Combining H_∞ and interval techniques to design robust low order controllers: application to piezoelectric actuators

Sofiane Khadraoui, Micky Rakotondrabe *Member, IEEE* and Philippe Lutz *Member, IEEE*

Abstract— **This paper addresses the design of robust controllers for interval systems. For that, the H_∞-standard approach is combined with interval techniques in order to derive controllers that ensure robust performances. The main advantages of the proposed approach are: 1) the natural and ease of modeling of the uncertain parameters thanks to intervals, 2) the low order of the controller since it can be lower than the order of the system, 3) and the fact that the structure of the controller can be fixed *a priori*. The paper is particularly focused on the design of a robust PID controller to control systems with order n. Experiments on a piezoelectric actuator (piezocantilever) are carried out and show the efficiency of the proposed method.**

I. INTRODUCTION

Piezoelectric materials are very common in the development of microsystems and microrobots. This recognition is due to the high resolution, high bandwidth and the high force density they can offer [1]. A common example of their applications is the piezoelectric microgripper which is dedicated to manipulate or to assemble small objects, i.e. to perform micromanipulation and microassembly tasks [2], [3]. A piezoelectric microgripper is composed of two piezoelectric actuators with cantilever structure (also called piezocantilevers) each one able to bend when a voltage is applied.

Despite the submicrometric resolution and the large bandwidth (more than 600Hz) of piezocantilevers, they exhibit strong nonlinearities (hysteresis and creep) and a badly damped vibration. Furthermore, they are very sensitive to environmental disturbances such as the manipulated objects or the ambient thermal variation. These characteristics lead to a loss of the general performances (accuracy, settling time, damping ratio) of the actuators and thus finally make micromanipulation/microassembly tasks fail. Another characteristic of piezocantilevers is their fragility. Thus, during a micromanipulation or a microassembly task, in case of breakage of a piezoelectric actuator, it is often desired to replace it with a similar one from the stock without re-identifying this new actuator and re-computing a new controller. To account all these characteristics and constraints, it has been demonstrated that linear models with uncertain or varying parameters can be used to model piezocantilevers [4]–[6] and thus robust controllers can be further designed to ensure the performances.

H_2, H_∞ and μ-*synthesis* approaches are among the existing robust control methods used to enhance the performances of piezocantilevers [4]–[6]. These methods provide a precise formulation and solution of the controller synthesis problem for which the H_∞-norm of a prescribed transfer function is minimized. The effeciency of these approaches has been proven in several kind of piezoelectric actuators (SISO and MIMO). However, the derived controllers (even reduced) are of high-order comparatively to the available implementation setup (embedded microsystems): controllers orders are more than 10 whilst the setup is a classical PIC microcontroller with a sampling time of $0.2ms$. Thus, due to the time consuming of the controllers, an unstability of the closed-loop often occurs. To bypass this limitation, we demonstrated in previous works [7]–[9] that combining intervals to model the uncertainties with classical controller design techniques may lead to low order controllers. Despite the computation time slightly higher than in classical robust control design approach (H_∞ and its extension), these proposed robust control design approaches based on intervals are very promising for embedded microsystems where the orders of the controllers are first essential.

Interval methods have been used to synthesize robust stabilizing controller for uncertain systems [10]–[12]. More than the robust stability, interval techniques can also deal with controllers synthesis that ensure robust performances. For instance, *Chen* and *Wang* [13] proposed a method to design a robust controller for interval systems in the state-space representation. In their work, two controllers were necessary: a robust controller stabilizing the feedback is computed first, then a pre-filter is introduced to ensure the wanted performances. A PID controller that ensures robust performances for parametric uncertain transfer functions was also proposed in [14]. However, its numerical application becomes difficult for high-order interval systems (numerically, it is limited to second order models). The authors in [15] have suggested a control algorithm prediction-based interval model that was efficiently applied to a welding process. In our previous works [7]–[9], robust control design methods for interval systems were presented. These latter works proved that interval analysis combined with a given linear control theory can be very promising for modeling and control piezoelectric microsystems where

Sofiane Khadraoui, Micky Rakotondrabe and Philippe Lutz are with the Automatic Control and Micro-Mechatronic Systems Department, AS2M,

FEMTO-ST Institute, UMR CNRS 6174/UFC/ENSMM /UTBM, 24 rue Alain Savary, Besançon 25000, France.

E-mail: {sofiane.khadraoui,mrakoton,plutz}@femto-st.fr

the models parameters are subjected to change during functioning.

PID controllers have been widely used in many industrial applications. Their popularity is due to their low-order, simplicity and efficiency even for nonlinear and for non-modelled systems. In this work, we propose an approach to enhance the capabilities of PID controllers to control piezocantilevers modeled by a transfer function with a priori unknown parameters but are assumed to lie within a given range. Interval modeling techniques [16] are thus introduced to bound these uncertain parameters. Contrary to the previous works [7]–[9], the approach in this paper is extended to the control of interval systems with more general specifications which can be easily transcribed into weighting functions during the synthesis. For that, the proposed approach consists to combine the standard H_∞-approach with interval techniques. Therefore, the approach allows to derive a robust PID controller for systems with order n and that ensures more generalized specified performances (tracking performances, input control limitation, etc.).

The paper is organized as follows. In section-II, preliminaries on interval analysis and systems are first presented. Section III is dedicated to the design of a PID control for interval systems. In section IV, we apply the proposed method to design a controller for piezocantilevers. Finally, we present in section V the controller implementation and some discussions relative to the experimental results.

II. Preliminaries on intervals

A. Basic Terms and Concepts on intervals

A closed interval number denoted by $[x]$ is a closed bounded such as:

$$[x] = [x^-, x^+] = \left\{ x \in R / x^- \leq x \leq x^+ \right\} \quad (1)$$

where x^- and x^+ are the left and right endpoints of $[x]$ respectively.

We say that $[x]$ is degenerate if $x^- = x^+$. By convention, a degenerate interval $[a, a]$ can be described with the real number a.

The width of an interval $[x]$ is given by $w([x]) = x^+ - x^-$. The midpoint of $[x]$ is given by $mid([x]) = \frac{x^+ + x^-}{2}$ and the radius of $[x]$ is defined by $rad([x]) = \frac{x^+ - x^-}{2}$.

B. Operations on intervals

The elementary mathematical operations can be extended to intervals. Let $[x] = [x^-, x^+]$ and $[y] = [y^-, y^+]$ be two intervals and let $\circ \in \{+, -, *, /\}$ be a law. We can write:

$$[x] \circ [y] = \{x \circ y \,|\, x \in [x], y \in [y]\} \quad (2)$$

Table I gives the details of the above interval operations.

TABLE I
CLASSICAL ARITHMETIC OPERATIONS ON INTERVALS [10], [17].

Operation	Definition
$+$	$[x] + [y] = [x^- + y^-, x^+ + y+]$
$-$	$[x] - [y] = [x^- - y^+, x^+ - y-]$
$*$	$[x] * [y] = [min\{x^- * y^-, x^+ * y^-, x^- * y^+, x^+ * y^+\},$ $max\{x^- * y^-, x^+ * y^-, x^- * y^+, x^+ * y^+\}]$
$/$	$[x]/[y] = [x] * [1/y^+, 1/y^-], 0 \notin [y]$

C. Interval systems

Definition 2.1: An interval model denoted by $[G](s, [\mathbf{a}], [\mathbf{b}])$ represents a family of systems:

$$[G](s, [\mathbf{a}], [\mathbf{b}]) = \frac{[N](s, [\mathbf{b}])}{[D](s, [\mathbf{a}])} = \frac{\displaystyle\sum_{j=0}^{m} [b_j] s^j}{\displaystyle\sum_{i=0}^{n} [a_i] s^i} \quad (3)$$

such as: $[\mathbf{b}] = [[b_0], ..., [b_m]]$ and $[\mathbf{a}] = [[a_0], ..., [a_n]]$ are two boxes (vectors of intervals) and s the Laplace variable. The system $[G]$ above generally represents a model with uncertain parameters.

D. Vertex polynomials and vertex systems

Given an interval system $[G](s, [\mathbf{a}], [\mathbf{b}])$ defined as in Definition 2.1 such that:

$$\begin{cases} [N](s, [\mathbf{b}]) = [b_0] + [b_1]s + [b_2]s^2 + ... + [b_m]s^m \\ [D](s, [\mathbf{a}]) = [a_0] + [a_1]s + [a_2]s^2 + ... + [a_n]s^n \end{cases} \quad (4)$$

Thus, the four Kharitonov vertex polynomials corresponding to $[N](s, [\mathbf{b}])$ and $[D](s, [\mathbf{a}])$ are:

$$\begin{aligned} N^{(1)}(s) &= b_0^- + b_1^- s + b_2^+ s^2 + b_3^+ s^3 + b_4^- s^4 + b_5^- s^5 + ... \\ N^{(2)}(s) &= b_0^- + b_1^+ s + b_2^+ s^2 + b_3^- s^3 + b_4^- s^4 + b_5^+ s^5 + ... \\ N^{(3)}(s) &= b_0^+ + b_1^- s + b_2^- s^2 + b_3^+ s^3 + b_4^+ s^4 + b_5^- s^5 + ... \\ N^{(4)}(s) &= b_0^+ + b_1^+ s + b_2^- s^2 + b_3^- s^3 + b_4^+ s^4 + b_5^+ s^5 + ... \end{aligned} \quad (5)$$

and

$$\begin{aligned} D^{(1)}(s) &= a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + a_4^- s^4 + a_5^- s^5 + ... \\ D^{(2)}(s) &= a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + a_4^- s^4 + a_5^+ s^5 + ... \\ D^{(3)}(s) &= a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + a_4^+ s^4 + a_5^- s^5 + ... \\ D^{(4)}(s) &= a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + a_4^+ s^4 + a_5^+ s^5 + ... \end{aligned} \quad (6)$$

respectively.

The sixteen Kharitonov (point) systems that corresponds to the interval system $[G]$ are the combination of these vertex polynomials. These sixteen Kharitonov systems are called vertex of $[G]$. We denote these sixteen Kharitonov vertex by $G^{(i)}$, with $i = 1 \rightarrow 16$.

E. H_∞-norm of an interval system

Theorem 2.1: Consider the interval system $[G](s, [\mathbf{a}], [\mathbf{b}])$ defined in Definition 2.1. The H_∞-norm of $[G]$ is the maximal among the H_∞-norm of the sixteen vertex, i.e.:

$$\|[G]\|_\infty = \max_{i=1\to16} \left\|G^{(i)}\right\|_\infty \qquad (7)$$

Proof: see [18], [19]. ∎

When the interval system $[G]$ is weighted by a weigthing function $W(s)$ which is a point, it is not advised to compute the multiplication $W[G]$ first and compute the H_∞-norm of the resulting interval plant afterwards. Indeed, developing the multiplication of the intervals polynomials produces a multi-occurance of the parameters and therefore a surestimation of the resulting intervals. Thus, the H_∞-norm of $W[G]$ is defined as follows:

$$\|W[G]\|_\infty = \max_{i=1\to16} \left\|WG^{(i)}\right\|_\infty \qquad (8)$$

In *Long-Wang* [20], the H_∞-norm of the sensitivity function of an interval system $[G](s, [\mathbf{a}], [\mathbf{b}])$ is proposed. The sensitivity of $[G]$ is defined by $[S] = \frac{1}{1+[G]} = \frac{[D]}{[N]+[D]}$, where $[N]$ and $[D]$ are the numerator and denominator defined in Definition 2.1. It has been then demonstrated that the sensitivity $[S]$ has only twelve vertex instead of sixteen vertex and thus its H_∞-norm is the maximal among the twelve norms:

$$\|[S]\|_\infty = \left\|\frac{[D]}{[N]+[D]}\right\|_\infty = \max_{i=1\to12}\left\|S^{(i)}\right\|_\infty \qquad (9)$$

III. CONTROLLER DESIGN METHOD

In this section, we propose to design robust PID controllers for interval systems. The robust performances are achieved by using H_∞-norm. For that, we combine H_∞-standard approach and the interval arithmetic.

A. Problem formulation

Consider the closed-loop pictured in Fig. 1, where $[G](s, [\mathbf{a}], [\mathbf{b}])$ is a SISO interval system to be controlled. $[C](s)$ is the controller to be designed. $[\mathbf{a}]$ and $[\mathbf{b}]$ are the interval parameters of the system. $y_c(t)$ is the reference input, $y(t)$ is the output signal and $u(t)$ is the input control signal.
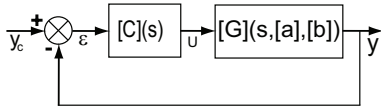

Fig. 1. Closed-loop control system.

We assume the family of system $[G](s, [\mathbf{a}], [\mathbf{b}])$ defined by:

$$[G](s, [\mathbf{a}], [\mathbf{b}]) = \frac{\sum_{j=0}^{m} [b_j]s^j}{\sum_{i=0}^{n} [a_i]s^i} \qquad (10)$$

Such as: $[\mathbf{a}] = [[a_0], ..., [a_n]]$ and $[\mathbf{b}] = [[b_0], ..., [b_m]]$, and $m \le n$.

In the sequel, the designed controller is a PID controller with adjustable parameters $[\theta] = [[K_p], [K_i], [K_d]]$:

$$[C](s, [\theta]) = [K_p] + [K_d]s + [K_i]\frac{1}{s} \qquad (11)$$

The objective is to find the set solution of PID parameters so that the closed-loop system respects some given performances whatever the parameters a_i and b_j ranging in their corresponding intervals $[a_i]$ and $[b_j]$. For that, the PID parameters will be properly adjusted using H_∞-criterion. Such criterion is defined as the H_∞-norm of some weighted transfer functions of the closed-loop to be less than or equal to one. The basic idea of our proposed method is therefore to combine the H_∞-standard approach with interval arithmetic.

B. Remind of the H_∞-standard principle

The H_∞-standard that considers the tracking performances and the input control limitation [21], [22] uses the standard block as pictured in Fig. 2-b. This standard scheme is derived from the weighted closed-loop in Fig. 2-a. The closed-loop control objectives are achieved through appropriate weighting functions $W_i(s)$ chosen to shape some important transfer functions of the closed-loop such as: the sensitivity function $S(s) = (1+C(s)G(s))^{-1}$ and the input transfer $C(s)S(s)$. In the figure, $P(s)$ is called the augmented system. It contains the system to be controlled with the different weightings.
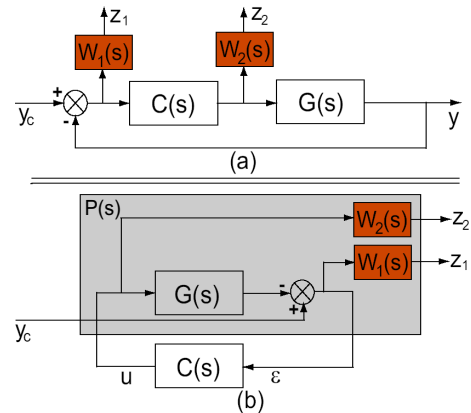

Fig. 2. Standard H_∞ control scheme.

The H_∞ problem consists to find a controller stabilizing the closed-loop and achieving the following H_∞-criterion:

$$\|F_l(P(s), C(s))\|_\infty \leq \gamma \qquad (12)$$

where γ is a positive scalar. If $\gamma \leq 1$, the nominal (specified) performances are achieved.

The linear fractionar transformation $F_l(P(s), C(s))$ is the transfer between the weighted outputs and the exogenous inputs of Fig. 2-a. That is:

$$F_l(P(s), C(s)) = z(s).y_c^{-1}(s) \qquad (13)$$

with $z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$

From Fig. 2-a $F_l(P(s), C(s))$ is given by:

$$F_l(P(s), C(s)) = \begin{pmatrix} W_1(s)S(s) \\ W_2(s)C(s)S(s) \end{pmatrix} \qquad (14)$$

Applying the H_∞ standard problem in (12) to (13) and (14), we obtain the following conditions to be satisfied:

$$\begin{cases} \|W_1(s)S(s)\|_\infty \leq \gamma \\ \|W_2(s)C(s)S(s)\|_\infty \leq \gamma \end{cases} \qquad (15)$$

C. H_∞ approach for interval systems

In our case the system is an interval model $[G](s, [\mathbf{a}], [\mathbf{b}])$, the controller to be designed is a PID controller (11). Since the system is interval, the augmented plant will also be interval: $[P](s, [\mathbf{a}], [\mathbf{b}])$. Moreover, the H_∞-criterion $\|F_l([P](s, [\mathbf{a}], [\mathbf{b}]), [C](s, [\theta]))\|_\infty \leq \gamma$ is given by:

$$\begin{cases} \|W_1(s)[S](s)\|_\infty \leq \gamma \\ \|W_2(s)[C](s, [\theta])[S](s)\|_\infty \leq \gamma \end{cases} \qquad (16)$$

In this case, if $\gamma \leq 1$, the robust performances are achieved.

The problem of finding the PID controller with tunable parameters $[\theta]$ can be formulated as follows:

Find the set Θ of PID parameter vector for which H_∞ performance holds for any positive number $\gamma \leq 1$, i.e.,

$$\Theta := \left\{ \theta \in [\theta] \, \middle| \, \begin{cases} \|W_1(s)[S](s)\|_\infty \leq \gamma \\ \|W_2(s)[C](s, [\theta])[S](s)\|_\infty \leq \gamma \end{cases} \right\} \qquad (17)$$

where $[S](s)$ depends on the PID parameters $[\theta]$ and of the boxes $[\mathbf{a}]$ and $[\mathbf{b}]$.

The problem given in (Eq.17) is known as a set-inversion problem which can be solved using set inversion algorithms. A set inversion operation consists to search the reciprocal image called subpaving of a compact set. One algorithm used to solve a set-inversion problem is the SIVIA algorithm ([10], [23]). By using SIVIA, it is possible to approximate with subpavings the set solution Θ described in (Eq.17). The subpaving Θ corresponds to the controller parameters for which the problem (Eq.16) is fulfilled. Table II presents the recursive SIVIA algorithm. It requires a search box $[\theta_0]$ also called initial box. The subpavings Θ is initially empty. ϵ represents the wanted accuracy of computation.

TABLE II

SIVIA Algorithm used to solve the set-inversion problem (Eq.17).

	SIVIA(in: $\|F_l([P](s, [\mathbf{a}], [\mathbf{b}]), [C](s, [\theta]))\|_\infty$, γ, $[\theta]$, ϵ; inout: Θ)
1	if $\|F_l([P](s, [\mathbf{a}], [\mathbf{b}]), [C](s, [\theta]))\|_\infty \leq \gamma$ then $\{\Theta := \Theta \bigcup [\theta]\}$ return;
2	if $width([\theta]) < \epsilon$ then $\{\Theta := \Theta\}$; return;
3	bisect $[\theta]$ into $L([\theta])$ and $R([\theta])$;
4	SIVIA($\|F_l([P](s, [\mathbf{a}], [\mathbf{b}]), [C](s, L([\theta])))\|_\infty$, γ, $L([\theta])$, ϵ; Θ); SIVIA($\|F_l([P](s, [\mathbf{a}], [\mathbf{b}]), [C](s, R([\theta])))\|_\infty$, γ, $R([\theta])$, ϵ; Θ).

However, the previous resolution requires the computation of the H_∞-norm of each term in (Eq.17) which are interval transfers. This computation can be done by using the preliminaries in section-II. The H_∞-norm $\|W_1(s)[S](s)\|_\infty$ is obtained by applying the definition in (Eq.8) and (Eq.9). We have:

$$\|W_1[S]\|_\infty = \max_{i=1\to12} \left\| W_1 S^{(i)} \right\|_\infty \qquad (18)$$

On the other hand, the H_∞-norm $\|W_1(s)[C](s, [\theta])[S](s)\|_\infty$ is obtained by applying the definition in (Eq.8) only, i.e.:

$$\|W_1[C][S]\|_\infty = \max_{i=1\to16} \left\| W_1 M^{(i)} \right\|_\infty \qquad (19)$$

where $[M] = [C][S]$ and $M^{(i)}$ ($i = 1 \to 16$) are the sixteen vertex of $[M]$.

Finally, contrary to the standard H_∞ problem (for point systems) where the optimal value of γ is found by dichotomy, we directly impose in this paper its value equal to one: $\gamma = 1$. The objective is to find directly the controller parameters with which the specified performances are respected.

IV. Application to piezocantilevers

The objective of this section is to apply the proposed method to control the deflection of piezocantilevers used in microgrippers. In this paper, we particularly consider a unimorph piezocantilever made up of a piezoelectric layer, often Lead-Zirconate-Titanate (PZT) ceramic, and one passive layer. When a voltage U is applied to the piezolayer, it expands/contracts resulting a global deflection (displacement) δ of the cantilever (see Fig. 3). Besides, applying a force F at its tip also results a deflection.

Piezocantilevers are widely used in micro/nano-positioning applications where the displacements magnitudes are small. These displacements must be highly accurate relative to a reference position. However, the behavior of piezocantilevers is uncertain due to their sensitivity to the environments (thermal variation, manipulated objects, etc.). Furthermore, it is essential in micromanipulation and microassembly to have a set of piezocantilevers (with the same dimensions) in a stock
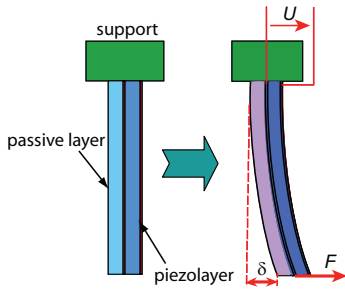
Fig. 3. A unimorph piezoelectric cantilever.

such that one can quickly and easily do an exchange in case of breakage during the tasks without calculating a new controller. Despite that they have the same general dimensions, differences at micrometric level still exist due to the imprecision of the microfabrication used to fabricate them. These micrometric differences make them have non-negligible differences in their models. All these characteristics and requirements lead to use models of piezocantilevers with uncertain parameters [7]–[9].

In this example, we construct the interval model $[G](s, [\mathbf{a}], [\mathbf{b}])$ using only two piezocantilevers instead of several. In order to have a *large* interval model, the lengths l_1 and l_2 of the two piezocantilevers are voluntarily chosen to be significantly different. Thus $[G](s, [\mathbf{a}], [\mathbf{b}])$ can be derived from two models $G_1(s)$ and $G_2(s)$ corresponding to the piezocantilever with lengths l_1 and l_2 respectively.

A. Presentation of the setup

The experimental setup is pictured in Fig. 4 and is based on:

- a unimorph piezocantilever having dimensions of $18mm \times 1mm \times 0.45mm$ (length, width and thickness),
- an optical sensors (Keyence LC-2420) with $10nm$ of resolution used to measure the deflections,
- a computer-DSpace hardware and the Matlab-Simulink software for the data-acquisition and control,
- and a high voltage (HV: $\pm 200V$) amplifier.

B. Modeling and identification

The linear relation between the deflection at the tip and the applied input voltage U is:

$$\delta = G(s)U \tag{20}$$

The two considered lengths of piezocantilevers are $l_1 = 16mm$ and $l_2 = 14mm$. To identify the two models $G_1(s)$ and $G_2(s)$ corresponding to the two piezocantilevers, a step response is used. For that, a second order model was chosen for each length of the piezocantilever. Indeed, the first mode is sufficient for micromanipulation and
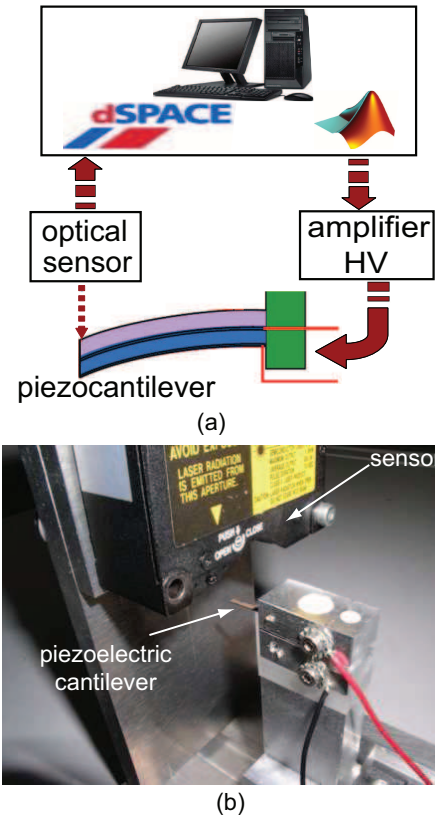

(a)

(b)

Fig. 4. The experimental setup.

mciroassembly tasks. Using the output error method and the matlab software, we obtain:

$$G_1(s) = 0.6587 \frac{3.533 \times 10^{-8}s^2 + 2.152 \times 10^{-4}s + 1}{3.374 \times 10^{-8}s^2 + 8.171 \times 10^{-6}s + 1}$$

$$G_2(s) = 0.45 \frac{3.336 \times 10^{-8}s^2 + 1.679 \times 10^{-4}s + 1}{2.119 \times 10^{-8}s^2 + 4.607 \times 10^{-6}s + 1} \tag{21}$$

Let us rewrite each model $G_i(s)$ for $i = 1, 2$ as follows:

$$G_i(s) = k_i \frac{b_{2i}s^2 + b_{1i}s + 1}{a_{2i}s^2 + a_{1i}s + 1} \tag{22}$$

such as: k_i and $\frac{b_{2i}s^2 + b_{1i}s + 1}{s^2 + a_{1i}s + 1}$ are the static gain and dynamic part of the piezocantilever with length l_i ($i = 1, 2$).

C. Derivation of the interval model

The interval model $[G](s, [\mathbf{a}], [\mathbf{b}])$ that represents a family of models is derived using the two point models $G_i(s)$. Considering each parameter of $G_1(s)$ and $G_2(s)$ as an endpoint of the interval parameter in $[G](s, [\mathbf{a}], [\mathbf{b}])$, we have:

$$[G](s, [\mathbf{a}], [\mathbf{b}]) = [K] \frac{[b_2]s^2 + [b_1]s + 1}{[a_2]s^2 + [a_1]s + 1} \tag{23}$$

such as:

$$[K] = [min(k_1, k_2), max(k_1, k_2)]$$
$$[b_1] = [min(b_{11}, b_{12}), max(b_{11}, b_{12})]$$
$$[b_2] = [min(b_{21}, b_{22}), max(b_{21}, b_{22})] \qquad (24)$$
$$[a_2] = [min(a_{21}, a_{22}), max(a_{21}, a_{22})]$$
$$[a_1] = [min(a_{11}, a_{12}), max(a_{11}, a_{12})]$$

After computation, we obtain:

$$[K] = [0.45, 0.6587]$$
$$[b_2] = [3.336, 3.533] \times 10^{-8}$$
$$[b_1] = [1.679, 2.152] \times 10^{-4} \qquad (25)$$
$$[a_2] = [2.119, 3.374] \times 10^{-8}$$
$$[a_1] = [4.607, 8.171] \times 10^{-6}$$

In order to increase the stability margin, if a controller exists, and to ensure that the interval model really contains the models (21), we propose to extend by 10% the width of each interval parameter in $[G](s, [\mathbf{a}], [\mathbf{b}])$. This choice is a compromise. If the widths are too large, it is difficult to find a controller that respects both the stability and performances of the closed-loop. Finally, the extended parameters used to compute the controller are:

$$[K] = [0.4395, 0.6691]$$
$$[b_2] = [3.326, 3.542] \times 10^{-8}$$
$$[b_1] = [1.655, 2.175] \times 10^{-4} \qquad (26)$$
$$[a_2] = [2.056, 3.436] \times 10^{-8}$$
$$[a_1] = [4.428, 8.349] \times 10^{-6}$$

D. Specifications

Piezocantilevers are very resonant (more than 90% of overshoot). Such overshoot is not desirable in micro-manipulation and microassembly tasks. Moreover, it is necessary to limit the applied voltage in order to avoid any damage of the piezoelectric actuators. The following specifications are therefore considered:

- closed-loop behavior with negligible (or without) overshoot,
- settling time $tr \leq 8ms$,
- static error $|\epsilon| \leq 1\%$,
- limited input voltage U. We choose a maximal voltage $U^{max} = 2.5V$ for each $1\mu m$ of reference.

E. Computation of the controller

In this experimental example, we consider a PI (proportional-Integral), i.e. K_d is set to zero in the PID structure:

$$[C](s, [\theta]) = [K_p] + \frac{[K_i]}{s} \qquad (27)$$

where the tunable parameters are $[\theta] = [[K_p], [K_i]]$.

Fig. 5-a presents the closed-loop scheme for the controller design, where weighting function $W_1(s)$ is for the tracking performances and $W_2(s)$ for the input control limitation.

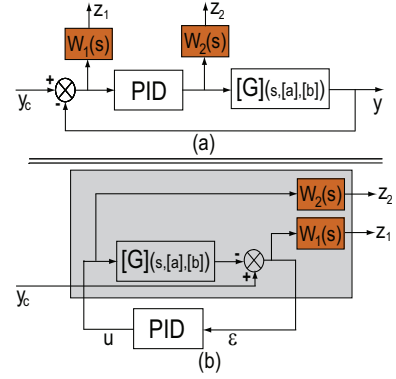From Fig. 5-a, we have:

Fig. 5.   a) The closed-loop scheme with the weighting functions. b) The standard scheme.

$$\begin{cases} z_1 = W_1(s)[S](s)y_c \\ z_2 = W_2(s)[C](s, [\theta])[S](s)y_c \end{cases} \qquad (28)$$

Such as $[S](s) = (1 + [C](s, [\theta])[G](s, [\mathbf{a}], [\mathbf{b}]))^{-1}$ is the sensitivity function.

From (28), the H_∞ standard problem becomes:

$$\begin{aligned} |[S](s)| \leq \left| \frac{\gamma}{W_1(s)} \right| \\ |[C](s)[S](s)| \leq \left| \frac{\gamma}{W_2(s)} \right| \end{aligned} \Leftrightarrow \begin{aligned} \|W_1(s)[S](s)\|_\infty \leq \gamma \\ \|W_2(s)[C](s)[S](s)\|_\infty \leq \gamma \end{aligned} \qquad (29)$$

where the aim consists to find the set-solution Θ of the PID parameters that ensures the H_∞ performance in (29), i.e.:

$$\Theta := \left\{ \theta \in [\theta] \left| \begin{cases} \|W_1(s)[S](s)\|_\infty \leq \gamma \\ \|W_2(s)[C](s, [\theta])[S](s)\|_\infty \leq \gamma \end{cases} \right. \right\} \qquad (30)$$

The weighting functions were chosen accordingly to the specifications (see Section IV-D). We chose:

$$\begin{cases} W_1(s) = \frac{0.002667s + 1}{0.002667s + 0.01} \\ W_2(s) = \frac{1}{2.5} \end{cases} \qquad (31)$$

Now we set $\gamma = 1$ and we solve the set-inversion problem in (30).

As described above, the problem (30) can be easily solved using the recursive algorithm presented in the Table II. The computation of the H_∞-norm of the two weighted interval plants $W_1(s)[S](s)$ and $W_2(s)[C](s, [\theta])[S](s)$ is based on the methods presented in [18]–[20].

Matlab-Software is used to implement the SIVIA algorithm. We choose an initial box for the controller parameters $[K_{p0}] \times [K_{i0}] = [0.4, 1.2] \times [400, 1200]$. The resulting subpaving is presented in Fig. 6. The dark colored subpaving Θ corresponds to the set parameters $[K_p]$ and $[K_i]$ of the controller (27) that ensures the performances defined by the H_∞-criterion (30).
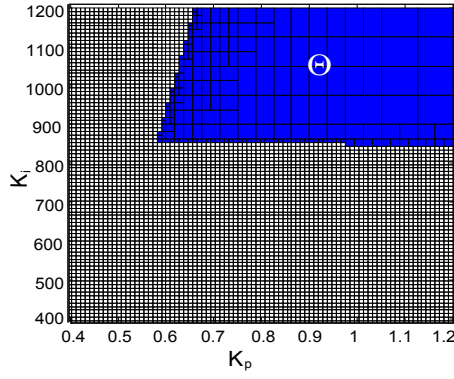
Fig. 6. Set-solution of the parameters $[K_p]$ and $[K_i]$ ensuring the wanted performances.

Fig. 7. Magnitudes of the bounds compared to the sensitivity $[S](s)$ and to the input transfer $C(s)[S](s)$.

Remark 4.1: We notice that any choice of the parameters $[K_p]$ and $[K_i]$ within the dark colored subpaving Θ (see Fig. 6) satisfies the conditions (30) and consequently ensures the required performances.

Remark 4.2: If $\Theta = \emptyset$ (i.e. no solution), the initial box of the parameters $[K_{p0}] \times [K_{i0}]$ must be changed and/or the specifications must be degraded and/or the structure of the controller must be modified (increase the order for example).

V. IMPLEMENTATION AND EXPERIMENTAL TESTS

The controller $C(s)$ to be implemented is chosen by taking any point parameters K_p and K_i within the set-solution Θ. In this example, we test two point controllers. We choose:

$$C_1(s) = \frac{0.8s+900}{s}$$

$$C_2(s) = \frac{s+1000}{s} \tag{32}$$

In order to prove that the inequalities (29) is satisfied, the magnitudes of the bounds $\left|\frac{1}{W_1(s)}\right|$ and $\left|\frac{1}{W_2(s)}\right|$ are compared to the magnitudes of the sensitivity function $|[S](s)|$ and of the transfer $|C(s)[S](s)|$ respectively (see Fig. 7) when using the implemented controllers (32).

The obtained results in Fig. 7 prove that the magnitudes of $[S](s)$ and $C(s)[S](s)$ are effectively bounded by that of $\frac{1}{W_1(s)}$ and $\frac{1}{W_2(s)}$ respectively for the both implemented controllers $C_i(s)$ $(i = 1, 2)$. This fact confirms that the specified performances are effectively ensured.

Now, we apply each controller $C_i(s)$ $(i = 1, 2)$ to the piezocantilever when its lengths $l = 16mm$ and when $l = 14mm$. Fig. 8 shows the experimental results when a step reference of $40\mu m$ is applied. As shown on the figure, the controllers (32) have played their roles since the closed-loop piezocantilevers satisfy the specifications. Indeed, experimental settling times obtained with $C_1(s)$ and $C_2(s)$ are about $tr_1 = 5.2ms$ when $l = l_1 = 16mm$ (Fig. 8-a) and $tr_2 = 7ms$ when $l = l_2 = 14mm$ (Fig. 8-b). The experimental overshoots and static errors are
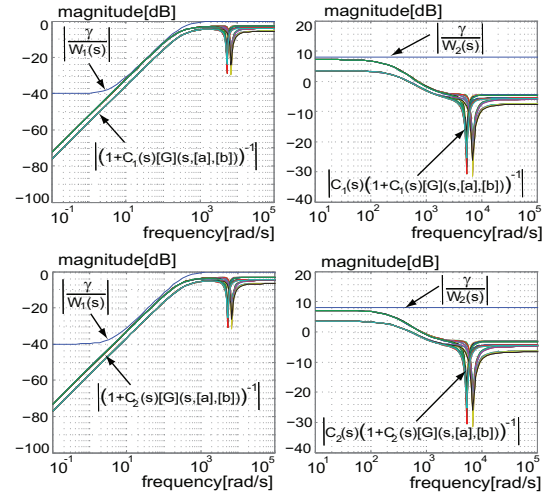
neglected ($D_{1,2} \approx 0$, $\varepsilon_{1,2} \approx 0 < 1\%$). Furthermore, the maximal voltages U applied to the both piezocantilevers are less than $40 \times 2.5 = 100V$, which should be the limit for a displacement of $40\mu m$. Indeed, the experiments shows that the maximal input voltage is $U^{max} = 97V$.
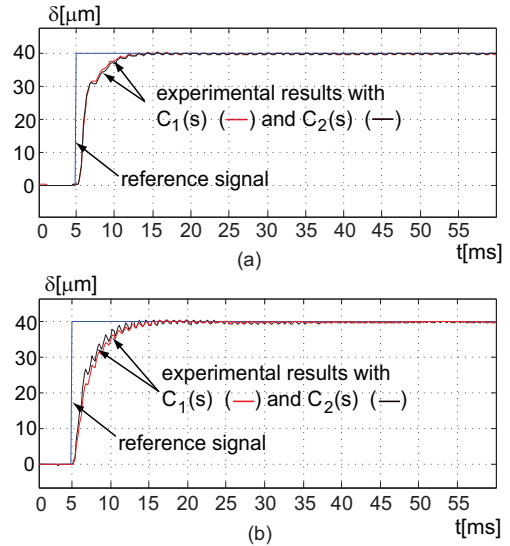

Fig. 8. Experimental step responses of the piezocantilever using $C_1(s)$ and $C_2(s)$. a: Piezocantilever with length $l = l_1 = 16mm$. b: Piezocantilever with length $l = l_2 = 14mm$.

VI. CONCLUSION

In this paper, interval techniques have been used to model the parametric uncertainties in piezoelectric actuators. Its main advantage is the ease and natural way to bound these uncertainties. Afterwards, we proposed to combine the H_∞-standard method with interval techniques to derive PID controllers that ensure the performances for the interval model. The main advantage

of the proposed control design is the possibility to derive low-order controllers for robust performances objective. The proposed approach was applied to piezocantilevers and the obtained experimental results effectively proved the efficiency of the approach.

REFERENCES

[1] S. Devasia, E. E. Eleftheriou, R. Moheimani, A survey of control issues in nanopositioning, *IEEE Transactions on Control System Technology (T-CST)*, Vol.15, No15, pp.802-823, 2007.

[2] Y. Haddab, N. Chaillet and A. Bourjault, "A microgripper using smart piezoelectric actuators", *IEEE/RSJ International Conference on Intelligent Robot and Systems (IROS)*, Takamatsu - Japan, 2000.

[3] J. Agnus, J. M. Breguet, N. Chaillet, O. Cois, P. de Lit, A. Ferreira, P. Melchior, C. Pellet and J. Sabatier, "A smart microrobot on chip: design, identification and modeling", *IEEE/ASME AIM*, Kobe Japan, pp.685-690, 2003.

[4] A. Sebastian, A. Pantazi, S. O. R. Moheimani, H. Pozidis, E. Eleftheriou, Achieving Subnanometer Precision in a MEMS-Based Storage Device During Self-Servo Write Process, *IEEE Transactions on Nanotechnology*, Volume 7, Number 5, 586-595, 2008.

[5] Micky Rakotondrabe, Cédric Clévy and Philippe Lutz, "Modelling and robust position/force control of a piezoelectric microgripper", *IEEE - CASE, (International Conference on Automation Science and Engineering)*, pp:39-44, Scottsdale AZ USA, Sept 2007.

[6] Micky Rakotondrabe, Yassine Haddab and Philippe Lutz, 'Quadrilateral modelling and robust control of a nonlinear piezoelectric cantilever', IEEE - Transactions on Control Systems Technology (T-CST), Vol.17, Issue 3, pp:528-539, May 2009.

[7] S. Khadraoui, M. Rakotondrabe and P. Lutz, Interval Modeling and Robust Control of Piezoelectric Microactuators, *IEEE Transactions on Control System and technology (T-CST)*, DOI.10.1109/TCST.2011.2116789, 2011.

[8] S. Khadraoui, M. Rakotondrabe and P. Lutz, "PID-Structured Controller Design for Interval Systems: Application to Piezoelectric Microactuators", *IEEE - ACC, (American Control Conference)*, accepted, San Francisco, California USA, June-July 2011.

[9] S. Khadraoui, M. Rakotondrabe and P. Lutz, "Robust control for a class of interval model: application to the force control of piezoelectric cantilevers", *IEEE - CDC, (Conference on Decision and Control)*, accepted, Atlanta Georgia USA, December 2010.

[10] L. Jaulin, M. Kieffer, O. Didrit, and E. Walter, *Applied Interval Analysis*. Springer, 2001.

[11] E. Walter, L. Jaulin, Guaranteed characterization of stability domains via set inversion, *IEEE Trans. on Autom. Control*, 39(4), 886-889, 1994.

[12] Ye. Smaginaa, Irina Brewerb, Using interval arithmetic for robust state feedback design, *Systems and Control Letters*, 46, 187-194, 2002.

[13] C.-T. Chen, M.-D. Wang, A two-degrees-of-freedom design methodology for interval process systems, *Computers and Chimical Engineering*, 23,1745-1751, 2000.

[14] J. Bondia, M. Kieffer, E. Walter, J. Monreal and J. Pict'o, "Guaranteed tuning of PID controllers for parametric uncertain systems", *IEEE Conference on Decision and Control*, 2948-2953, 2004

[15] K. Li, Y. Zhang, Interval Model Control of Consumable Double-Electrode Gas Metal Arc Welding Process, *IEEE - Transactions on Automation Science and Engineering (T-ASE)*, 10.1109/TASE, 2009.

[16] Keel, L. H., Bhattacharyya, S. P., "Control system design for parametric uncertainty", *International Journal of Robust and Nonlinear Control*, Vol.4, 87-100, 1994.

[17] R. E. Moore, *Interval Analysis*, Prentice-Hall, Englewood Cliffs N. J., 1966.

[18] Sen-Jian An, Lin Huang, Enping Wang, On the parametric H_∞ problems of weighted interval plants, *IEEE Transactions on Automatic Control*, Vol.45, 332-335, 2000.

[19] Sen-Jian An, Xiheng Hu, Branka Vucetic, Wanquan Liu, "Vertex results for parametric shifted H_∞ performance of weighted interval plants", *IEEE Conference on Decision and Control*, Vol.5, 4195-4196, 2000.

[20] Wang, L., H_∞ Performance of Interval Systems, *eprint arXiv:math/0211013*, Vol.1, 1-8, 2002.

[21] K. Zhou, J. Doyle, and K. Glover, *Robust and optimal control*, Prentice-Hall, 1996.

[22] Balas, G. J., Doyle, J. C., Glover, K., Packard, A. and Smith, R., *μ -synthesis and synthesis toolbox*, The Mathworks User's Guide-3, 2001.

[23] L. Jaulin and E. Walter, Set inversion via interval analysis for nonlinear bounded-error estimation, *Automatica*, 29(4), 1053-1064, 1993.