.2 W</td>
</tr>
<tr>
<td>$p_r$</td>
<td>Rotor losses in the SPM</td>
<td>0.17 W</td>
</tr>
<tr>
<td>$p_{pm}$</td>
<td>PM loss in the SPM</td>
<td>6.5 W</td>
</tr>
<tr>
<td>$p_{sd}$</td>
<td>Stator slot losses in the SPM</td>
<td>18.13 W</td>
</tr>
<tr>
<td>$p_{sr}$</td>
<td>Stator losses in the IPM</td>
<td>4.07 W</td>
</tr>
<tr>
<td>$p_{pr}$</td>
<td>Rotor losses in the IPM</td>
<td>0.31 W</td>
</tr>
<tr>
<td>$p_{pm}$</td>
<td>PM losses in the IPM</td>
<td>7.94 W</td>
</tr>
<tr>
<td>$p_{sd}$</td>
<td>Stator slot losses in the IPM</td>
<td>18.12 W</td>
</tr>
<tr>
<td>$l_t$</td>
<td>Convection coefficient inside the rotor</td>
<td>100 W/(m² °K)</td>
</tr>
<tr>
<td>$T_{int}$</td>
<td>Temperature inside the rotor</td>
<td>70 °C</td>
</tr>
<tr>
<td>$l_s$</td>
<td>Convection coefficient outside the stator</td>
<td>100 W/(m² °K)</td>
</tr>
<tr>
<td>$T_{ext}$</td>
<td>Temperature outside the stator</td>
<td>70 °C</td>
</tr>
</tbody>
</table>
A. Thermal Results of the SPM and Validation

For the 6-slots/4-poles SPM, the temperature distribution at speed of 500 rpm in the whole machine is shown in Fig. 6. We can observe that the temperature is higher inside the stator slots where power loss is higher [see Table II]. In the middle of air-gap, the temperature and heat flux components distribution calculated by the developed analytical method and Fem using the parameters and power losses of Table II are given in Fig. 7.

![Fig. 6: Temperature distribution in the SPM.](image)

![Fig. 7: Temperature and heat flux components distribution in the middle of air-gap.](image)
To show the ability of the analytical method to predict the temperature distribution in the PMs and the stator slots, the temperature curves in the $\theta$- and $r$-direction obtained analytically are represented in Figs. 8–9 and compared with Fem. The analytical results are in very good agreement with the Fem results. A parametric analysis with varying the convective coefficients $h_r$ and $h_s$ is also done. In Fig. 10, the temperature distribution in the machine when the convective coefficients $h_s=20$ W/(m$^2$°K) and $h_r=100$ W/(m$^2$°K) is shown. The corresponding air-gap temperature distribution is represented in Fig. 11. It can be observed that the temperature is higher than the case with $h_r=100$ W/(m$^2$°K). For the case with $h_s=20$ W/(m$^2$°K) and $h_r=100$ W/(m$^2$°K), in Figs. 12–13, we show the temperature distribution in the machine obtained by using Fem and temperature distribution in the middle of air-gap obtained by using the developed analytical method and Fem. In this case also, the analytical results are very close to those of Fem. The variation of temperature in the center of PM and stator slot when the convective coefficients $h_r$ and $h_s$ varies is presented in Figs. 14–15. The comparison of the analytical method results with those obtained by Fem confirms the validity of the proposed analytical method to predict the temperature distribution in the SPM with a very good accuracy.
Fig. 12: Temperature distribution in the SPM for $h_r=20\,\text{W/(m}^2\,\text{K)}$ and $h_s=100\,\text{W/(m}^2\,\text{K)}$.

Fig. 13: Temperature distribution in the middle of air-gap for $h_r=20\,\text{W/(m}^2\,\text{K)}$ and $h_s=100\,\text{W/(m}^2\,\text{K)}$.

Fig. 14: Temperature variation with varying $h_s$ and $h_r=100\,\text{W/(m}^2\,\text{K)}$ in a point at the center of PM and stator slot.

Fig. 15: Temperature variation with varying $h_r$ and $h_s=100\,\text{W/(m}^2\,\text{K)}$ in a point at the center of PM and stator slot.

B. Thermal Results of the IPM and Validation

Studied IPM machine has the same dimensions and parameters than SPM machine with small difference in power losses [see Tables I – II]. The temperature distribution in the entire machine predicted using Fem is given in Fig. 16. The temperature and heat flux components distribution in the middle of air-gap obtained with the analytical method is shown in Fig. 17. We can observe a very good agreement between analytical and Fem results. The temperature distribution in the middle of first PM and stator slot in the $\theta$- and $r$-direction [see Figs. 18 – 19] obtained analytically and with Fem confirm the accuracy of the proposed analytical method to predict heat transfer in those regions where the effect of temperature is very important.

Fig. 16: Temperature distribution in the IPM.
When the cooling outside the IPM is not sufficient ($h_s=20$ W/(m$^2$ °K)), the heat is not evacuated and the temperature is very high in the stator and air-gap [see Figs. 20–21]. The same observation can be done in the case of not sufficient cooling in the rotor shaft with $h_r=20$ W/(m$^2$ °K). In this case, the rotor temperature is high but lower than the case of low value of $h_s$ [see Figs. 22–23]. The variation of temperature in the center of first PM and stator slot with the convection coefficient $h_s$ and $h_r$ is represented in Figs. 24–25. Those curves are very important for the design of insulation of stator winding and PM where the characteristics depend on temperature. The effect of convection coefficient of air-gap (i.e., $h_e$) on the PM and the stator slot is shown in Figs. 26–27. It can be observed that the temperature in the middle of PM decrease with small values of $h_e$ and the temperature in the middle of the stator slot increase for small values of $h_e$. With low values of convective coefficient in the air-gap, the air-gap is considered as a barrier to heat transfer between stator slot (where the heat source is higher) and PM.

![Fig. 17: Temperature and heat flux components distribution in the middle of air-gap.](image)

(a) In the $\theta$-direction.

(b) In the $r$-direction.

![Fig. 18: Temperature distribution in the middle of the first PM.](image)

(a) In the $\theta$-direction.

(b) In the $r$-direction.

![Fig. 19: Temperature in the middle of the first stator slot.](image)

(a) In the $\theta$-direction.

(b) In the $r$-direction.
Fig. 20: Temperature distribution in the IPM for $h_s=20 \text{ W/(m}^2\text{ K)}$ and $h_r=100 \text{ W/(m}^2\text{ K)}$.

Fig. 21: Temperature distribution in the middle of air-gap for $h_s=20 \text{ W/(m}^2\text{ K)}$ and $h_r=100 \text{ W/(m}^2\text{ K)}$.

Fig. 22: Temperature distribution in the IPM for $h_r=20 \text{ W/(m}^2\text{ K)}$ and $h_s=100 \text{ W/(m}^2\text{ K)}$.

Fig. 23: Temperature distribution in the middle of air-gap for $h_r=20 \text{ W/(m}^2\text{ K)}$ and $h_s=100 \text{ W/(m}^2\text{ K)}$.

Fig. 24: Temperature variation with varying $h_s$ and $h_r=100 \text{ W/(m}^2\text{ K)}$ in a point at the center of PM and stator slot.

Fig. 25: Temperature variation with varying $h_r$ and $h_s=100 \text{ W/(m}^2\text{ K)}$ in a point at the center of PM and stator slot.
C. Thermal Results of the IM and Validation

As explained above, the IM configuration with 6 stator slots and 4 rotor bars studied in this paper does not exist and used only for showing the validity of the proposed analytical model for thermal modeling. The difference between the analyzed IM and IPM in term of thermal modeling is the conductivity of PM and cage rotor bars. This latter which is equal to 394 W/(m °K) is very high than the conductivity of PM equal to 9 W/(m °K). The heat sources in the PMs of the IPM are replaced with the heat sources of cage rotor bar in the IM. The dimensions of cage rotor bars are higher than the dimensions of PMs in this example. The temperature distribution in the machine obtained by using Fem is represented in Fig. 28. It can be observed that the essential of heat is located in the stator slots. The temperature and heat flux components distribution in the middle of air-gap is shown in Fig. 29. The analytical results are very close to those issued from Fem.

The temperature distribution in the θ- and r-direction in the middle of the first cage rotor bar and the first stator slot is shown in Figs. 30 ~ 31. The accuracy of the new analytical model is established also in those subdomains where it is important to know the heat transfer for the insulation design. The effect of cooling outside the IM and inside the rotor shaft...
is represented with the convective coefficients $h_s$ and $h_r$ respectively. For $h_s$ equal to 20 W/(m$^2$ °K), which is small, we represent in Figs. 32 – 33 the temperature distribution in the whole machine and in the middle of air-gap. The high temperature is located inside the stator and the temperature in the air-gap is higher also compared to $h_s$ equal to 100 W/(m$^2$ °K). For $h_r$ equal to 20 W/(m$^2$ °K) compared to $h_r$ equal to 100 W/(m$^2$ °K), the essential of heat is located in the rotor [see Fig. 34] and the temperature distribution in the air-gap [see Fig. 35] is higher than the case with $h_r$=100 W/(m$^2$ °K). Low values of convective coefficients represent a barrier for heat transfer outside the stator and inside the rotor.
Fig. 35: Temperature distribution in the middle of air-gap for $h_r=20$ W/(m$^2$K) and $h_s=100$ W/(m$^2$K).

D. Thermal Results of the Spoke Machine and Validation

The exact analytical model for the Spoke machine has less number of regions and the rotor convective heat transfer in the rotor represent a BC of PMs and rotor teeth subdomains. The main dimensions and thermal parameters are given in Tables I – II with a difference in height and opening of PMs and rotor teeth. The heat sources used for the thermal model are the same as in the IPM. The analytical results are obtained with the number of harmonics for $m_n$, $m_m$, $k_k$, $m_m$, $k_k$, and $k_k$ equal to 200, 70, 70, 60 and 60 respectively. The harmonics number is chosen to get accurate results with a reasonable time of calculation and to avoid numerical errors.

In Fig. 36, the temperature distribution in the machine obtained with Fem is shown. It can be observed that the temperature is higher in the slots due to Joule losses which are important in the slots. The temperature and heat flux components distribution in the middle of air-gap is represented in Fig. 37. The analytical results are very close to those obtained with Fem. To confirm the validity of the proposed analytical method for spoke-type machines, the temperature distribution in the center of the first PM and stator slot in both directions $r$ and $\theta$ are shown in Figs. 38 and 39. In the PM and stator slots subdomains, the analytical results are also in very good agreement with those obtained with Fem.

Fig. 36: Temperature distribution in the Spoke machine.

Fig. 37: Temperature and heat flux components distribution in the middle of air-gap.

(a) In the $\theta$-direction.
optimization process that include the effect of the variation of thermal conductivities, convective coefficients and power losses in temperature distribution. The analytical results are in very good agreement with those obtained by Fem.

Heat transfer using convection coefficient in the air-gap is also investigated and the results are very interesting.

REFERENCES


[16] F. Dubas, and K. Boughrara, “New Scientific Contribution on the 2-D Subdomain Technique in Polar Coordinates: Taking into Account of
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