Analytical Prediction of Iron Core Losses in Flux-Modulated Permanent-Magnet Synchronous Machines

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In this paper, an analytical approach allowing the prediction of the iron core losses in flux-modulated permanent-magnet synchronous machines (FM-PMSMs). According to the Maxwell-Fourier method (viz., the multi-layer model) and Cauchy’s product theorem, the magnetic field distribution is determined in different parts of machine by considering the iron permeability on the basis of the solving both Poisson’s and Laplace’s equations. Next to the direct current (DC) bias flux density and the minor hysteresis loops, which are taken into account, the Bertotti’s model and the flux variation locus (FVL) are employed to calculate the iron core losses under load and no-load conditions in post-processing from the magnetic field analytical calculation. Finally, in order to validate the proposed analytical method, the results are verified by the comparison with finite-element method (FEM). The comparisons show good results of the proposed model.

Index Terms— Convolution theorem, flux-modulated permanent-magnet synchronous machines, iron core losses, DC bias, hysteresis loop, flux variation locus, Maxwell-Fourier method, numerical.

I. INTRODUCTION

The flux-modulated permanent-magnet (PM) synchronous machine (FM-PMSM) is one of the many type of magnetic geared machines, is typically used for wind power systems, because it could provide high-torque applications at low-speed and also characterized by a direct-drive electrical machine without employing the mechanical gearbox [1]-[3].

However, the existence of the flux-modulation layer in the FM-PMSM, augments not only the fundamental wave but also many high-frequency harmonics field and meanwhile a large harmonic iron losses [4]. Therefore, in order to compute the iron losses, the detailed knowledge of the magnetic field distribution in the iron part of this machine is very important.

The finite-element method (FEM) is a powerful tool used for calculate both the spatial and temporal variations of the magnetic flux density and consequently can be analyze the iron losses of electrical machines [5]-[6]. Generally, the iron loss calculation is based on the loss decomposition approach (i.e., hysteresis/eddy-current/excess losses) [7] which can be implemented in the post-processing step of the FEM. Huang et al. (2012) [8] calculated the core iron losses of PMSM with flux variation locus (FVL) and FEM. The result give a good agreement with the experimental ones without the use of any correction factor. The influence of direct current (DC) bias flux density has been taken into consideration by Simão et al. (2009) [9] to calculate the iron core losses under DC magnetic bias, where the classical Steinmetz equation has been improved. Zhang et al. (2017) [10] focused to use the Fourier analysis to predict the iron losses in switched reluctance machine. Usually, the iron losses are computed by FEM from the magnetic field associated to each element in the mesh. However, Chen et al. (2017) [11] divided the core of the motor in different parts representative points are used to fast iron loss calculation and different modified factors are applied to take into account the losses due to minor loops. The temperature impact in the iron losses under different flux densities, frequencies, and DC bias flux density are investigated by Xue et al. (2017) [12], where the improved iron loss model obtained by FEM have been validated by tests on a steel lamination as well as an electrical machine.

Nevertheless, the FEM can provide high precision of iron losses it consumes large computation time. In other hand, the motor designers like better a more fast method. The analytical models deliver the solution to this problem, because they are distinguished by the quick time saving. Many publications have dealt with the iron loss calculations by different analytical models. Sheikh-Ghalavand et al. (2010) [13] and Tariq et al. (2017) [14] developed a magnetic equivalent circuit (MEC) to calculate these losses in linear and interior PMSMs, respectively. Messal et al. (2017) [15] proposed an original approach of iron loss prediction exploiting the dynamic hysteresis model and the magnetic flux density waveforms resulting from generalized nonlinear adaptive MEC using a mesh-based formulation. Tian et al. (2017) [16] used the complex relative air-gap permeance in surface-mounted PMSMs to estimate the iron losses by using the modified Steinmetz equation. Liang et al. (2018) [17] used an analytical approach based on the subdomain technique for the prediction of no-load stator iron losses in spoke-type PMSMs. The iron parts are considered to be infinitely permeable, so that the saturation effect is neglected. Nevertheless, the influence of bridge saturation has been considered by using a nonlinear simple MEC. The average magnetic densities of the tooth and yoke for the iron loss evaluation have been obtained on the basis of the continuity of magnetic flux in the air-gap. Desvaux et al. (2017) [18] used a hybrid method between the subdomain technique and the discretized linear MEC in the pole piece (i.e., modulation layer) for the optimization of coaxial magnetic gear. This enabled them to calculate the magnetic flux density distribution and the

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iron losses in ferromagnetic parts (i.e., internal/external yoke and pole pieces).

Recently, new semi-analytical contributions are developed to take into account the solution of the magnetic field distribution in the iron part. Spranger et al. (2016) [19] developed a first work based on the Maxwell-Fourier method (viz., the multi-layer model) using the convolution theorem (i.e., Cauchy’s product theorem) to determine the solution of the magnetic field distribution in iron parts. The adjacent regions (e.g., rotor or/stator slots/teeth) are assumed to be one homogeneous region with a relative permeability developed as a Fourier series expansion. This method is used in different machines, viz.: switched reluctance machine [20], synchronous reluctance machine [21], and PMSMs [22]-[23]. Meanwhile, Dubas et al. (2017) [24]-[25] developed a first exact subdomain technique in polar and Cartesian coordinates considering finite soft-magnetic material permeability, which has been applied to an air- or air-cored coil supplied by a constant current. The subdomains connection is performed directly in both directions. The general solutions of Maxwell’s equations are deduced by applying the principle of superposition. Recently, this novel scientific contribution has been implemented for radial-flux electrical machines [26]. For the same reason, another technique based on subdomain technique and Taylor polynomial has been applied in spoke-type PMSMs [27].

The infinitely permeable assumption in the iron parts used in the various models that are based on the Maxwell-Fourier method does not give the solution of the magnetic field in these parts of iron, which make the iron core loss calculation be incorrect from the continuity of magnetic flux in the air-gap and in the slots. Because of that, the rate of error in the calculation is significant [16]-[17]. In other hand, the directly calculation of iron core losses have been done successfully in [22]-[23] by divided the stator into different segments. However, the stator division procedure in different segments have been done only in the tangential direction and the iron core losses are calculated with negligent both of the iron losses in the rotor part, the DC bias flux density and the minor hysteresis loops, which make the calculation of the iron core losses relatively inaccurate. Moreover, the losses have been obtained at no-load condition and only for one rotor speed. Therefore, the accuracy of analytical model for the calculation of this losses type have not well explained for this different condition.

In this paper, a new analytical model of FM-PMSM is developed for the iron core loss computation with different rotor speed and in both conditions (i.e., no-load and on-load of armature reaction field), the DC bias flux density and the minor hysteresis loops have been taken into account. The developed analytical model is based on the Maxwell-Fourier method (viz. the multi-layer model) and Cauchy’s product theorem. The magnetic field distribution in all regions of FM-PMSM are predicted. The iron core losses are calculated by divided the core of the machine in different parts (i.e., in radial and tangential direction) and by used FVL method and Bertotti’s model. Finally, all the results from the proposed analytical model are validated by the FEM [28].

II. MOTOR CONFIGURATION

The topology of the studied FM-PMSM with two air-gap is given in [3] and is shown in Fig. 1. It includes: i) an inner stator with \( Q_s = 12 \) slots and three-phases (\( q = 3 \)) non-overlapping winding with all teeth wound (viz., the double-layer concentrated winding distribution), which give \( p_s = Q_s / q = 4 \) stator pole-pair number, and ii) an outer rotor PMs surface-mounted by radially magnetized patterns with \( p = 17 \) rotor pole-pair number.

For the modulation layer, there are \( Q_n = p + p_s = 21 \) pole pieces placed between the stator and the rotor. The gear ratio \( G_r = -p / p_s = -4.25 \). The relation between the stator rotary field frequency \( f_e \) and the machine mechanical rotation frequency \( F \) is \( F = f_e / G_r \) with \( f_e = f_{el} / p_e \) where \( f_{el} \) is the electrical frequency. In order to get 50 Hz currents in the stator armature, then the stator rotary field frequency will be \( f_e = 12.5 \) Hz (i.e., 750 rpm). Therefore, the outer-rotor frequency it must be to 2.94 Hz (i.e., 176 rpm) in the opposite direction.

However, in the configuration presented in Fig. 1, the stator winding is placed in the clockwise direction and the rotor is rotated in the anti-clockwise direction. Thus, we take the value absolute of the gear ratio \( |G_r| \) during the calculations procedures (i.e., the indication of the rotation direction are taken physically not mathematically).

![Fig. 1. Configuration of the studied FM-PMSM.](image)

![Fig. 2. Studied FM-PMSM (a) Simplified model and (b) Different areas and feature points for magnetic flux density and iron loss calculation.](image)
III. PROBLEM FORMULATION ASSUMPTIONS
The model is subedit in two-dimensional polar coordinate system and has been partitioned into 8 regions as shown on Fig. 2(a). Region I and VIII represented the stator and rotor yoke respectively, Region II represented the stator slots and teeth, Region III represented the stator isthmus-opening and tooth-tips, Region IV and VI are represented the inner and outer air-gap respectively, Region V represented the modulation layer, and Region VII represented the PMs. During each region, the magnetic field is assumed periodic. The model is formulated in magnetic vector potential with the following assumptions:

• The magnetic materials are considered as isotropic with constant magnetic permeability corresponding to linear zone of the B(H) curve;
• All electrical conductivities of materials are supposed as nulls (i.e., the eddy-current reaction field in the materials are neglected);
• The stator slots/teeth have radial sides;
• The end-effects are neglected.

The angular position of the \(i_s^m\) modulation layer elements (i.e., Region V) and \(i_r^n\) stator elements (i.e., Region II and Region III) are defined respectively by

\[
\alpha_s^s = \frac{2\pi}{q_s}(i_s - 1) \tag{1}
\]
\[
\alpha_r^n = \frac{2\pi}{q_r}(i_r - 1) \tag{2}
\]

IV. ANALYTICAL MODEL
Magnetic field calculation in all regions is calculated from by solving the following Laplace’s and Poisson’s matrix equations

\[
\frac{\partial^2 A_k^s}{\partial r^2} + \frac{1}{r} \frac{\partial A_k^s}{\partial r} - \left(\frac{V^k}{r}\right)^2 A_k^s = 0, \quad k = I, III, IV, V, VI, VII and VIII
\]
\[
\begin{cases}
-\mu_0^k \tilde{A}_k, & k = II \\
-j \frac{\mu_0^k}{r} N \tilde{M}_r, & k = VII
\end{cases}
\]

where

\[
N = \text{diag}[N_1 \ldots N]
\]
\[
V^k = \begin{pmatrix} N_1 & \cdots & N_k \end{pmatrix}
\]
\[
M_r = \text{diag}\left(\tilde{M}_{r,n} \cdots \tilde{M}_{r,1}\right)
\]

\[
\tilde{M}_{r,n} = \frac{2p B_{\text{perm}}}{n \eta 
\tilde{M}_{r,n} = \frac{2p B_{\text{perm}}}{n \eta}
\]
\[
J_z = \begin{bmatrix} J_{z,-n} & \cdots & J_{z,n} \end{bmatrix}^T
\]

where

\[
\begin{cases}
\frac{1}{2n \eta} \sum_{m=1}^{2n} l_{m} e^{-j m \frac{\pi}{2}} (e^{j m \frac{\pi}{2}} - 1) \\
+ j \tilde{A}_k \left(1 - e^{-j m \frac{\pi}{2}} \right) e^{j m \alpha_s} n \neq 0 \\
0 \quad n = 0
\end{cases}
\]

where \(j = \sqrt{-1}\); \(n = 1, 2, \ldots, N\) is spatial harmonic orders; \(I_{j,1} \& I_{j,2}\) are the stator current densities of slots for two tangential parts for three-phases current are given in Appendix.

Using the separation of variables method in \(r, \theta\), the general solution of (3) for each regions are formulated as

\[
A_k^s |_r = \left(\frac{r}{R_s}\right)^{\frac{k}{2}} a^l + \left(\frac{R_s}{r}\right)^{\frac{k}{2}} b^l \tag{10}
\]
\[
A_k^s |_r = W^\mu \left(\frac{r}{R_s}\right)^{\frac{k}{2}} a^m + \left(\frac{R_s}{r}\right)^{\frac{k}{2}} b^m \tag{12}
\]
\[
A_k^s |_r = \left(\frac{r}{R_s}\right)^{\frac{k}{2}} a^v + \left(\frac{R_s}{r}\right)^{\frac{k}{2}} b^v \tag{15}
\]

\[
\begin{align}
A_k^s |_r &= \left(\frac{r}{R_s}\right)^{\frac{k}{2}} a^\nu + \left(\frac{R_s}{r}\right)^{\frac{k}{2}} b^\nu \tag{17}
\end{align}
\]

where \(a^l \& b^l\) are the column vectors of the unknown coefficients, \(W^k\) is the diagonal eigenvalue matrix of \(V^k\). \(\lambda^k\) is the diagonal eigenvalue matrix of \(V^k\) if \(k = II, III\) and \(V\) or \(\lambda^k = |N|\) if \(k = I, IV, VI, VII\) and \(VIII\), and \(I\) is a diagonal identity matrix with same size as \(N\).

The application of the Dirichlet boundary conditions to the inner boundary of Region I (\(A_k^s |_{r=R_s} = 0\)) and the outer boundary of Region VIII (\(A_k^s |_{r=R_9} = 0\)), (10) and (17) are reduced to

\[
A_k^s |_r = \left(\frac{r}{R_s}\right)^{\frac{k}{2}} a^l + \left(\frac{R_s}{r}\right)^{\frac{k}{2}} b^l \tag{18}
\]
\[
\begin{align}
A_k^s |_r &= \left(\frac{r}{R_s}\right)^{\frac{k}{2}} a^\nu + \left(\frac{R_s}{r}\right)^{\frac{k}{2}} b^\nu \tag{19}
\end{align}
\]

where \(k\) represent the all regions, and \(\mu_{c, inv}\) is the convolution matrices of the inverse magnetic permeability in the Region \(II, III\) and \(V\) given by [19] and [22].

\[
\mu_{c, inv} = \begin{bmatrix} \mu_0^{inv} \cdots \mu_0^{inv} \\
\vdots \vdots \\
\mu_0^{2n} \cdots \mu_0^{2n} \end{bmatrix}
\]
where \( \{ \theta_{s}, \theta_{f}, Q, i, a_{i} \} \) are replaced in the second region by \( \{ \theta_{s}, \theta_{f}, Q, i, a_{i}^{f} \} \), in the third region by \( \{ \theta_{tp}, \theta_{st}, \theta_{s}, \theta_{f}^{2} \} \) and in the fifth region by \( \{ \theta_{p}, \theta_{ps}, Q_{n}, i, a_{ip}^{n} \} \). the naming of this opening angle are given in Table I. In the other regions \( \mu_{c, inv} = [\mu_{iron}]^{-1} \) for \( k = I \) and \( VIII \), and \( \mu_{c, inv} = [\mu_{p}]^{-1} \) for \( k = IV, VI \) and \( VII \).

The system of fourteen \( [B_{regions} \times 2] \) boundary conditions matrix equations (20) and (21) could be used to determine the unknown coefficients of magnetic vector potentials in the all regions according to [20]. Then, the radial and tangential magnetic flux density \( B_{r} \) and \( B_{0} \) in the all regions can be deduced from the magnetic vector potentials matrix by

\[
B_{r}|_{r} = -j \frac{1}{r} \mathbf{N} \mathbf{A}_{2}|_{r}
\]

and with \( N_{s} \) points in the tangential direction by

\[
B_{r}(r, \theta) = \mathbb{R}e \left( \mathbf{N}_{r}^{T} \mathbf{e}^{-jN_{s} \theta} \right)
\]

and with \( N_{s} \) points in the tangential direction by

\[
B_{0}(r, \theta) = \mathbb{R}e \left( \mathbf{N}_{0}^{T} \mathbf{e}^{-jN_{s} \theta} \right)
\]

where

\[
\mathbf{N}_{r} = \text{repmat}(-\theta, 2N + 1)
\]

and \( \mathbf{N}_{0} = \text{repmat}(-\theta, 2N + 1) \)

\[
\mathbf{N}_{r} = \left[ \begin{array}{cccc}
-\theta_{1} & -\theta_{2} & \cdots & -\theta_{N_{s}}\\
-\theta_{1} & -\theta_{2} & \cdots & -\theta_{N_{s}}\\
\vdots & \vdots & \vdots & \vdots \\
-\theta_{1} & -\theta_{2} & \cdots & -\theta_{N_{s}}
\end{array} \right] \quad 1
\]

and

\[
\mathbf{N}_{0} = \left[ \begin{array}{cccc}
\theta_{1} & \theta_{2} & \cdots & \theta_{N_{s}}\\
\theta_{1} & \theta_{2} & \cdots & \theta_{N_{s}}\\
\vdots & \vdots & \vdots & \vdots \\
\theta_{1} & \theta_{2} & \cdots & \theta_{N_{s}}
\end{array} \right] \quad 2
\]

and

\[
\theta_{1} = 0 \quad (rad)
\]

\[
\theta_{2} = \theta_{1} + \Delta \theta
\]

\[
\theta_{N_{s}} = 2\pi \quad (rad)
\]

\[
\theta_{N_{s}} = 2\pi / (N_{s} - 1)
\]

V. VALIDATION OF MAGNETIC FIELD DISTRIBUTION BY THE FEM

To verify the validity of the proposed model, the spatial variations of magnetic flux densities (i.e., radial and tangential components of magnetic field distribution) in the inner and outer air-gap as well as the temporal variations of magnetic flux densities at three different points (green color in Fig. 2(b)) were calculated with the analytical model and compared with those obtained from the FEM. The main parameters of the FM-PMSM can be found in Table I.

A. Comparison of Spatial Magnetic Field Distribution

Magnetic flux density waveforms in the inner and outer air-gap are shown in Fig. 3 and Fig. 4, which are the function of the tangential position \( \theta \), for no-load and on-load condition, respectively (i.e., with only PM and with both PM and armature reaction field) through \( l_{c} = \bar{I} \sin(0) \), \( l_{b} = \bar{I} \sin(-2\pi/3) \) and \( l_{c} = \bar{I} \sin(2\pi/3) \) where \( \bar{I} = 16 A \).

The computation time required for the magnetic field calculation in linear analytical model and FEM (linear and nonlinear) of FM-PMSM in one rotor position are shown in Table II. In linear analytical model, the resulting system of equations has 1,608 and 2,408 unknown coefficients for \( N = 100 \) and \( N = 150 \) respectively. The linear and nonlinear FEM for 6,380 boundary and 119,108 elements of first order.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Parameters</th>
<th>Values (Units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_{r, rem} )</td>
<td>PMs remanence flux density</td>
<td>1.2 (T)</td>
</tr>
<tr>
<td>( \mu_{r} )</td>
<td>PMs relative permeability</td>
<td>1</td>
</tr>
<tr>
<td>( Q_{s} )</td>
<td>Number of stator slots</td>
<td>12</td>
</tr>
<tr>
<td>( p )</td>
<td>Number of pole-pairs</td>
<td>17</td>
</tr>
<tr>
<td>( Q_{n} )</td>
<td>Number of modulation layer</td>
<td>21</td>
</tr>
<tr>
<td>( N_{e} )</td>
<td>Conductor number of slot coil</td>
<td>72</td>
</tr>
<tr>
<td>( w_{r} )</td>
<td>Rotor speed</td>
<td>18.47 (rad/s)</td>
</tr>
<tr>
<td>( R_{1} )</td>
<td>Shaft Radius</td>
<td>45 (mm)</td>
</tr>
<tr>
<td>( R_{2} )</td>
<td>Inner radius of the slot</td>
<td>70 (mm)</td>
</tr>
<tr>
<td>( R_{3} )</td>
<td>Inner radius of the slot- openings</td>
<td>90 (mm)</td>
</tr>
<tr>
<td>( R_{4} )</td>
<td>Radius of the stator surface</td>
<td>93 (mm)</td>
</tr>
<tr>
<td>( R_{5} )</td>
<td>Inner radius of the modulation layer</td>
<td>94 (mm)</td>
</tr>
<tr>
<td>( R_{6} )</td>
<td>Outer radius of the modulation layer</td>
<td>104 (mm)</td>
</tr>
<tr>
<td>( R_{7} )</td>
<td>Inner radius of the PMs</td>
<td>105 (mm)</td>
</tr>
<tr>
<td>( R_{8} )</td>
<td>Rotor bore radius</td>
<td>111 (mm)</td>
</tr>
<tr>
<td>( R_{9} )</td>
<td>Radius of the external rotor surface</td>
<td>125 (mm)</td>
</tr>
<tr>
<td>( L_{s} )</td>
<td>Stack length</td>
<td>100 (mm)</td>
</tr>
<tr>
<td>( \theta_{s} )</td>
<td>Stator slot angle</td>
<td>1.25(\pi/Q_{s}) (rad)</td>
</tr>
<tr>
<td>( \theta_{ts} )</td>
<td>Stator teeth opening angle</td>
<td>2(\pi/Q_{s}) - \theta_{s} (rad)</td>
</tr>
<tr>
<td>( \theta_{ta} )</td>
<td>Stator slot opening angle</td>
<td>(1/3)\theta_{b} (rad)</td>
</tr>
<tr>
<td>( \theta_{tp} )</td>
<td>Stator teeth tips opening angle</td>
<td>2(\pi/Q_{s}) - \theta_{ta} (rad)</td>
</tr>
<tr>
<td>( \theta_{ml} )</td>
<td>Modulation layer slot opening angle</td>
<td>\pi/Q_{n} (rad)</td>
</tr>
<tr>
<td>( \theta_{mla} )</td>
<td>Modulation layer opening angle</td>
<td>2(\pi/Q_{s}) - \theta_{ml} (rad)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>COMPUTATION TIMES.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Analytical Model</td>
<td>FEM</td>
</tr>
<tr>
<td>( N=100 )</td>
<td>( N=150 )</td>
</tr>
<tr>
<td>Time [sec]</td>
<td>2.71</td>
</tr>
</tbody>
</table>
For $N_{p} = 500$ points and using $N = 100$ spatial harmonics, the resulting system of equations has $[(2N + 1)B_{\text{regions}}]$ unknown coefficients. Fig. 5 shows another presentation of the magnetic field distribution in all machines regions where the magnitude of $\mathbf{B}$ is plotted. It is observed that the analytical model has excellent similar accuracy to the FEM. Instead of the iron core relative permeability value that are used in the other results ($\mu_{r,\text{iron}} = 4,000$), Fig. 6 shows the capability of the model to give good results of two different values of the iron core relative permeability (100 and 4,000). The saturation effect appears clearly in the results of the radial and tangential component of the magnetic flux density in the inner air-gap [see Fig. 6(a) and 6(b)]. However, in Fig. 6(c) and 6(d) the saturation effects are less in the outer air-gap region, due to proximity of this region to the PM region. The comparison between the linear analytical model and the nonlinear FEM is illustrated in Fig. 7, where the radial magnetic flux density in the inner and outer air-gap has been presented under load condition. The resultants show a strong convergence between the two models. This is because the type of studied machine which is working at the linear part of $B(H)$ curve.
For the modulation layer, in the Fig. 8(b) and Fig. 9(b) the comparison shows analytical and FEM waveforms of the radial and tangential magnetic flux density matches very well except little phase shift in the tangential component can be well observed in the Fig. 10(b) and Fig. 11(b), because the Gibbs’ phenomenon contributes to the error. The same problem have bas been observed before by Liang et al. [17]. However, the influence of the tangential magnetic flux density on the iron losses is much smaller than the radial component.

![Graphs showing magnetic flux density comparison](image)

Fig. 7. Radial magnetic flux density of FM-PMSM with linear analytical model and nonlinear FEM at on-load condition in the middle of the (a) inner air-gap, and (b) outer air-gap.

![Graphs showing radial and tangential components](image)

Fig. 8. No-load evolution in the radial and tangential components of magnetic flux density for a point on the: (a) stator, (b) modulation layer, and (c) rotor.

Fig. 10(c) and Fig. 11(c) show the waveforms and loci comparison of the results in the rotor, respectively, the analytical and FEM waveforms of the both radial and tangential magnetic flux density are identical with different frequency to the other results in the fixed element (i.e., stator and modulation layer). We can see the exist of the dc bias flux density in radial component. This is because the presence of PM in the rotor part. Nevertheless, the magnetic flux density values are small compared to the other values in the other part of the machine.
Fig. 9. On-load evolution in the radial and tangential components of magnetic flux density for a point on the: (a) stator, (b) modulation layer, and (c) rotor.

Fig. 10. Loci of magnetic flux density at no-load condition for a point on the: (a) stator, (b) modulation layer, and (c) rotor.

Fig. 11. Loci of magnetic flux density at on-load condition for a point on the: (a) stator, (b) modulation layer, and (c) rotor.

VI. IRON LOSS COMPUTATION

For a no-sinusoidal excitation, the calculation of iron core losses \( P_{\text{iron}} \) in the FM-PMSM is defined by the Bertotti’s model expressed in three contributions [7], taking into account the influence of minor hysteresis loops [30] and the DC bias flux density [9]. The iron core losses is expressed by:

\[
P_{\text{iron}} = P_{\text{hys}} + P_{\text{edd}} + P_{\text{ex}} = k_{\text{hys}} \cdot f \left( \frac{B_m}{\mu_0} \right)^2 \cdot C_f \cdot \epsilon(B_{dc}) + \frac{\sigma d_i f}{12} \int_0^T \left( \frac{dB(t)}{dt} \right)^2 dt
\]

where \( P_{\text{hys}} \), \( P_{\text{edd}} \), and \( P_{\text{ex}} \) are respectively the hysteresis, eddy-current and excess losses; \( B \) is the iron core magnetic density; \( B_m \) is the peak value of the magnetic flux density in the iron core; \( T = 1/f \) is the iron core magnetic flux density period with \( f \) the frequency; \( k_{\text{hys}} \), \( k_{\text{ex}} \), and \( \alpha = 2 \) are respectively the coefficient of hysteresis losses, excess losses and the Steinmetz constant; \( \sigma \) is the electrical conductivity; and \( d_i \) is the laminations thickness. The correction factor \( C_f \) used to take the total loss depend on the magnitude of every local minor loops is given by:

\[
C_f = 1 + \frac{k}{B_{\text{irrot}}} \sum_{i=1}^{n_i} \Delta B_i
\]

where \( k \) a coefficient in the range 0.6 to 0.7, \( n_i \) is the number of minor loops and \( \Delta B_i \) is the magnitude of the \( i \)th minor loop are calculated by working on all the local maximum and minimum points in the flux density waveforms [see Fig. 18].

The correction factor \( \epsilon(B_{dc}) \) used to take the influence of dc bias flux density [see Fig. 18(b)] is defined as

\[
\epsilon(B_{dc}) = 1 + k_{\text{dc}}B_{dc}^\alpha
\]

where \( B_{dc} \) is the DC bias flux density, \( k_{\text{dc}} \) and \( \alpha_{dc} \) are the DC bias coefficients and can be obtained by fitting the measured results.

A. Flux Variation Locus Method

Because the iron core loss is affected not only by the alternating flux, but also the rotational flux in the FM-PMSM. It is useful to use the FVL method where the rotational flux effect can also be taken into account [8]. Fig. 12 show the elliptical locus in the rotating machine, where \( B_T \) and \( B_L \) are respectively the major and minor of magnetic flux density which can be determined from \( \{ B_T; B_L \} \) by using (39) and the algorithm shown in Fig. 13. The angle \( \phi \) are varied from 0° to 180° with a step 0.5°, then \( \{ B_T; B_L; \gamma \} \) corresponding to \( \{ B_{r,\phi}; B_{\theta,\phi}; \phi \} \) when \( B_{r,\phi} \) equal the minimum value from the all maximum values (\( B_{\theta,\phi} \)) of \( B_{r,\phi} \).

\[
\begin{bmatrix} B_{r,\phi}(t) \\ B_{\theta,\phi}(t) \end{bmatrix} = M^{\text{rot}}(\phi) \begin{bmatrix} B_r(t) - B_{r,\text{aver}(t)} \\ B_\theta(t) - B_{\theta,\text{aver}(t)} \end{bmatrix}
\]

\[
M^{\text{rot}}(\phi) = \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix}
\]

Fig. 12. Elliptical locus for the iron loss evaluation.
The application of FVL method is carried out by the algorithm presented in Fig. 13 to the temporal magnetic field variations at three different points in the FM-PMSM presented in Fig. 8 11 are shown in Fig. 14 17. The algorithm give a good results in both models (i.e., analytical and FEM) and the little phase shift in the tangential component which appeared in Fig. 10(b) and Fig. 11(b) has disappeared [see Fig. 16(b) and Fig. 17(b)].

Fig. 16. Loci of magnetic flux density at on-load condition for a point on the: (a) stator, (b) modulation layer, and (c) rotor.

Fig. 17. Loci of magnetic flux density at no-load condition for a point on the: (a) stator, (b) modulation layer, and (c) rotor.

Fig. 18. Hysteresis loop with minor loops (a) without DC bias flux density and (b) with DC bias flux density.

In (39), the ripples values of \{B_r; B_θ\} are used by subtracted their average values to calculate the iron losses. This case appears in the rotor part where the average values of \{B_r\} are not null [see Fig. 8(c), Fig. 9(c), Fig. 10(c) and Fig. 11(c)].
The iron core losses are calculated by taken into account the rotational flux effect in the FM-PMSM, in different areas $S_{np}$ [see Fig. 2(b)] where $n_p = 1, 2 \ldots N_p$ from the magnetic flux density in different points $a_{np}$ where (35) ~ (36) are extended to

$$P_{hys} = k_{hys} V_{np} f \left[ B_{r,m} \alpha C_{f.r.c} \varepsilon(B_{r,dc}) \right]$$

$$P_{edd} = \frac{\sigma d t}{12} V_{np} f \left[ \left( \frac{dB_1(t)}{dt} \right)^2 + \left( \frac{dB_2(t)}{dt} \right)^2 \right]$$

$$P_{ex} = k_{ex} V_{np} f \left[ \left( \frac{dB_1(t)}{dt} \right)^2 + \left( \frac{dB_2(t)}{dt} \right)^2 \right]$$

where $V_{np} = S_{np} L_u$ are the different volume for different iron parts loss calculation, and $B_{LM}$ & $B_{L,m}$ are respectively the peak values of the major and minor magnetic density in the iron core.

VII. VALIDATION OF IRON LOSSES BY THE FEM

Using the coefficients are given in Table III, the iron losses are calculated under no-load and on-load condition with three-phases sinusoidal currents where $w = 2\pi f_e$, $I$ varied from 0 to 16 A. The rotor speed are varied from the half to seven times of (176 rpm). The Bertotti’s model are also used by FEM. Fig. 19 and Fig. 20 show the variation of hysteresis, eddy current and excess loss with speed in the no-load and on-load condition, respectively. It is observed that the analytical results are agrees well with the FEM despite a little difference where the analytical results are a little bigger than the FEM one in the high speed.

![Fig. 19. No-load analytically and FEM predicted iron losses: (a) Hysteresis, (b) Eddy-current, and (c) Excess loss.](image1)

![Fig. 20. On-load analytically and FEM predicted iron losses: (a) Hysteresis, (b) Eddy-current, and (c) Excess loss.](image2)

![Fig. 21. Iron loss evolution in the various ferromagnetic parts (modulation layer, rotor and stator) at (a) no-load and (b) on-load condition.](image3)

In Fig. 21(a) and (b), the iron loss evolutions in the modulation layer, rotor and stator according to rotor speed in both conditions (i.e., no-load and on-load of armature reaction field) are illustrated, respectively. As expected, in the no-load condition the iron losses are higher in the modulation layer than in the stator and very small in the rotor. However, in the on-load condition one can observe that the iron losses in the stator are increased and in the other (i.e., rotor and modulation layer), the iron losses are not much affected. The comparison between the proposed analytical model and FEM show a good agreement in the no-load condition [see Fig. 21].

The iron core losses in the FM-PMSM are shown in Fig. 22(a) according to supply stator current windings with three rotor speed (viz., 176, 617 and 1,235 rpm), and in Fig. 22(b) according to the rotor speed with three different supply current (viz., 0, 8, and 16 A). A good agreement can be seen between the linear analytically obtained and the linear FEA results. In the nonlinear FEM can see a little error in the iron core losses compared to the two linear models results with 6.4 % and 5.2 % at 1,235 rpm and 16 A for linear analytical model and linear FEM respectively [see Fig. 21]. However, the error is very small especially if we take into consideration, the small calculation time taken by the linear models compared to the calculation time taken by the nonlinear FEM [see Table II].
Maxwell-Fourier method (viz. the multi-layer model) and Cauchy’s product theorem. The FM-PMSM is divided into eight regions and standard boundary conditions are used to solve the Poisson’s and Laplace’s equations. The magnetic field distribution in all regions of machine are predicted.

According to Bertotti’s model, and taking into account the minor hysteresis loops and the DC bias flux density, the FVL method are used in the post-processing procedure to produce the iron losses in the different parts of machine. The results showed that the analytical method give satisfactory results with FEM, which confirms the validity of the proposed model. It can be noted that the iron core losses are affected by the increase in the supply current and/or rotor speed as the results showed.

The next step will be extended to calculate the other losses, energy balance sheet and electromagnetic performances of this machine to be used in optimization procedures, in order to start making the proceeding to fabrication and manufacture of FM-PMSM for wind power applications.

APPENDIX

The stator current densities of slots for two tangential parts for three-phases current are defined as

\[ J_{(1)} = \frac{N_c}{S} \mathbf{C}_{(1)}^T [i_a \ i_b \ i_c] \]  
\[ J_{(2)} = \frac{N_c}{S} \mathbf{C}_{(2)}^T [i_a \ i_b \ i_c] \]

where \{i_a; \ i_b; \ i_c\} are the armature currents of three-phases, \(N_c\) is conductor number of slot coil, \(S = \theta_s (R_2^2 - R_1^2) / 4\) is the surface of the stator slot coil in kth region, and \(\mathbf{C}_{(1)}^T \ & \mathbf{C}_{(2)}^T\) are the transpose of the connection matrix linking the three-phases and the stator slots that represent the distribution of stator windings in the slots. The connection matrices \(\mathbf{C}_{(1)} \ & \mathbf{C}_{(2)}\) are given by

\[ \mathbf{C}_{(1)} = [\mathbf{C} + [\mathbf{C}] ] / 2 \]
\[ \mathbf{C}_{(2)} = [\mathbf{C} - [\mathbf{C}] ] / 2 \]
\[ \mathbf{C} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix} \]

These connection matrices can be generated automatically by using ANFRACTUS TOOL developed in [29].

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