Vibration Energy Localization from Nonlinear Quasi-Periodic Coupled Magnets

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Abstract. The present study investigates the modeling of the vibration energy localization from a nonlinear quasi-periodic system. The periodic system consists of n moving magnets held by n elastic structures and coupled by a nonlinear magnetic force. The quasi-periodic system has been obtained by mistuning one of the n elastic structures of the system. The mistuning of the periodic system has been achieved by changing either the linear mechanical stiffness or the mass of the elastic structures. The whole system has been modeled by forced Duffing equations for each degree of freedom. The forced Duffing equations involve the geometric nonlinearity and the mechanical damping of the elastic structures and the magnetic nonlinearity of the magnetic coupling. The governing equations, modelling the quasi-periodic system, have been solved using a numerical method combining the harmonic balance method and the asymptotic numerical method. This numerical technique allows transforming the nonlinearities present in the governing equations into purely polynomial quadratic terms. The obtained results of the stiffness and mass mistuning of the quasi-periodic system have been analyzed and discussed in depth. The obtained results showed that the mistuning and the coupling coefficients have a significant effect on the oscillation amplitude of the perturbed degree of freedom.

Keywords: Energy localization · Nonlinear dynamics · Quasi-periodic system

1 Introduction

Over the last few years, energy harvesting from ambient energy has received increased attention. Several research projects have been oriented towards the design and the modeling of various harvesting systems. This trend of scavenging the ambient energy is related to the reduction of the required power supply for such microsystems and to the replacement of the battery which is limited by its life-time and requires maintenance. The harvesting approach is considering as a promising approach for innovation, miniaturization, respect for ecological issues and is part of the theme of renewable energies as well.

Diverse ambient energy sources are available in our environment and their conversion into electrical energy is a major challenge to increase the autonomy of isolated

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or abandoned systems. Each environment can correspond to one or more energy sources such as sunlight, wind, thermal gradients, and mechanical vibrations. For each of these sources, one or more conversion principles exist for generating electricity. Mechanical vibration sources provide potential energy that can be scavenged for charging self-powered systems. In several researches, design of mechanical to electrical energy devices, based on different conversion mechanisms, has been attempted (El-Hami et al. 2001; Erturk and Inman 2011; Cassidy et al. 2011; Yang et al. 2014). Currently, the most existing solutions for vibration-to-electricity transduction are accomplished by electrostatic (Roundy et al. 2003; Mitcheson et al. 2004), piezoelectric (Anton and Sodano 2007), and electromagnetic applications (Yang et al. 2009).

The purpose of this study is to investigate and to analyze the modeling of a quasi-periodic system. The effects of the mistuning and nonlinearities of the proposed system are discussed. The damping factor of the quasi-periodic system was estimated experimentally by the half-power bandwidth method (Papagiannopoulos and Hatzigeorgiou 2011). The geometric and magnetic nonlinearities introduced in the model as well as the mistuning effect of the mechanical stiffness allow enlarging the bandwidth and localize the energy.

2 System Modeling

The quasi-periodic system presented in this survey was inspired by existing published works (e.g. The nonlinearity was inspired by Mann and Sims (2009), Mahmoudi et al. (2014), Ping et al. (2015), and Abed et al. (2016) while the mistuning effect and the vibration localization was inspired by Yoo et al. (2003) and Malaji and Ali (2015)). However, the main drawback of the previous harvesting systems is mainly the large mechanical damping factor. This significant damping is due to the friction of the lateral surface of the center moving magnet with the inner surface of the coil holder which affects directly the oscillation amplitude and then the harvested power.

The concept proposed in this paper uses quasi-periodic structure in order to take advantage of the multimodal approach and the vibration localization, while the mechanical and magnetic forces have been used to guide and couple the center moving magnets as well as reducing the mechanical damping factor. The considered system is composed of $n + 2$ magnets (two fixed magnets and n moving magnets). The poles of the whole magnets have been oriented to repel each other. The center moving magnets are mechanically attached to structure with a very low damping factor. The coils have been placed next to the n moving magnets. The separating distance between the $n + 2$ magnets can be tuned via threaded mechanism in order to adjust the magnetic coupling force as well as the linear resonance.

2.1 Magnetic Force

The resulting magnetic force has been estimated numerically by the 2D finite element method (Meeker 2006) while varying the gap between the magnets. Figure 1a shows the FEMM model for one degree of freedom while Fig. 2b shows the numerical estimation of the top and bottom of the magnetic force as a function of the separation distance (gap d) between two magnets (Fig. 3).

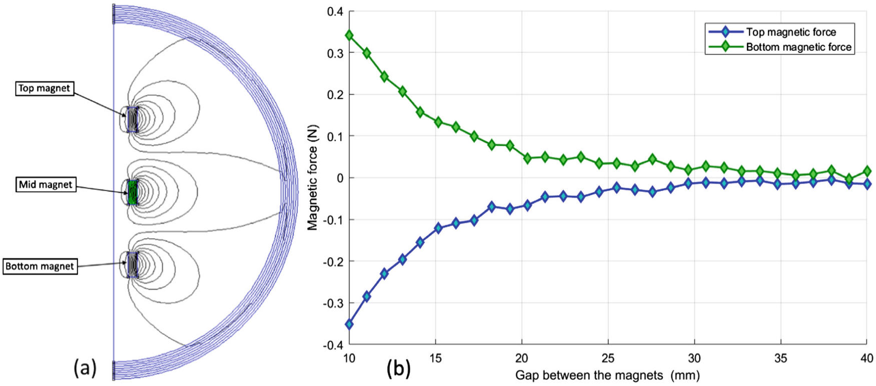

Fig. 1. Equivalent model for two moving magnets.

The numerical results of the magnetic force estimated by FEMM have been fitted for several values of gap d using a least-squares procedure. So, the total magnetic force can be identified as:

$$F^{mg}(x) = k_1^{mg}x + k_3^{mg}x^3, \tag{1}$$

where $k_1^{mg} = 2\lambda_1 + 4d\lambda_2 + 6d^2\lambda_3$ is the linear stiffness coefficient and $k_3^{mg} = 2\lambda_3$ is the cubic nonlinear stiffness coefficient in which d is the gap between the magnets. x is the displacement of the moving magnet. The FEMM result of the total magnetic force as a function of the displacement of the mid magnet and the fitting data for the gap equal to d = 40 mm as well as the magnetic linear stiffness k_1^{mg} deduced from the fitting FEMM data of different separating distance value.

The estimated parameters for the magnetic linear stiffness at d = 40 mm are $\alpha_1 = 313.71\ \mathrm{N\,m^{-1}}$, $\alpha_2 = -4.1e^{+3}\ \mathrm{N\,m^{-2}}$, and $\alpha_3 = 3.06e^{+3}\ \mathrm{N\,m^{-3}}$.

The accuracy of the fitted data has been checked by an overlay of the numerical data. The magnetic field B of the permanent magnets has been obtained analytically by the expression developed for ring magnets in reference (Camacho and Sosa 2013).

$$B(d) = \frac{\mu_0 M}{2}\left[\left(\frac{d}{\sqrt{d^2 + r_{out}^2}} - \frac{d-h}{\sqrt{(d-h)^2 + r_{out}^2}}\right) - \left(\frac{d}{\sqrt{d^2 + r_{int}^2}} - \frac{d-h}{\sqrt{(d-h)^2 + r_{\mathrm{int}}^2}}\right)\right], \tag{2}$$

where d stands for the gap between two magnets, r_{int} and r_{out} are the inner and outer radius respectively.

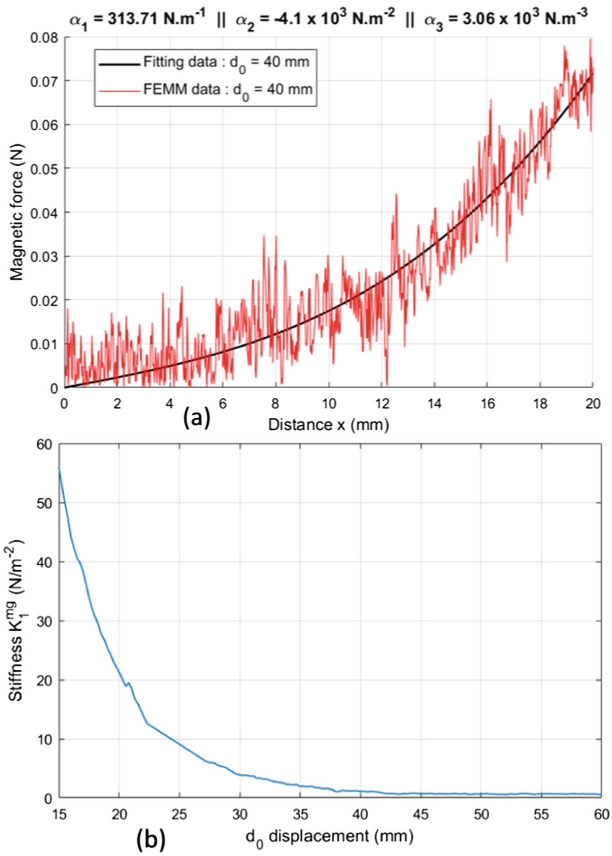

Fig. 2. (a) The FEMM result and the fitting data for $d = 40$ mm. (b) The linear stiffness k_1^{mg} estimated by fitting the FEMM data for each separating distance value.

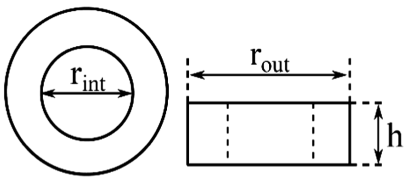

Fig. 3. Geometrical parameters of the magnet.

2.2 Governing Equations

In the present section, two center moving magnets are considered as illustrated in the equivalent mechanical and electrical model (Fig. 4). The proposed harvesting devise is

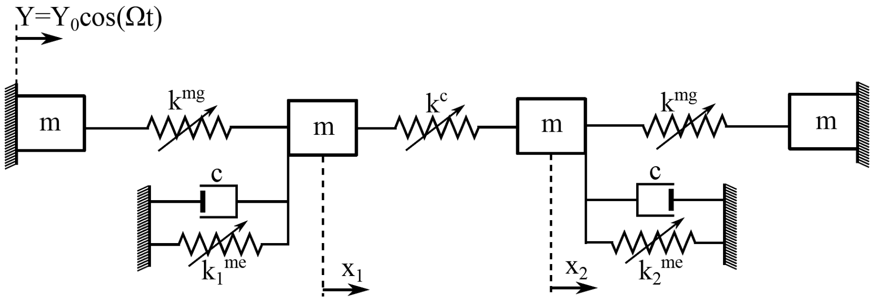

Fig. 4. Equivalent mechanical and electrical model for two moving magnets.

modeled using two forced duffing equations. So, the governing equation of the designed harvester can be written as:

$$m\ddot{x}_j + c\dot{x}_j + F_j^{me}(x) + F_j^{mg}(x) = -m\ddot{Y}; \quad with \quad j = 1, 2, \tag{3}$$

where c stands for the mechanical damping factors respectively. F_j^{me} and F_j^{mg} are the mechanical and magnetic forces for each moving magnet. $\ddot{Y}$ is the excitation acceleration of the support as shown in Fig. 4. It is assumed that the two center moving magnets have the same mass, mechanical and electrical damping.

$$\begin{cases} \ddot{x}_1 + 2\xi\omega_1\dot{x}_1 + \omega_1^2(1+2\beta)x_1 - \beta x_2 + \gamma x_1^3 - \beta_{NL}x_2^3 = -\ddot{Y} \\ \ddot{x}_2 + 2\xi\omega_1\dot{x}_2 + \omega_1^2(\alpha+2\beta)x_2 - \beta x_1 + \gamma x_2^3 - \beta_{NL}x_1^3 = -\ddot{Y} \end{cases}, \tag{4}$$

$$2\xi\omega_1 = \frac{c}{m}, \beta = \frac{k_c^L}{k_1^{me}}, \beta_{NL} = \frac{k_c^{NL}}{m}, \omega_1^2 = \frac{k_1^{me}}{m}, \alpha = \frac{k_2^{me}}{k_1^{me}},$$

where α and β are the stiffness mistuning and coupling coefficients, respectively.

The solving procedure uses the classical harmonic balance method combined with the asymptotic numerical method (Cochelin and Vergez 2009). This technique allows transforming the nonlinearities present in the governing equation (Eq. 4) into purely polynomial quadratic terms.

3 Results and Discussion

In the present section, several numerical simulations have been performed in the case of two moving magnets. These simulations enable us to highlight the importance of the nonlinearity and mistuning of the designed harvesting device. The mistuning coefficients α represents the ratio of the mechanical linear stiffness of the second moving magnet to the ones of the first moving magnet. It is assumed in the present simulation that $k_{mg} = k_c$.

Figure 5 represents the frequency response for periodic and quasi-periodic structures with $\beta = 0.0083$ and an acceleration $a = 0.006\ g$. Stiffness of the first moving

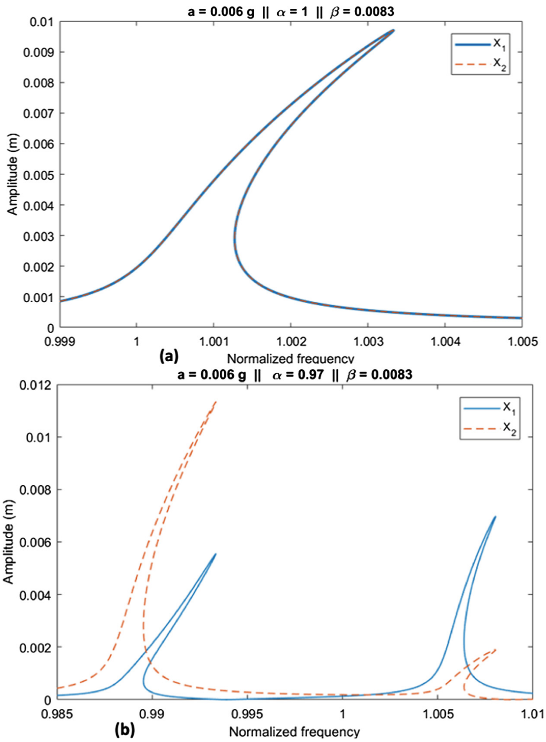

Fig. 5. Frequency response without (a) and with (b) stiffness perturbation.

magnet is taken as nominal stiffness. The mistuning was achieved by varying the stiffness of the second moving magnet. As shown in Fig. 5b, the amplitude of the perturbed dof was increased significantly with respect to the first dof. In addition, the bandwidth of the whole system was increased.

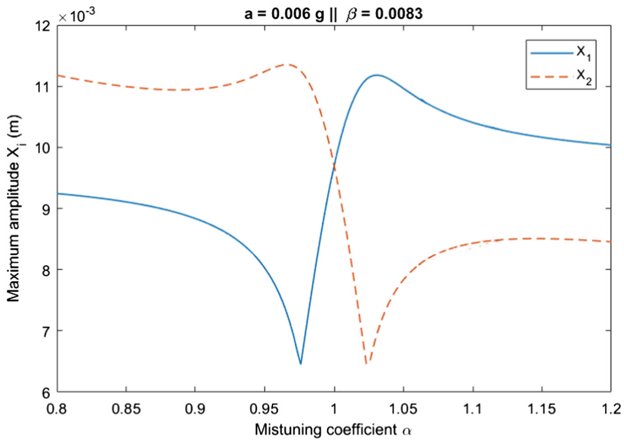

Fig. 6. Effect of the variation of the mistuning coefficient α on the maximum amplitudes.

Figure 6 shows the variation of the maximum amplitudes of the quasi-periodic system due to the variation of the mistuning coefficient α with an acceleration $a = 0.006\ g$ and $\beta = 0.0083$. As shown in this figure, when the mistuning coefficient α is less than 1, the amplitude of the perturbed dof increases with respect to the first dof. However, when $\alpha > 1$ the first dof represents an important amplitude compared to the perturbed dof.

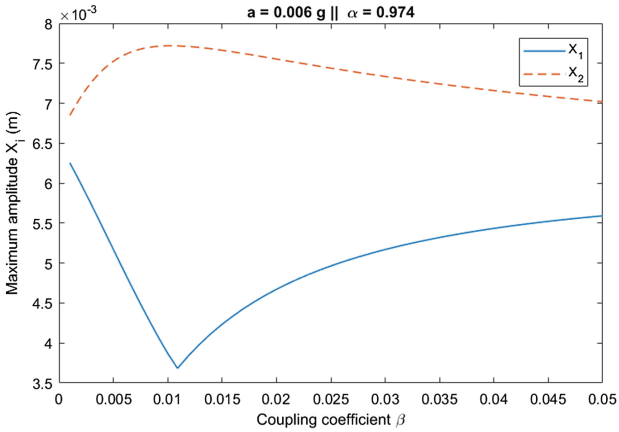

Fig. 7. Effect of the variation of the coupling coefficient β on the maximum amplitudes.

Figure 7 shows the variation of the maximum amplitudes of the present structure due to the variation of the coupling coefficient β with an acceleration $a = 0.006\ g$ and $\alpha = 0.97$. As shown in this figure, the coupling coefficient β has a significant effect on the oscillation amplitude of the proposed system.

4 Conclusion

In this paper, we studied the effect of the mistuning and coupling coefficients as well as the nonlinearity on the frequency response of a periodic structure. The obtained results show that the perturbation of one of the moving magnet, the magnetic coupling coefficient, and the nonlinearity increase the oscillation amplitude of the periodic system and enlarge the bandwidth as well. Thus, we can take advantage of these aspects to enhance the harvested power of a vibration energy harvesting mechanism. The proposed approach can be generalized to a large-scale quasi-periodic system.

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