Analysis of an Integrated 4-DoF Parallel Wrist for Dexterous Gripping

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***Abstract*— The paper presents the kinematic analysis of an integrated four degrees of freedom (DoFs) parallel surgical wrist. This robot is capable of grasping/cutting thanks to a folding platform that is fully controlled by external actuators. To formally investigate the mobilities of the proposed structure, the equivalent twist graph concept is employed to characterize the constraints and the actuation wrenches that are applied to the configurable moving platform. Moreover, screw theory is used to formulate the Jacobian matrix in a simple and integrated form under a unified framework. This approach adapts and extends the application of screw theory to robots with configurable platforms, whose configuration is not described by a Special Euclidian group (SE3) element. Finally, an insight of singular configurations is presented.**

I. Introduction

During the three last decades, many parallel mechanisms have been proposed with different degrees of freedom (DoFs). PMs have attracted much attention from academic and industrial communities due to their notable characteristics, such as high stiffness, load-to-weight ratio, and dynamic performances. However, compared to serial manipulators, PMs suffer from a limited workspace volume, complex control, and more singularities inside the accessible workspace.

To identify the capabilities of the PMs, many approaches have been used in the structural analysis of the PMs' motion of the moving platform: constraint methods [1], evolutionary morphology [2] [3], group theory [4] [5], and screw theory [6] [7] [8].

For instance, based on the theory of reciprocal screws [9], Amine *et al*. analyzed the synthesis of a 4-DoF manipulators (3T1R) [10]. Grassmann-Cayley algebra (GCA) was used to obtain the geometric singularity conditions for these manipulators. Hao and McCarthy used screws to define conditions for line-based singularities for parallel manipulators of the Gough-Stewart type [11]. One can note that in all the foregoing works, the end-effector was considered as a rigid platform.

For manipulators with a configurable platform, many works have been proposed on the analysis of singularities of the H4 robot using screw theory [12] [13] [14]. Inspired by the design of H4, a systematic method was developed in [15] for the type synthesis of 4-DoF parallel mechanisms 3T1R .

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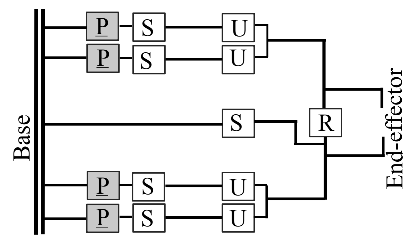

Fig. 1. Architectural scheme of the manipulator (from [18]). The joints are represented by rectangles and the links between those joints are represented by lines. Let P, R, U, and S represent prismatic, revolute, universal, and spherical joints, respectively.

Mohamed and Gosselin developed and analyzed kinematically redundant parallel manipulators with a closed loop configurable platform designed for grasping tasks [16]. In 2014, Lambert *et al* presented some results regarding the mobility, over-constraints, Jacobian matrix, singularities, and type synthesis of parallel robots with a closed loop configurable platform via graph theory and screw theory [17].

This paper focuses on the analysis of motion of an original 4-DoFs parallel wrist dedicated to minimally invasive surgery presented in [18]. The wrist belongs to a new family of parallel robots with a foldable platform. Its architectural scheme is given in Fig. 1. The wrist is composed of four moving legs connected to a common base and linked to an end-effector. Each leg is driven by a linear actuator fixed on the base. A central leg, also fixed on the base and connected to one part of the foldable platform through a spherical joint, is used to constrain the three translations of the manipulator. The configurable platform is composed of two bodies linked by a revolute joint. The basic concept of the wrist is described by a schematic drawing presented in Fig. 2.

The manipulator can provide the three DoFs in rotation in a compact and stiff design. This new robotic wrist can undertake the tasks of dexterous grasping or cutting the target tissues inside the human body via two fingers or scissors located on both sides of the foldable platform.

On the basis of the screw and reciprocal theory [10], the constraint conditions of this manipulator, also known as the platform constraint system, are analyzed to possess a given number of DoFs[1]. Similarly, infinitesimal

[1]The criterion of Grübler-Kutzbach, used by a great number of engineers to formulate the mobility of serial mechanisms, fails to analyze some over-constrained mechanisms [19]. In fact, the geometric arrangement of the constraints imposed by the joints in space must be considered.

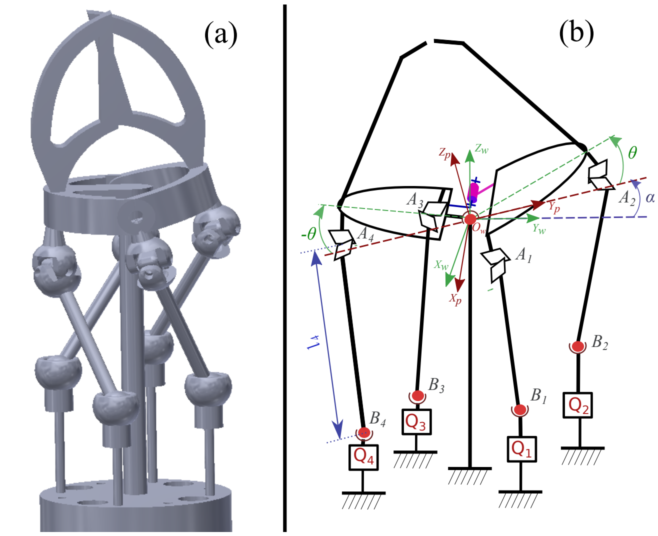

Fig. 2. (a) Schematic drawing of the manipulator. (b) The kinematic architecture of the wrist (from [18]).

kinematics by means of reciprocal screw theory is developed and used to formulate the Jacobian matrix. This manipulator class cannot be presented by the parameters of a simple twist. As a matter of fact, each part of the end-effector could be presented by an attached frame $\mathcal{F} \in \mathrm{SE}(3)$. Hence, the twists associated with each part of the platform are combined to get a single matrix form for the Jacobian. For such manipulators, the concept of screw theory is introduced to simplify the parametrization of the constraints of the manipulator and allows better designing for applications such as minimally invasive surgery.

II. Constraint analysis

The architecture has four leg structures and a central bar connected via a spherical joint to the left part of the platform shown in Fig.2. For convenience, we define the following parameters for describing the geometry of the kinematic model. As shown in Fig. 2, the global coordinate system is located at the center of the spherical joint of the central leg $\mathcal{F}_w = (O_w, \mathbf{x}_w, \mathbf{y}_w, \mathbf{z}_w)$. Let $\mathcal{F}_p = (O_w, \mathbf{x}_p, \mathbf{y}_p, \mathbf{z}_p)$ represent the frame defined by O_w, $\mathbf{x}_p$, and the symmetrical axis $\mathbf{z}_p$ between the left and the right parts of the platform.

The struts have a total length l_i, where $i = 1, .., 4$ denotes the legs. The left part of the configurable platform is connected to the right part via a revolute joint, and we define the relative rotation angle by θ. The central strut is attached to the left part of the platform via a spherical joint and fixed to the base on the other end. The other struts are attached on one end to the platform through a universal joint ${}^{w}A_i$ and to the moving rod on the other end via a spherical joint ${}^{w}B_i$. The configurable platform carries the two-finger gripper. Each rod is attached to a linear actuator placed remotely from the wrist.

The configuration of the platform is defined by the angular folding value θ, and the orientation of the platform that is identified by the three angles roll, pitch, and yaw, (α, β, γ).

A spatial mechanism consists of links and kinematic joints. All kinematic joints can be expressed as different screws; let ε denote a twist and τ denote a wrench. A zero pitch twist ε_0 represents a pure rotation corresponding to a revolute joint, and an infinite pitch twist ε_∞ represents a pure translation of a prismatic joint. A pure moment is represented by an infinite pitch wrench τ_∞ and a pure force is represented by a zero pitch wrench τ_0. Therefore, a spherical joint can be expressed by the combination of three non-coplanar revolute joints, and the universal joint is the combination of two revolute joints.

In this section, to validate the mobility of the wrist, first, the actuators are considered as passive joints. Hence, the DoFs of the foldable platform are obtained by means of screw theory. Then, the actuators are blocked, so the manipulator must be fully constrained and cannot provide any motion.

A. *Constraint wrench system*

This section shows that the proposed parallel wrist produces the three rotations and a relative DoF in rotation between two parts of the platform along $\mathbf{x}_p$.

Every spherical joint is considered as three intersecting non-coplanar revolute joints $\hat{\varepsilon}_{0i2}$, $\hat{\varepsilon}_{0i3}$, and $\hat{\varepsilon}_{0i4}$. Let $\hat{\varepsilon}_{ij}$ be a unit twist associated with the j^{th} joint of the i^{th} leg. The axis of $\hat{\varepsilon}_{0i2}$ is aligned with the $\mathbf{x}_w$ axis ; the axis of $\hat{\varepsilon}_{0i4}$ is aligned with the longitudinal axis of the leg; and the axis of $\hat{\varepsilon}_{0i3}$ is perpendicular to both $\hat{\varepsilon}_{0i2}$ and $\hat{\varepsilon}_{0i4}$. Let $\hat{\varepsilon}_{\infty i1}$ denote a pure translation associated with the direction of the linear actuator.

Overall, there are six joint screws associated with each leg (for $i = 1, .., 4$), and they could be presented as $\hat{\varepsilon}_{\infty i1} = \begin{bmatrix} 0 \\ \mathbf{s}_{i1} \end{bmatrix}$, $\hat{\varepsilon}_{0i2} = \begin{bmatrix} \mathbf{s}_{i2} \\ \mathbf{r}_{B_i} \times \mathbf{s}_{i2} \end{bmatrix}$, $\hat{\varepsilon}_{0i3} = \begin{bmatrix} \mathbf{s}_{i3} \\ \mathbf{r}_{B_i} \times \mathbf{s}_{i3} \end{bmatrix}$, $\hat{\varepsilon}_{0i4} = \begin{bmatrix} \mathbf{s}_{i4} \\ \mathbf{r}_{B_i} \times \mathbf{s}_{i4} \end{bmatrix}$, $\hat{\varepsilon}_{0i5} = \begin{bmatrix} \mathbf{s}_{i5} \\ \mathbf{r}_{A_i} \times \mathbf{s}_{i5} \end{bmatrix}$ and $\hat{\varepsilon}_{0i6} = \begin{bmatrix} \mathbf{s}_{i6} \\ \mathbf{r}_{A_i} \times \mathbf{s}_{i6} \end{bmatrix}$,

where $\mathbf{s}_{ij}$ denotes a unit vector with $\mathbf{s}_{i1} = [0\ 0\ 1]^t$, $\mathbf{s}_{i2} = [1\ 0\ 0]^t$, $\mathbf{s}_{i4} = (\mathbf{r}_{A_i} - \mathbf{r}_{B_i})/||\mathbf{r}_{A_i} - \mathbf{r}_{B_i}||$, $\mathbf{s}_{i3} = \mathbf{s}_{i2} \times \mathbf{s}_{i4}$, $\mathbf{s}_{15} = \mathbf{s}_{35} = \mathbf{x}_p$, $\mathbf{s}_{25} = \mathbf{s}_{45} = \mathbf{y}_p$, and $\mathbf{s}_{i6} = \mathbf{s}_{i4} \times \mathbf{s}_{i5}$. Moreover, $\mathbf{r}_{A_i}$ and $\mathbf{r}_{B_i}$ are, respectively defined as the vectors directed along the axis $\overrightarrow{O_w{}^{w}A_i}$ and $\overrightarrow{O_w{}^{w}B_i}$.

We define the twist $\mathcal{T}_i$ associated with the strut's joints, $\mathcal{T}_i = \mathrm{span}(\hat{\varepsilon}_{\infty i1}, \hat{\varepsilon}_{0i2}, \hat{\varepsilon}_{0i3}, \hat{\varepsilon}_{0i4}, \hat{\varepsilon}_{0i5}, \hat{\varepsilon}_{0i6})$. In a non-singular configuration, there is no wrench $\mathcal{W}_i$ reciprocal to the twists $\mathcal{T}_i$. Thus, each moving strut has 6 DoFs:

$$\mathcal{W}_i = \mathcal{T}_i^{\perp} = \{\varnothing\}\,. \tag{1}$$

In general, a strut applies no constraint on the platform. However, if $\mathbf{s}_{i4} \perp \mathbf{s}_{i1}$, the manipulator is in a serial singular configuration

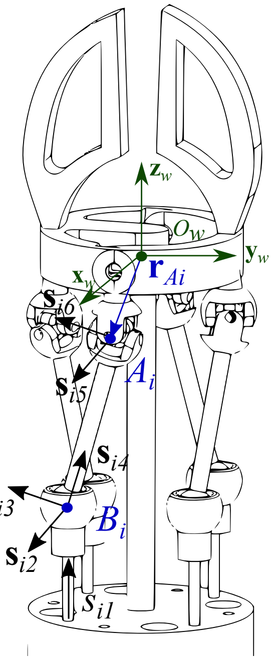

Fig. 3. Parameterization of the i^{th} leg of the wrist.

and $\mathcal{W}_i = \mathcal{T}_i^{\perp} = \begin{bmatrix} \mathbf{s}_{i4} \\ \mathbf{r}_{A_i} \times \mathbf{s}_{i4} \end{bmatrix}$.

Furthermore, the twists associated with the central bar connected to the left part of the platform are presented below:

$\hat{\varepsilon}_{0c1} = \begin{bmatrix} \mathbf{x}_p \\ 0 \end{bmatrix}$, $\hat{\varepsilon}_{0c2} = \begin{bmatrix} \mathbf{y}_p \\ 0 \end{bmatrix}$, $\hat{\varepsilon}_{0c3} = \begin{bmatrix} \mathbf{z}_p \\ 0 \end{bmatrix}$.

We define the twist $\mathcal{T}_c$ associated with the central bar as

$$\mathcal{T}_c = \text{span}(\hat{\varepsilon}_{0c1}, \hat{\varepsilon}_{0c2}, \hat{\varepsilon}_{0c3}). \tag{2}$$

The central bar provides three constraint forces on the left moving platform. A three-unit wrench of zero pitch $\mathcal{W}_c$ written in $\mathcal{F}_w$ assumes the following general form:

$$\mathcal{W}_c = \mathcal{T}_c^{\perp} = \text{span}\left(\begin{bmatrix} \mathbf{x}_p \\ 0 \end{bmatrix}, \begin{bmatrix} \mathbf{y}_p \\ 0 \end{bmatrix}, \begin{bmatrix} \mathbf{z}_p \\ 0 \end{bmatrix}\right). \tag{3}$$

Let us consider the left part of the platform separately from the right one. Hence the three constraint forces, applied by the central bar, span the constraint wrench system $\mathcal{W}_l$ of the left part of the platform, namely,

$$\mathcal{W}_l = \text{span}(\mathcal{W}_c, \mathcal{W}_1, \mathcal{W}_2) = \mathcal{W}_c. \tag{4}$$

One can note that the constraint forces only constrain the three translations of the left part of the platform.

Let us now consider the right part of the platform separately from the left part and the central bar; the twist $\mathcal{T}_r$ associated with the right part is only defined by the two twists $\mathcal{T}_i$ for $i = 3$ and 4. Thus, the constraint wrench system $\mathcal{W}_r$ of the right part of the platform is,

$$\mathcal{W}_r = \text{span}(\mathcal{W}_3, \mathcal{W}_4) = \{\varnothing\}. \tag{5}$$

Accordingly, the right part of the platform, independent from the left one, has the full six DoFs. However, if $\mathbf{s}_{i1}$ becomes perpendicular to $\mathbf{s}_{i4}$ the right part of the platform loses the displacement DoF along $\mathbf{s}_{i4}$.

We define the twist system of the revolute joint as $\hat{\varepsilon}_{0r} = \begin{bmatrix} \mathbf{s}_p \\ 0 \end{bmatrix}$ and $\mathbf{s}_p = \mathbf{x}_p$.

Since the left part of the platform has three rotational DoFs, the right part has six DoFs motion, and the revolute joint connected the two parts provides a rotational motion; the moving foldable platform can provide three rotational DoFs and a relative folding angle between the two parts of the platform [15].

B. Actuation wrench system

Let us now consider that the actuator joints are locked. In this case, the wrist should be fully constrained. The twists $\hat{\varepsilon}_{i1}$ for $i = 1, .., 4$ are null and a set of additional constraint wrenches appears for every leg. These additional wrenches are zero-pitch wrenches whose axes are parallel to $\mathbf{s}_{i4}$ (along the leg axis) and cross the axis of the unactuated universal joint. Hence, the leg wrench force can be written as

$$\mathcal{W}_i = \mathcal{T}_i^{\perp} = \begin{bmatrix} \mathbf{s}_{i4} \\ \mathbf{r}_{A_i} \times \mathbf{s}_{i4} \end{bmatrix} \text{ (for } i = 1, .., 4). \tag{6}$$

Note that this result obtained with the S-U kinematic chain leg is similar to the well known S-S chain. The only difference is the internal mobility of the struts.

If we consider the left part of the platform, the constraint forces, applied by the central bar and the two legs, span the constraint wrench system of the left part of the platform, namely,

$$\mathcal{W}^l = \text{span}(\mathcal{W}_c, \mathcal{W}_1, \mathcal{W}_2). \tag{7}$$

The five constraint forces together restrict the three translation DoFs and two rotational DoFs.

Hence, according to the reciprocity conditions of screws presented in [10], the mobility of the left part of the platform $\mathcal{T}^l$ is defined by the reciprocal screw of $\mathcal{W}^l$, which is a rotation twist. It allows to find the line $(O_w, \mathbf{s}_L)$ through the point O_w which crosses the axes of the wrenches $\mathcal{W}_1$ and $\mathcal{W}_2$. The line is defined by the intersection of the two planes $(O_w,\ (\overrightarrow{O_w{}^w A_1}, \mathbf{s}_{14}))$ and $(O_w,\ (\overrightarrow{O_w{}^w A_2}, \mathbf{s}_{24}))$ as shown in Fig. 4..

$$\mathcal{T}^l = \mathcal{W}^{l\perp} = \begin{bmatrix} \mathbf{s}_L \\ \mathbf{r}_{O_w} \times \mathbf{s}_L \end{bmatrix} = \begin{bmatrix} \mathbf{s}_L \\ 0 \end{bmatrix}, \tag{8}$$

Let us now consider the right part of the platform. According to the above wrench rule [10], the two legs $i = 3, 4$ generate one constraint force each. The two

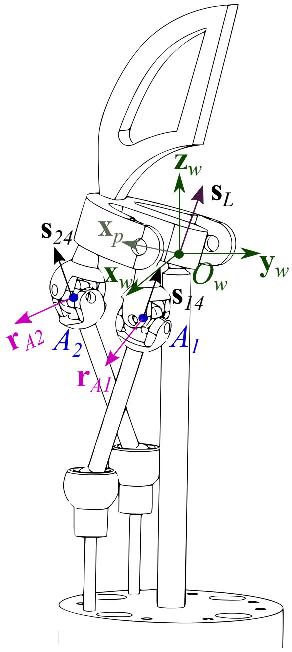

Fig. 4. Mobility of the left part of the foldable platform of the wrist.

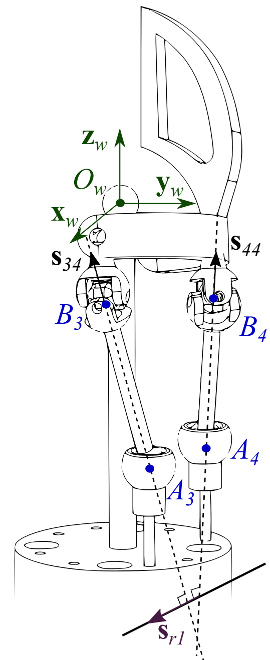

Fig. 5. Mobility of the right part of the foldable platform of the wrist

constraint forces together restrict the translation along two axes.

$$\mathcal{W}^r = \text{span}(\mathcal{W}_3, \mathcal{W}_4) \tag{9}$$

The motions of the right part of the platform $\mathcal{T}^r$ are defined by the reciprocal screw of $\mathcal{W}^r$, which can be expressed according to the reciprocity conditions by the following screw system:

$$\mathcal{T}^r = \mathcal{W}^{r\perp} = \text{span}\left(\begin{bmatrix} 0 \\ \mathbf{s}_{r1} \end{bmatrix}, \begin{bmatrix} \mathbf{s}_{34} \\ \mathbf{r}_{A_3} \times \mathbf{s}_{34} \end{bmatrix}, \begin{bmatrix} \mathbf{s}_{44} \\ \mathbf{r}_{A_4} \times \mathbf{s}_{44} \end{bmatrix}, \begin{bmatrix} \mathbf{s}_{r1} \\ \mathbf{r}_{A_3} \times \mathbf{s}_{r1} \end{bmatrix}\right), \tag{10}$$

where $\mathbf{s}_{r1} = \mathbf{s}_{34} \times \mathbf{s}_{44}$ and the three rotation screws form a variety of a twist which is of of dimension three.

At this stage, the two parts of the wrist are decoupled. From equations (8) and (10), using the revolute joint to connect the two parts will lead to the general twist system of the two parts of the foldable platform, which is provided by the intersection of the twist systems:

$$\begin{cases} \mathcal{T}_f^r = \mathcal{T}^r \cap \text{span}(\mathcal{T}^l, \hat{\varepsilon}_{0r}) \\ \mathcal{T}_f^l = \mathcal{T}^l \cap \text{span}(\mathcal{T}^r, \hat{\varepsilon}_{0r}) \end{cases} \tag{11}$$

In the general case, the central leg of the wrist applies three constraint forces to the moving platform, thus the manipulator is constrained all translations. In addition, the four moving legs constrain the three rotations and the relative rotational mobility between the two parts of the platform. Under these geometric constraints, the foldable platform of the wrist is fully constrained inside the accessible workspace excluding its singular configurations. Hence, according to equation (11), in singular configurations, the wrist may lose some constraints and the foldable platform gains one or several DoFs.

III. Jacobian matrix of the wrist using screw theory

In this section, the velocity analysis of the wrist is carried out by means of screw theory. The input of the manipulator is provided by the velocity of the four linear actuators attached to the base denoted by $\dot{\mathbf{q}} = [\dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4]^t$. The output variables are the three rotation angles and the relative rotation angle between the two parts of the platform.

Let $\mathbf{t}_r = [\boldsymbol{\omega}^r, 0_{1\times 3}]^t = [\omega_x^r, \omega_y^r, \omega_z^r, 0, 0, 0]^t$ and $\mathbf{t}_l = [\boldsymbol{\omega}^l, 0_{1\times 3}]^t = [\omega_x^l, \omega_y^l, \omega_z^l, 0, 0, 0]^t$ be, respectively, the twists of the left and the right part of the foldable platform with respect to the base frame. Additionally, $\omega_x^{l,r}, \omega_y^{l,r}$, and $\omega_z^{l,r}$ are, respectively, the angular velocity associated to the roll, pitch, and yaw rotations of each part of the platform.

There are six screws associated with each leg, and the first joint is the only actuated joint; the remaining five are passive, as presented in Eq. (1). We denote $\dot{\delta}_{ij}$ as the angular velocity associated with the j^{th} passive joint of the i^{th} leg.

The twists $\mathbf{t}_r$ and $\mathbf{t}_l$ could be written in screw form [6] as follows:

$$\begin{cases} \mathbf{t}_r = \hat{\varepsilon}_{\infty i1}\dot{q}_i + \hat{\varepsilon}_{0i2}\dot{\delta}_{i2} + \hat{\varepsilon}_{0i3}\dot{\delta}_{i3} + \hat{\varepsilon}_{0i4}\dot{\delta}_{i4} + \hat{\varepsilon}_{0i5}\dot{\delta}_{i5} + \hat{\varepsilon}_{0i6}\dot{\delta}_{i6} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(for } i = 1, 2) \\ \mathbf{t}_l = \hat{\varepsilon}_{\infty i1}\dot{q}_i + \hat{\varepsilon}_{0i2}\dot{\delta}_{i2} + \hat{\varepsilon}_{0i3}\dot{\delta}_{i3} + \hat{\varepsilon}_{0i4}\dot{\delta}_{i4} + \hat{\varepsilon}_{0i5}\dot{\delta}_{i5} + \hat{\varepsilon}_{0i6}\dot{\delta}_{i6} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(for } i = 3, 4). \end{cases} \tag{12}$$

Let $\hat{\varepsilon}_{0ir}$ be the unit screw that is reciprocal to all the passive joint screws except $\hat{\varepsilon}_{\infty i1}$, which is an active joint screw. The reciprocal screw $\hat{\varepsilon}_{0ir}$ can be identified as a zero-pitch screw as follows:

$$\hat{\varepsilon}_{0ir} = \begin{bmatrix} \mathbf{s}_{i4} \\ \mathbf{r}_{A_i} \times \mathbf{s}_{i4} \end{bmatrix}. \tag{13}$$

If $\hat{\varepsilon}_{0ir} \circ \hat{\varepsilon}_{ij} = 0$, where $\circ$ represents the reciprocal product between two screws, then $\hat{\varepsilon}_{ir}$ is reciprocal to $\hat{\varepsilon}_{ij}$. Taking the reciprocal product of both sides of Eq. (12), we obtain

$$\begin{cases} \hat{\varepsilon}_{0ir} \circ \mathbf{t}_r = \hat{\varepsilon}_{0ir} \circ \hat{\varepsilon}_{\infty i1}\, \dot{q}_i = \mathbf{s}_{i4}^t\, \mathbf{s}_{i1}\, \dot{q}_i \text{ (for } i = 1, 2) \\ \hat{\varepsilon}_{0ir} \circ \mathbf{t}_l = \hat{\varepsilon}_{0ir} \circ \hat{\varepsilon}_{\infty i1}\, \dot{q}_i = \mathbf{s}_{i4}^t\, \mathbf{s}_{i1}\, \dot{q}_i \text{ (for } i = 3, 4). \end{cases} \tag{14}$$

In fact, since the platform of this robot is composed of two kinematically related bodies (through a revolute joint along $\mathbf{s}_p$), the platform's configuration cannot be expressed as an SE3 element (4 rotations is not an SE3 element). Instead, the two twists $\mathbf{t}_r$ and $\mathbf{t}_l$ are combined together to have a velocity vector for the configurable platform $\mathbf{t} = [\boldsymbol{\omega}, \omega_\theta, 0_{1\times 3}]^t = [\omega_x, \omega_y, \omega_z, \omega_\theta, 0, 0, 0]^t$. To keep a symmetrical representation of both sides of the platform, we present the angular velocity by the vector $\boldsymbol{\omega} = (\boldsymbol{\omega}^l + \boldsymbol{\omega}^r)/2$ and $\pm\omega_\theta = \pm\dot{\theta}/2$ gives the scalar angular velocity of each platform segment in relation to a reference orientation of the combined platform as $\omega_\theta\, \mathbf{s}_p = \boldsymbol{\omega} - \boldsymbol{\omega}^r = \boldsymbol{\omega}^l - \boldsymbol{\omega}$.

$$\begin{bmatrix} \boldsymbol{\omega}^l \\ \boldsymbol{\omega}^r \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{3\times 3} & \mathbf{s}_p \\ \mathbf{I}_{3\times 3} & -\mathbf{s}_p \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \omega_\theta \end{bmatrix} \tag{15}$$

Finally, casting equation (14) into a matrix and vector form

$$\mathbf{J}_l \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \omega_\theta \end{bmatrix} = \mathbf{J}_r \dot{\mathbf{q}}, \tag{16}$$

where

$$\mathbf{J}_l = \begin{bmatrix} (\mathbf{r}_{A_1} \times \mathbf{s}_{14})^t & (\mathbf{r}_{A_1} \times \mathbf{s}_{14})^t\, \mathbf{s}_p \\ (\mathbf{r}_{A_2} \times \mathbf{s}_{24})^t & (\mathbf{r}_{A_2} \times \mathbf{s}_{24})^t\, \mathbf{s}_p \\ (\mathbf{r}_{A_3} \times \mathbf{s}_{34})^t & -(\mathbf{r}_{A_3} \times \mathbf{s}_{34})^t\, \mathbf{s}_p \\ (\mathbf{r}_{A_4} \times \mathbf{s}_{44})^t & -(\mathbf{r}_{A_4} \times \mathbf{s}_{44})^t\, \mathbf{s}_p \end{bmatrix} \in R^{4\times 4} \tag{17}$$

and

$$\mathbf{J}_r = \begin{bmatrix} \mathbf{s}_{14}^t\, \mathbf{s}_{11} & 0 & 0 & 0 \\ 0 & \mathbf{s}_{24}^t\, \mathbf{s}_{21} & 0 & 0 \\ 0 & 0 & \mathbf{s}_{34}^t\, \mathbf{s}_{31} & 0 \\ 0 & 0 & 0 & \mathbf{s}_{44}^t\, \mathbf{s}_{41} \end{bmatrix} \in R^{4\times 4}. \tag{18}$$

The matrix $\mathbf{J}_l$ defined in Eq. (17) could be expressed differently by applying the permutation property of the vector triple product along with the commutative property of the dot product:

TABLE I

Some singularity conditions of the PM wrist

Case	Singularity condition	Figure
1	$\mathbf{s}_{i4} \perp \mathbf{s}_{i1}$	
2	$\mathbf{s}_{14} \parallel \mathbf{s}_{24} \parallel \mathbf{s}_{34} \parallel \mathbf{s}_{44} \parallel \mathbf{z}_w$	6 (a)
3	$\mathbf{r}_{A_i} \parallel \mathbf{s}_{i4}$	6 (b)
4	$(O_w, \mathbf{r}_{A_3}) \in plane(O_w, \mathbf{s}_{34}, \mathbf{s}_{44})$	6 (c)

$$\mathbf{J}_l = \begin{bmatrix} (\mathbf{s}_{14} \times \mathbf{x}_w)^t \mathbf{r}_{A_1} & (\mathbf{s}_{14} \times \mathbf{y}_w)^t \mathbf{r}_{A_1} & (\mathbf{s}_{14} \times \mathbf{z}_w)^t \mathbf{r}_{A_1} & (\mathbf{s}_{14} \times \mathbf{s}_p)^t \mathbf{r}_{A_1} \\ (\mathbf{s}_{24} \times \mathbf{x}_w)^t \mathbf{r}_{A_2} & (\mathbf{s}_{24} \times \mathbf{y}_w)^t \mathbf{r}_{A_2} & (\mathbf{s}_{24} \times \mathbf{z}_w)^t \mathbf{r}_{A_2} & (\mathbf{s}_{24} \times \mathbf{s}_p)^t \mathbf{r}_{A_2} \\ (\mathbf{s}_{34} \times \mathbf{x}_w)^t \mathbf{r}_{A_3} & (\mathbf{s}_{34} \times \mathbf{y}_w)^t \mathbf{r}_{A_3} & (\mathbf{s}_{34} \times \mathbf{z}_w)^t \mathbf{r}_{A_3} & -(\mathbf{s}_{34} \times \mathbf{s}_p)^t \mathbf{r}_{A_3} \\ (\mathbf{s}_{44} \times \mathbf{x}_w)^t \mathbf{r}_{A_4} & (\mathbf{s}_{44} \times \mathbf{y}_w)^t \mathbf{r}_{A_4} & (\mathbf{s}_{44} \times \mathbf{z}_w)^t \mathbf{r}_{A_4} & -(\mathbf{s}_{44} \times \mathbf{s}_p)^t \mathbf{r}_{A_4} \end{bmatrix} \tag{19}$$

The Jacobian matrix relating the four actuator inputs $\dot{Q}$ to the motion twist is thus

$$\begin{bmatrix} \boldsymbol{\omega} \\ \omega_\theta \end{bmatrix} = \mathbf{J}\, \dot{\mathbf{q}} = \mathbf{J}_r^{-1} \mathbf{J}_l\, \dot{\mathbf{q}}. \tag{20}$$

By analyzing the Jacobian matrix $\mathbf{J}$ obtained in Eq. (20), some singularity conditions could be observed. According to the classification of singularities in [10], the constraint singularities correspond to mechanism postures where $\mathbf{J}_r$ becomes rank deficient, whereas, the architectural singularities are obtained when $\mathbf{J}_l$ becomes rank deficient.

Table I expands some singularity conditions observed by analyzing matrices $\mathbf{J}_r$ and $\mathbf{J}_l$. Case 1 comprehensively describes the geometric condition of the architectural singularities. The two versions of $\mathbf{J}_l$ provide different insights into the geometric conditions for the constraint singularities. Case 2, where all limbs are directed along $\mathbf{z}_w$ as seen in Fig. 6 (a), makes $\mathbf{J}_l$ rank deficient where the third column is zero. In case 3, presented in Fig. 6 (b), the vector $\mathbf{r}_{A_i}$ becomes collinear to the axis direction $\mathbf{s}_{i4}$, which results in $\mathbf{r}_{A_i} \times \mathbf{s}_{i4} = 0$ and makes the i^{th} raw of $\mathbf{J}_l$ zero. Fig. 6 (c) shows the singularity in Case 4 where the vectors $\mathbf{r}_{A_3} \times \mathbf{s}_{34}$ and $\mathbf{r}_{A_4} \times \mathbf{s}_{44}$ are collinear, which makes raw 4 a linear multiple of raw 3.

IV. Conclusion and outlook

In this paper, the mobility of a 4-DoFs wrist with a foldable platform structure has been analyzed. This was performed using the theory of screws and reciprocal screws. First, constraint analysis was achieved to validate the mobility of the wrist. Then, a wrench analysis of each part of the foldable platform in the three-dimensional protective space was presented. Accordingly, the constraint condition of the end-effector of the wrist was formulated and analyzed in order to provide the geometric condition for some singular configurations. Then, instantaneous kinematics by means of screw theory was introduced. In this step, the screw theory was adapted and extended to model a non-rigid body, namely the configurable robot platform. Finally, the Jacobian matrix

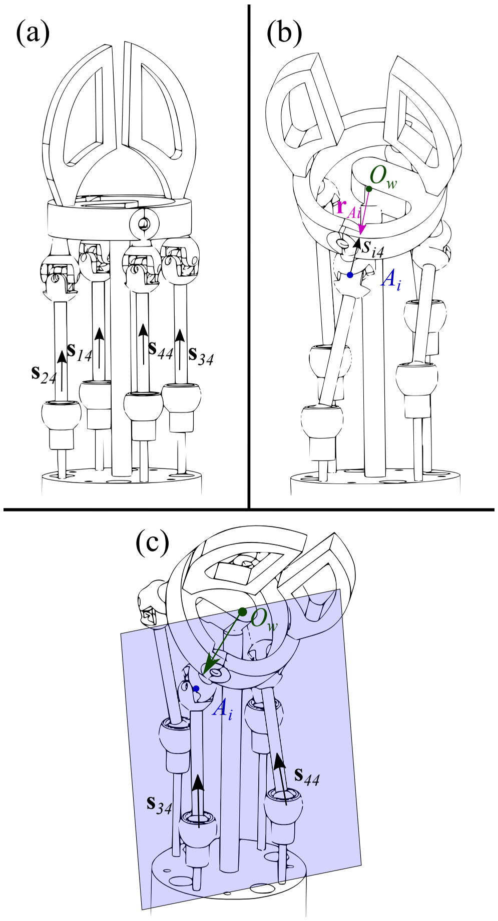

Fig. 6. Some singularity configurations given in Table I

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was presented through the formulation of the instantaneous kinematics relationship.

Future works will include performing an optimized design of the wrist at the targeted scale and its validation for minimally invasive surgery. Furthermore, exhaustive determination of the singularity condition of the PM wrist, using GCA and Grassmann geometry(GG), will be performed.

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