Investigation for the analysis of the vibrations of quasi-periodic structures

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Abstract

Periodic structures are well known for the possibility to exhibit band gap effects. This work aims at investigating vibration behaviors of quasi-periodic structures. In this paper, the quasi-periodic structure is defined as a type of beam with an impedance mismatch generated by Fibonacci orders with non-symmetrical translation in geometry, acting as a waveguide. Two types of quasi-periodicity are considered, namely finite, and infinite Fibonacci sequences using super unit cell. Considering flexural elastic waves in above mentioned quasi-periodic models, the frequency ranges corresponding to band gaps are investigated, using either spectral analysis of infinite structures or frequency response functions of finite structures. Fibonacci beams exhibit multi stop bands with short widths in different frequency ranges, whereas periodic and its super unit cells-based structures have only one stop band frequency with larger frequency extension.

1 Introduction

The vibroacoustic response in many types of structures is complex. Structures are shaped continuous, periodic, aperiodic or random. These types of structures have different vibroacoustic responses with respect to their geometrical and materials properties. Some of them can be designed as periodic or quasi-periodic structures. The desirable periodic structures that have identical repetition can be modelled by using materials and geometrical periodicity [1].

Many research activities show that vibrations responses of periodic and quasi-periodic structures exhibit frequency band gap effects [1,2,3,4,5]. In periodic structures, the impedance mismatch generated by periodic discontinuities in the geometry, acting as a waveguide, and/or in the constituent material, causes destructive wave interference phenomena over specific frequency bands called “stop band” or “band gaps” [6]. For quasi-periodic structures, Fibonacci sequences are considered in several researches [7]. Spectral fragmentation of elastic waves represents the replication of primary and secondary gaps in Fibonacci sequence [7]. In other research paper, Gei. M. et al [8] claims that in an elementary repeated unit cell, generated adopting the Fibonacci sequence, in cases of axial and flexural vibration by higher order generation, the number of stop bands changes. The concern of the present paper is to model a quasi-periodic beam utilized by two different cross sections of 3D 2 node-beam finite element following the Fibonacci series.

The main issue is that the anti-symmetric condition of quasi-periodic structures cannot be applied by the wave-based approach using spectral analysis of Bloch-Floquet theorem, unless using an elementary cell/volume representative using Fibonacci lower order replications. In fact, the structural dynamics, model analysis and Frequency Response Function (FRF) of the structures are considered for understanding the dynamics structural behavior of the system to ensure a safe, efficient and affordable design.

In this research, an isotropic beam with a finite number of cells and a wave guide section with infinite number of cells is considered in the analysis. The quasi-periodicity of the span is translated according to the Fibonacci sequences [8]. The identical double cross sections (A) and (B) are translated using Fibonacci generating order from 4^{th} to 10^{th} orders (Figure 1).

The analysis has been carried out in two main sections. The first section presents the finite element analysis of Frequency Response Function [9,10], whereas the second section describes a wave guide section model, using combination of wave and finite element using transfer matrix method [11,12,13]. This wave guide section is modelled following super unit cell. Super unit cell is a cell hosting the first 4^{th} generated order of Fibonacci sequence for Wave Finite Element Method (WFEM) analysis. Stiffness and mass matrix of the given unit super cell is extracted using MATLAB and APDL-ANSYS platform. Then, the extracted matrices are used into WFEM FRF and spectral analysis of Bloch-Floquet boundary conditions [14].

2 Theoretical approach

2.1 Spatial elastic waves

In infinite solid medium there are two types of waves that propagate with different velocities [15]. It is known from vector analysis that any vector field can be represented as sum of two vectors, one of which has a scalar potential and the other a vector one:

$$\mathbf{u} = \mathbf{u}_l + \mathbf{u}_t = \mathrm{grad}\varphi + \mathrm{rot}\boldsymbol{\psi} \tag{2.1.1}$$

Taking into consideration that rot $u_l = \mathrm{div}\, u_t = 0$ and substituting equation (2.1.1) into the wave equation for elastic medium with the application of operations rot and div, we get

$$\frac{\partial^2 \mathbf{u}_l}{\partial t^2} - c_l^2 \nabla^2 \mathbf{u}_l = 0 \quad c_l = \sqrt{(\Lambda + \frac{2\mu}{\rho}}, \tag{2.1.2}$$

and

$$\frac{\partial^2 \mathbf{u}_t}{\partial t^2} - c_l^2 \nabla^2 \mathbf{u}_t = 0 \quad c_t = \sqrt{\frac{\mu}{\rho}} \tag{2.1.3}$$

$\mathbf{u}_l$ and $\mathbf{u}_t$ vectors correspond to the longitudinal and transversal waves, Λ and μ are Lame constants and ρ is a density. In the other hand c_l and c_t are the longitudinal and transversal wave velocities.

Hence the vector $\mathbf{u}_l$ in equation (2.1.2) is called a longitudinal wave or a wave of expansion-compression which, the vibration direction in it coincides with its propagation direction. The wave $\mathbf{u}_t$ in equation (2.1.3) in which the direction of vibrations is perpendicular to the direction of wave propagation and in which deformations are shear is called a transversal or shear wave.

2.2 Methods and tools

The modern tools for investigating the most significant impacts of quasi-periodic structures on the vibrational response of a given structure are codes based on conventional FE analysis, combination of wave and finite element, spectral finite element, and transfer matrix methods.
Theoretical background for modelling quasi periodicity and its significant impacts on wave analysis is carried out with mathematical series named Fibonacci sequence, Thue-Morse, Rodin-Shapiro and Penrose lattice. Spectral analysis of these types of subsystems are shown in phononic crystals, anisotropic beams, rods and Bravais lattices [16]. In this portion of work, a finite element beam with lower 4^{th} and higher with

10th generated order of Fibonacci sequence is modelled using MATLAB in parallel with APDL-ANSYS. The lower order is used as a super unit cell waveguide section for infinite structures, whereas the higher order is used as one span of finite beam in FE analysis [Table 1].

Fibonacci orders	**4th**	**5th**	**7th**	**8th**	**9th**	**10th**
Number of cells	5	8	21	34	55	89

Table 1: Geometrical distribution of cells according to Fibonacci generated orders.

Figure 1 shows a 2-node beam type 188 following Fibonacci orders. The geometrical characteristics of this beam is a combination of two different cross section. The distance between each key point is 100 mm, every segment between these two points are one of abovementioned two cross section with three finite elements. Section (A) and (B) can be seen in figure 1. Embedded Fibonacci sequence starts from $\boldsymbol{S_1}$ the first order and ends to $\boldsymbol{S_n}$ the $\boldsymbol{N_{th}}$ order of the sequence. The mathematical expressions following the Fibonacci sequences in this beam are defined in the following algorithm:

$$\boldsymbol{S}_1 = \{\boldsymbol{A}\}, \qquad \boldsymbol{S}_2 = \{\boldsymbol{AB}\},\ \boldsymbol{S}_3 = \{\boldsymbol{ABA}\},\ \ldots,\ \boldsymbol{S}_{\boldsymbol{n}} = \boldsymbol{S}_{\boldsymbol{n}\text{-}1}\boldsymbol{S}_{\boldsymbol{n}\text{-}2}$$

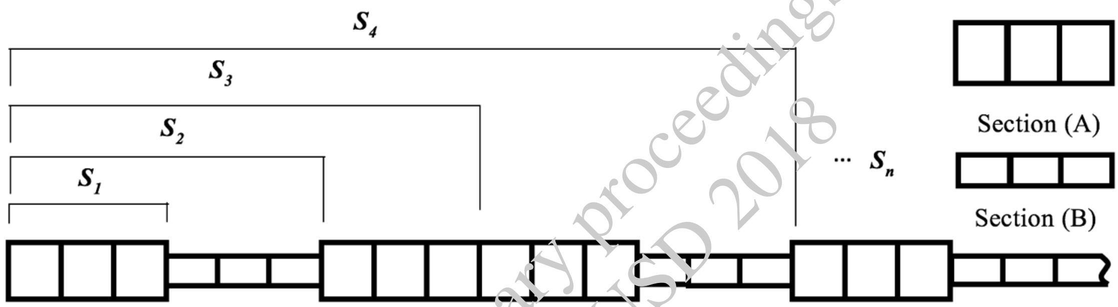

Figure 1: Numerical model of quasi-periodic model following Fibonacci sequence.

2.3 Finite Element Dynamic Analysis

Commercial FE software's APDL-ANSYS is used to perform the frequency response function of the quasi-periodic beam model. For damped structure, the dynamic finite element model is usually described by the system:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}(\omega) \tag{2.3.1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ in equation (2.3.1) are the mass, damping and stiffness matrices.

The associated eigenvalue problem is:

$$(\mathbf{K}\text{-}\omega_\nu^2\mathbf{M})\,\psi_\nu = 0, \quad \nu = 1, 2, \ldots \tag{2.3.2}$$

let $\boldsymbol{\Psi} = [\boldsymbol{\psi}_1, \boldsymbol{\psi}_2, \boldsymbol{\psi}_3, \ldots, \boldsymbol{\psi}_N]$ be the orthonormalized model basis, and $\boldsymbol{\Lambda} = \mathbf{diag}\ (\boldsymbol{\omega}_{\boldsymbol{v}}^2)$ the spectral matrix; the model projection $\mathbf{x} = \boldsymbol{\Psi}\ \mathbf{q}(\omega)$ and the mode superposition method allows to express, in the case of proportional damping, the dynamic response in the following form:

$$\mathbf{x}(\omega) = \sum_{i=1}^{N} \boldsymbol{\psi}_i \frac{\boldsymbol{\psi}_i^T \mathbf{f}(\omega)}{\omega_i^2\text{-}\omega^2 + 2j\xi\omega_i\omega}. \tag{2.3.3}$$

The FRF for displacement of the system is given by equation (2.3.4), where $\mathbf{x}(\omega)$ is the response of a system and $\mathbf{F}(\omega)$ is its excitation force.

$$\mathbf{H}(\omega) = \frac{\mathbf{x}(\omega)}{\mathbf{F}(\omega)} \tag{2.3.4}$$

The FRF for the k^{th} node degree of freedom with a single excitation force at j^{th} degree of freedom can be simulated by equation (2.3.5).

$$\mathrm{H}_{\mathrm{kj}}(\omega) = \sum_{\mathrm{i}=1}^{\mathrm{N}} \frac{\psi_{\mathrm{ik}}\ \psi_{\mathrm{ij}}}{\omega_{\mathrm{i}}^2\text{-}\omega^2 + 2\mathrm{j}\xi\omega_{\mathrm{i}}\omega} \tag{2.3.5}$$

Modal analysis and frequency response function of the quasi-periodic beam are computed. The aim is to investigate the frequency band gap like behavior in finite span with generating order of Fibonacci (number of cells) of beam type 2-node 188, by considering structural damping.

The conventional FE analysis is used for the frequency response function of the entire span with 10^{th} generated order of Fibonacci. The number of elements in numerical model are designed according to the known criteria which of one sixth of the wave-length. The description of the mechanical properties is provided in Table 2.

Material properties	**Aluminium**	**Polyethylene**
Modulus of elasticity	$2\text{x}10^{11}$ Pa	$7.2\text{x}10^{8}$ Pa
Poisson’s ratio	0.3	0.3
Mass density	7800 kg/m^3	935 kg/m^3

Table 2: Mechanical properties of quasi-periodic bi-material beam.

2.4 Forced response analysis

In this section, the forced response function of a quasi-periodic beam is analysed. The analysis is done with a free-free Boundary Condition (BC). A white spectrum is applied in one end of the beam in the vertical direction and the response is simulated in the other end of the beam. The mesh size in this problem is optimized to a maximum frequency range of 5KHz. The choice of the free-free (BC) is interesting to visualize the energy transfer through the waveguide span. This type of wave guides is intended to use as a junction between two infinite mediums which the mechanics of those materials are not known. The results show band gaps in different ranges of frequencies. Multiple stop bands appear by the positioning of cross sections according to the Fibonacci sequence. The reason is that in this quasi-periodic structure the direction of elastic wave is considered in x direction, therefore an interference of reflected and incident waves creates frequency stop bands (i.e. in this frequency ranges the waves “cannot freely propagate”). This type of stop bands could be predicted as one type of passive wave filter in a quasi-periodic beam.

Figure 2 shows a Fibonacci super unit cell that contains 5 cross sections [ABAAB]. This cell shows the end of part of span with 50 cm length. The span is considered with (210,340, 550, and 890) cm long on the frequency ranges of [0-150] kHz. Using MATLAB and APDL-ANSYS the harmonic analysis is carried out. The allocated point response is received at the head of the span. The results of this dynamic analysis show that the full energy is transferred through the start and end of the span which can be represented as a junction between two infinite media. There is multiple frequency stop bands in different ranges of frequencies. Each frequency stop band has some localized wave modes that rapidly attenuates inside the band gap [Figure 3].

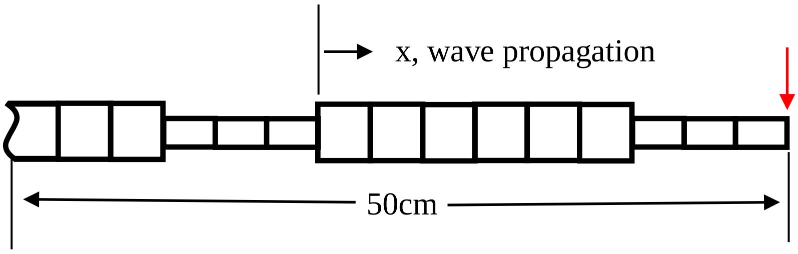

Figure 2: Quasi-periodic structure super unit cell; this cell is the Nth cell of the span with applied unit force at the end node.

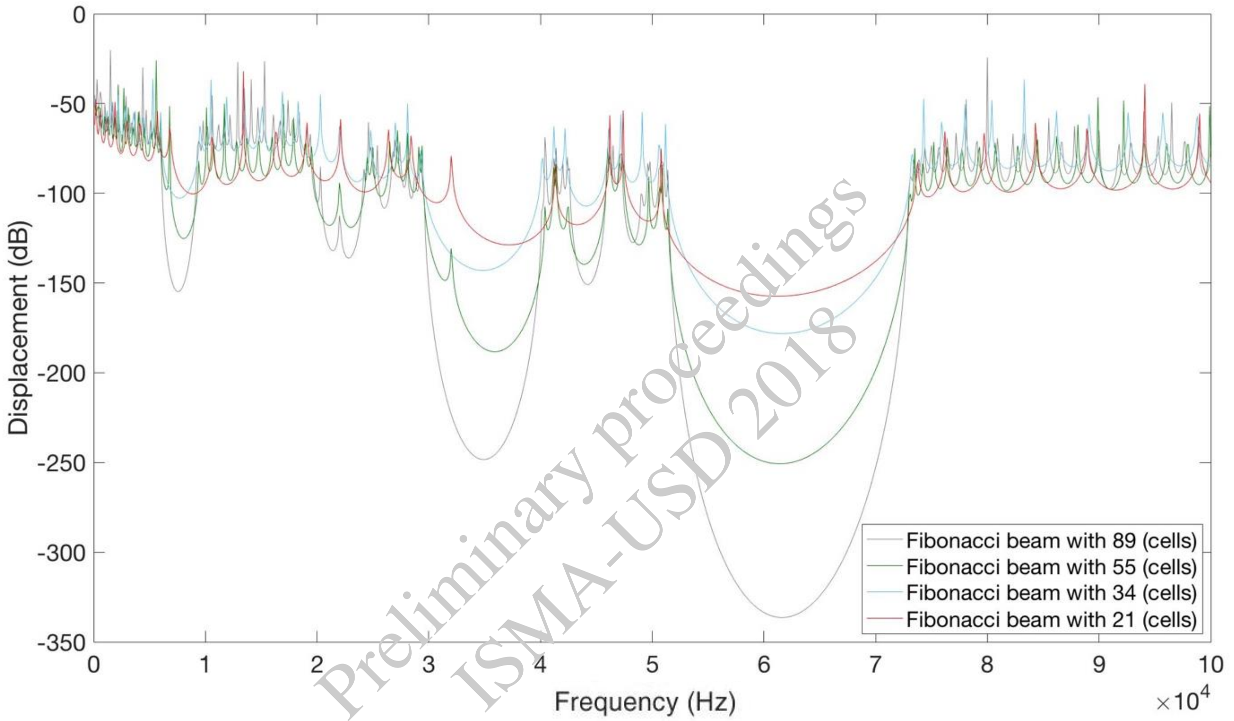

Figure 3: Frequency response function of a finite quasi-periodic beam with length increment.

3 Spectral Analysis of infinite structure

3.1 Substructures dynamics

A substructure (super unit cell) waveguide is considered as a unit cell of the periodicity pattern. In this part of an investigation carried out to obtain an algorithm for quasi-periodic function that permits to the repetition of unit cell with disturbance inside the periodicity.

The substructure in figure 4 is coded as a unit cell in MATLAB and the generated script is then run in APDL-ANSYS for extracting the transfer matrices. The extracted stiffness and mass matrices are used to generate results based on FRF and spectral analysis of Bloch-Floquet boundary condition.

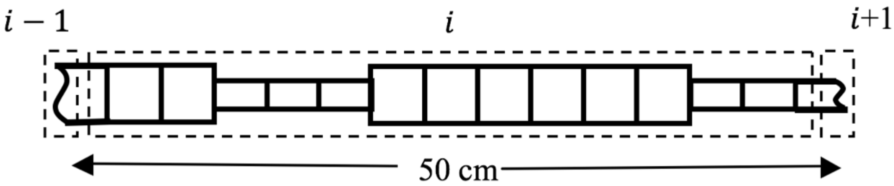

Figure 4: Waveguide section for spectral analysis of Floquet-Bloch boundary condition.

The FRF analysis is achieved in frequency range [0-5000] Hz, for a structural damping of $\eta = 0.001$. The dynamic response of the unit cell is described by:

$$\mathbf{Dq} = \mathbf{F} \tag{3.1.1}$$

Where $\mathbf{D}$, $\mathbf{q}$, and $\mathbf{F}$ define the dynamic stiffness matrix, displacement vector and force vector. The dynamic stiffness matrix, in equation (3.1.2), with complex modulus of elasticity becomes

$$\mathbf{D} = -\omega^2\mathbf{M} + (1 + \mathrm{i}\eta)\mathbf{K} \tag{3.1.2}$$

$\mathbf{M}$, $\mathbf{K}$, and η are respectively the mass matrix, the stiffness matrix and the loss factor.

Dynamic stiffness matrix for the general case can be written for internal and external degrees of freedom. The internal degrees of freedom can be condensed and the left and right boundary displacements and forces are used in the analysis.

$$\begin{bmatrix} \widetilde{\mathbf{D}}_{\mathbf{II}} & \widetilde{\mathbf{D}}_{\mathbf{IL}} & \widetilde{\mathbf{D}}_{\mathbf{IR}} \\ \widetilde{\mathbf{D}}_{\mathbf{LI}} & \widetilde{\mathbf{D}}_{\mathbf{LL}} & \widetilde{\mathbf{D}}_{\mathbf{LR}} \\ \widetilde{\mathbf{D}}_{\mathbf{RI}} & \widetilde{\mathbf{D}}_{\mathbf{RL}} & \widetilde{\mathbf{D}}_{\mathbf{RR}} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_{\mathbf{I}} \\ \mathbf{q}_{\mathbf{L}} \\ \mathbf{q}_{\mathbf{R}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{f}_{\mathbf{L}} \\ \mathbf{f}_{\mathbf{R}} \end{Bmatrix} \tag{3.1.3}$$

3.2 Transfer matrix

A unit cell script file is coded and, then, transformed for post processing using commercial software's. The same procedure could be done directly through sub-structuring analysis (CMS) for extracting transfer matrices by utilizing commercial software's. Using MATLAB code, both mass and stiffness matrices of size [48x48] are extracted by 4th generated order of Fibonacci which has 5 unit cells and each cell has 3 elements. So, there a total of 15 elements in one super unit cell. Then the global matrices are extracted through APDL-ANSYS for wave finite element analysis.

$$\mathbf{S} = \begin{bmatrix} -\mathbf{D}^{*\,-1}_{\mathbf{LR}}\mathbf{D}^{*}_{\mathbf{LL}} & -\mathbf{D}^{*\,-1}_{\mathbf{LR}} \\ \mathbf{D}^{*}_{\mathbf{RL}}-\mathbf{D}^{*}_{\mathbf{RR}}\mathbf{D}^{*\,-1}_{\mathbf{LR}}\mathbf{D}^{*}_{\mathbf{LL}} & \mathbf{D}^{*}_{\mathbf{RR}}\mathbf{D}^{*\,-1}_{\mathbf{LR}} \end{bmatrix}. \tag{3.2.1}$$

$\mathbf{S}$ is the transfer matrix and $\boldsymbol{D}^*$ components inside the transfer matrix are the condensed internal degrees of freedom inside the unit cell that, defines, all the boundary internal nodes and boundary external nodes in equation (3.2.2)

$$\mathbf{D}^* = \mathbf{D}_{\mathbf{BB}}-\mathbf{D}_{\mathbf{BI}}\mathbf{D}_{\mathbf{I}}^{-1}\mathbf{D}_{\mathbf{IB}} \tag{3.2.2}$$

Equation (3.2.2) is condensed dynamic stiffness matrix. In equation (3.2.3) $\boldsymbol{S}$ is the transfer matrix which is symplectic and J is a 2nx2n matrix with real entries (non-singular matrix) or commonly called skew matrix.

$$\mathbf{S}^{\mathrm{T}}\mathbf{J}\mathbf{S} = \mathbf{J} = \begin{bmatrix} \mathbf{0} & \mathbf{I_n} \\ \mathbf{-I_n} & \mathbf{0} \end{bmatrix}. \tag{3.2.3}$$

4 Numerical Results

4.1 Frequency response function of finite structures

FRF of finite structure is analysed in two different parts. The first part corresponds to the conventional FE analysis of the quasi-periodic beam with isotropic and homogenous materials. Whereas the second part is analysed by considering inhomogeneous material variation including the Fibonacci cross section variation as in the first part. The quasi-periodicity in the first case is inscribed by cross section variations, according to the Fibonacci sequence.

4.1.1 FRF Results-geometrical variation

The first results are shown by finite element analysis of the FRF for the four different orders of the Fibonacci sequence beam. The 7th order corresponds to 21 number of cells, whereas the 8th, 9th, and 10th order follows the 34, 55, and 89 cells. The cells are in two different cross sections (A) and (B).

Figure 5 shows an FRF of the Fibonacci beam with increasing length by generated orders of the sequence. The results of flexural waves are plotted on the 1500 frequency steps in the range of [0-5000] Hz. The results show multiple stop bands alongside the frequency ranges for instance around [600-800] Hz, [2100-2500] Hz and followed a large frequency stop band around [3100-4200] Hz. There is also a medium band from [4500-4800].

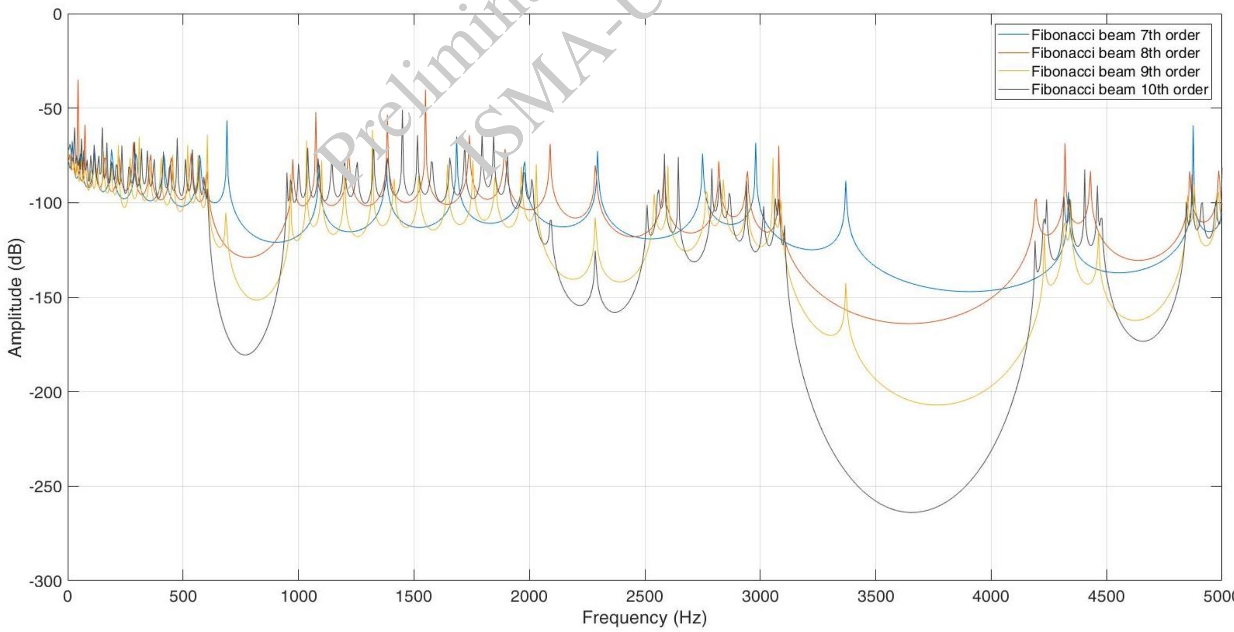

Figure 5: Finite element analysis of the FRF for fourth different orders of Fibonacci beam.

The number of frequency stop bands in the flexural wave analysis of finite quasi-periodic beam depends on the correspondence frequency range. It can be seen that four frequency stop bands are appearing in the frequency ranges of [0-5] kHz. These band gap like behaviours has narrow widths and if there is an increase in the number unit cells, the depth of the stop band frequency grows deeper according to displacement decibels.

4.1.2 FRF Results of the beam with two materials

In this part of FRF analysis the same criteria of the Fibonacci beam are analysed with bi-material distribution all alongside the beam span. The first material is aluminium and the second is polyethylene. Cross section (A) is made of aluminium and cross section (B) is made of polyethylene.

The length of the structure starts from 2.1m span to 8.9 m explicitly ranging from [7^{th},8^{th},9^{th}, 10^{th}] orders of Fibonacci with [21,34,55,89] cells. The FRF analysis has been carried out with free-free boundary condition for all types of orders. A unit spectrum white force is applied in the head of the span (First node) and the response is simulated from the tip of the span (last node). Figure 6 shows two very wide frequency stop bands in the ranges of [0-5000] Hz. The first gap starts from [250-1750] Hz, following the second stop band from [1800-4100] Hz. These wider gaps are due to the bi-material distribution all over the span. The very promising results are the frequency stop bands and its depths by increasing the length of the span/the number of cells. The more increase in the generating orders of Fibonacci will give a result with more depth in the band gap of maximum order. In contrary lower orders has shorter frequency stop bands with low amplitude in (dB).

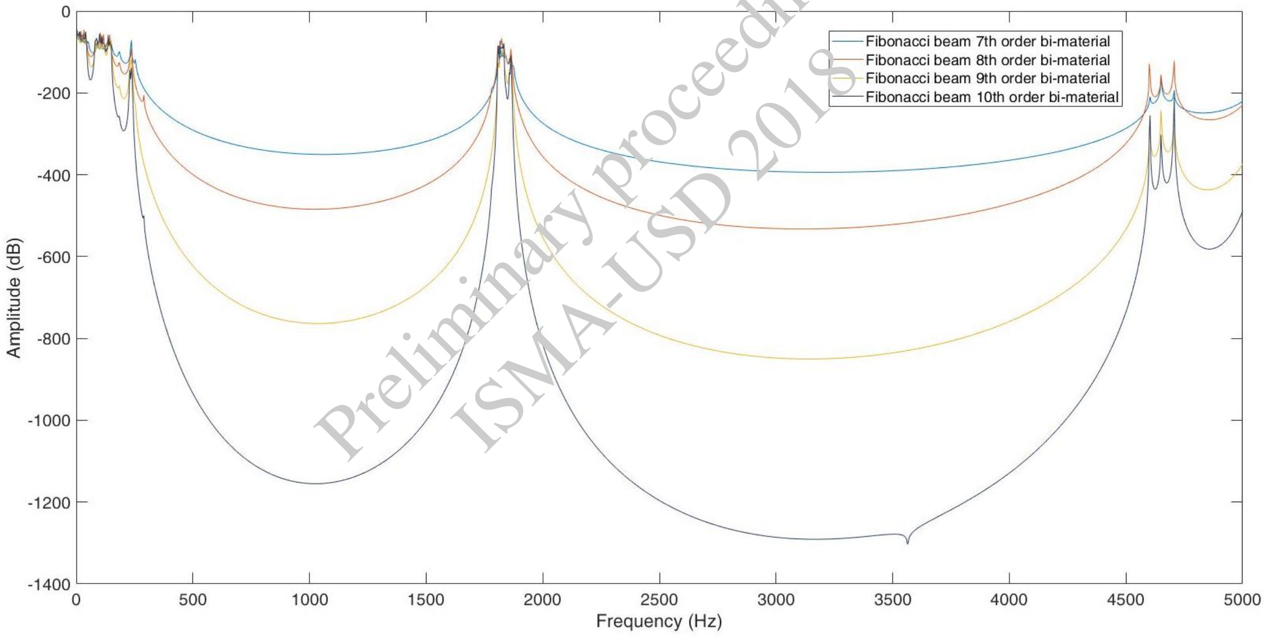

Figure 6: FRF of the Fibonacci beam with bi-materials using Finite element analysis.

4.2 Frequency response function using WFEM

These results correspond to the spectral analysis of the infinite structure. The mass and stiffness matrices are extracted and used as an input for the WFEM using transfer matrix method. The 4^{th} order of the abovementioned sequence is taken as a unit super cell and Floquet-Bloch boundary condition is applied to balance the left and right-side displacement and force vectors of the unit cell. The internal degrees of freedom are eliminated using condensation method and the transfer matrix is modelled with entrees from boundary-boundary and internal-boundary nodes. The super unit cell model can be seen in the Figure 7. The first result shows an FRF analysis of the unit cell by 20 replications of the same cell either in the right or left of the core cell.

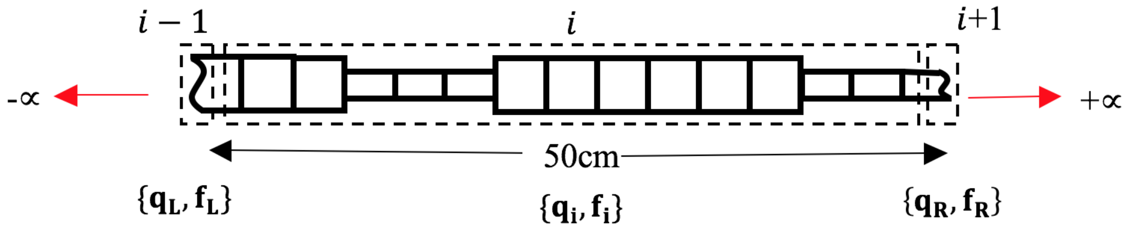

Figure 7: waveguide section of 4th order Fibonacci sequence

Flexural waves are investigated in this analysis. These types of waves can be visualized in the results of FRF using WFEM. Beside that, the findings in the spectral analysis also provide the longitudinal waves as well as flexural ones and for validating such kind of waves, WFEM analysis is used to plot the FRF of longitudinal wave and then compared with spectral analysis of Bloch-Floquet waves. Figure 8 shows a wave based finite element FRF of the Fibonacci 4th order. The repeated unit cell shows a promising filtering conditions in the different frequency ranges in the limit of [0-5000] Hz. Figure 9 shows a spectral analysis of the Floquet-Bloch boundary condition into a Fibonacci 4th order cell. The first branch of the plot in red describes the bending, and compressional waves. In contrary the second branch in blue shows the evanescent waves with curve shapes that corresponds to the stop bands of the real part of the wave numbers in the red branch of the plot. The red branch of the plot at Figure 9 starts with bending waves that propagates from [0-162] Hz and then a frequency stop band appears in [162-187] Hz frequency range. The waves are propagating until the next frequency stop band between the two portions of the bending waves. Another stop band appears between [653-1014] Hz, following the middle one around [1809-2298] Hz. The last frequency stop band is a combination of longitudinal/ compressional and flexural waves. The first mode starts form [3263-4557] Hz that belongs to the last flexural wave, whereas the following second blue curve with higher imaginary numbers in the negative zone of wave numbers corresponds to the longitudinal band gap [3523-4522] Hz.

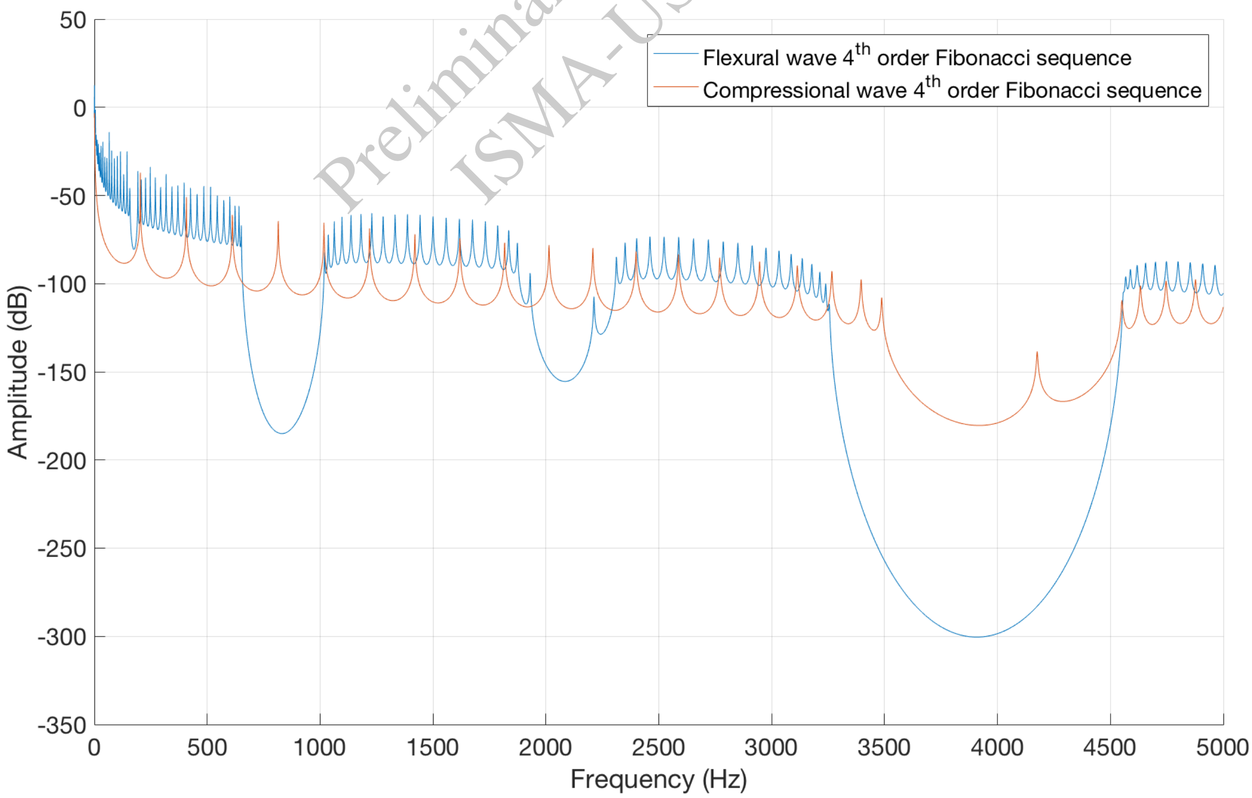

Figure 8: FRF of Fibonacci beam 4TH order analysis by (WFEM) using transfer matrix

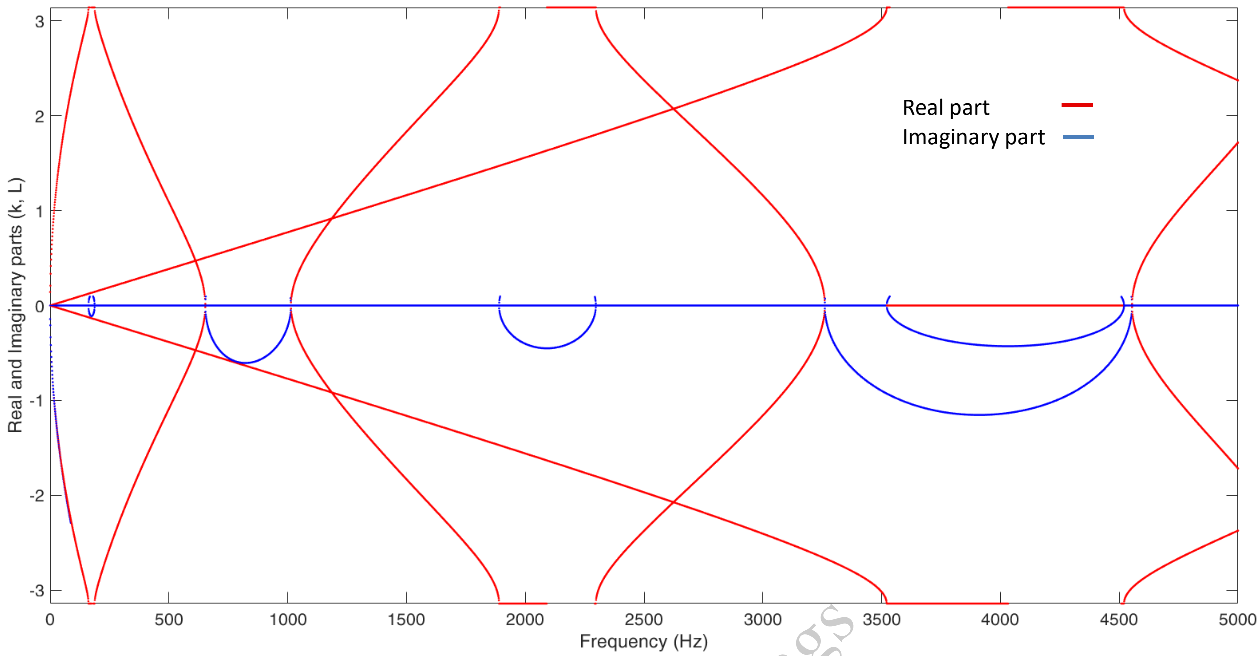

Figure 9: Spectral analysis of Floquet-Bloch boundary condition on the Fibonacci 4th order.

Table 3 shows CPU elapsed time of numerical simulation for three types of analysis; model FRF by FE analysis, model FRF by WFEM analysis and a spectral analysis of Floquet-Bloch waves. The three types of analysis are carried out using MATLAB®, APDL-ANSYS® and Intel® Core™ i7-6700 CPU @3.40GHz (8 CPUs). The elapsed time for maximum frequency 5kHz in spectral analysis of Bloch waves is expected to be very costly among the list. It seems that there is a huge difference almost around 52.7% between the spectral and FE analysis and 97.9% between spectral and WFEM analysis. All three types of analysis follow different frequency steps; thus, they are solved by two different solution methods, inverse and full VT (Variational Technology) harmonic analysis methods.

If we consider the spectral analysis as a reference in Table 3, the FE and WFEM are 47.3% and 2.1% of the CPU timing used in spectral analysis of Bloch waves. It is obvious that WFEM is one of the most useful technique for saving computation time, but still there are open topics in model order reduction to save more computational time.

Model FRF/Spectral analysis of Bloch waves	Methods	Steps number	Elapsed time (sec)	CPU ratio (%)
Spectral analysis of Bloch waves over the wave guide section	Inverse	1	266.1	100%
FE FRF analysis of wave guide section	VT Full	1500	126.22	47.3%
WFEM FRF analysis of a wave guide section	Inverse	1	5.61	2.1%

Table 3: CPU time of numerical simulations.

5 Conclusion

In a perfect periodic beam the effects of vibration are very different from the quasi-periodic one. In quasi-periodic beam, the impedance mismatch generated by four types of Fibonacci orders, causes destructive wave interference and thus generates the band gaps. The geometrical cross section variation, following Fibonacci sequence in the analysis, creates, multi frequency stop bands with narrow band gaps in which, waves cannot freely propagate. These narrow band gaps not only appear in the medium frequency ranges as Bragg type frequency stop bands in periodic beams, but it also covers the lower and higher ranges of frequencies in between [0-5000] Hz.

By increasing the number of Fibonacci generated orders; the frequency stop bands ranges do not change, while in contrary the depth of the frequency stop bands grows deeper.

Using material variation in the quasi-periodic beam can significantly provide, frequency stop bands with larger widths. The effects of wider stop bands can help the structure to be a useful filter in real engineering problems. The very simple models herein presented allow continuing to study the configurations even considering the variation of the materials.

Acknowledgements

This paper containing intermediate investigation, carried out in the framework of the VIPER project (VIbroacoustic of PERiodic media). This project has received funding from the European Union’s Horizon 2020 research and innovation program under Marie Curie grant agreement No 675441 and EUR EIPHI (ANR 17-EURE-0002) project.

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