DOI : 10.1109/PHM.2012.6228964

Scheduling predictive maintenance in flow-shop

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***Abstract*—Availability of production equipments is one major issue for manufacturers. Predictive maintenance is an answer to prevent equipment from risk of breakdowns while minimizing the maintenance costs. Nevertheless, conflicts could occur between maintenance and production if a maintenance operation is programmed when equipment is used for production.**

The case studied here is a flow-shop typology where machines could be maintained once during the planning horizon. Machines are able to switch between two production modes. A nominal one and a degraded one where machine run slowly but increase its remaining useful life. We propose a mixed integer programming model for this problem with the makespan and maintenance delays objective. It allows to find the best schedule of production operation. It also produces, for each machine, the control mode and if necessary the preventive maintenance plan.

I. Introduction

Availability of production equipments is one major issue for manufacturers.

Predictive maintenance is an answer to prevent equipment from risk of breakdowns while minimizing the maintenance costs. Nevertheless, conflicts could occur between maintenance and production if a maintenance operation is programmed when equipment is used for production.

In this paper, we propose a new model that proposes the best predictive maintenance policies. We consider a workshop production context where machines are continuously monitored. The machine could be control with two modes :

- a nominal mode for which machine has a fixed throughput rate ;
- a degraded mode where machines run slower but with less constraints and then components are less touch by degradation. Using this mode, the risk of breakdown is reduced and the remaining useful life increases.

Since machine are continuously monitored, we suppose that a prognostic system evaluates, for each machine, the remaining useful life for the two different modes. The remaining useful life for a machine is consider as a threshold that should not be crossed. Before this date a known preventive maintenance has to be planned. Then, a decision problem appears. What should be the best control mode for each machine and the best predictive maintenance policies to achieve a given production profile in the shortest duration.

The case studied here is a flow-shop typology where machine could be maintained once during the horizon of planning. We develop a mixed integer programming model for this problem. This model deals with the makespan objective. It allow to find the best schedule of production operation. It also produces, for each machine, the control mode and if necessary the preventive maintenance plan. In the paper we also propose some exact solution for various cases.

Works were previously proposed in the last decade. We can quote first a state of the art presented by Schmidt [5]. In his paper, the author reviews the results related to scheduling problems where machines are subjected unavailability constraints. Single and multi machine problems are analysed and criteria such as completion times and due dates are surveyed. More recently Ma *et al.* [4] also propose a survey of scheduling with deterministic machine availability constraints. This article is only dedicated on fixed maintenance period. All scheduling problem typologies are reviewed (parallel machine, flow-shop, job-shop and open-shop). Most of such kind of scheduling problems have been shown to belong to NP-Hard class. Then, some exact algorithms were proposed to solve smaller cases, otherwise dedicated heuristics are often proposed.

In the field of prognostics recent works proposed new original approaches. Prognoses a failure consists in estimating the time before the failure on a given equipment. It is also called the Remaining Useful Life (RUL). This task can be realized by using three main approaches: model-based prognostic, data-driven prognostic and experience-based prognostic as exposed in Lebold and Thurston [3]. In our approach, we suppose that machines are monitored. Then, prognostic based on data are the best adapted to our context. Some works were developed recently in this field [2]. We can cite for instance the contribution of Tobon-Mejia et al. [6]. The authors used Dynamic Bayesian Network, after a learning phase, to estimate the RUL of an equipment. In this paper, we will consider that this problem is already solved and we concentrate our works on decision problem that appears when the prognostic system gives its results (the RUL).

This paper is composed of 3 sections. First, we present the scheduling problem. On the one side the classical flow-shop case and on the other side the maintenance scheduling problem. Then, in section 3 we develop our model and the mathematical formulation of the problem. Finally, an exact resolution approach is presented. It gives some results on random generated problem.

II. Problem presentation

One of the main issues of maintenance is to guaranteed the availability of production systems. To reach this goal, maintenance departments define their own policies. For each machine to be maintained preventive and corrective maintenance policy is proposed. The resulting maintenance plan can be performed and leads to minimize the maintenance cost. Nevertheless, maintenance planning is often changed due to production constraints or unexpected breakdowns and so on. Then, conflicts appear between maintenance and production departments. One way to solve these conflicts is to schedule both maintenance tasks and production tasks together. This is known in the literature as the "joint maintenance and production scheduling problem" or "scheduling problem with availability constraints".

In this context, the problem we deal with is focussed on the flow-shop case. We describe, first, the constraints of this kind of shop. Then, we propose a simplified model for maintenance planning using prognostic information.

A. Production scheduling problem

The flow-shop problem is a well-known scheduling problem. This kind of production shop is composed of m machines organize in a line. n jobs have to be performed. each job production consists m tasks that have to be performed on the m machines in the same order, machine 1, machine $2,\ldots$, machine m. each machine is able to realize only one task at a time. We consider here the permutation problem. It means that when the order is defined for the first machine, the tasks are ordered with the same order for all other machines.

For each job i $(1 \leq i \leq n)$, processing time on machine j $(1 \leq j \leq m)$ is known and denoted by $p_{i,j}$.

Generally, the main objective in scheduling problem is to minimize the total duration of the schedule. This goal is called the makespan. Minimizing the makespan leads to maximize the use of machines.

B. Maintenance scheduling problem

To prevent risks of breakdown, preventive maintenance policy can be carried out. Generally, preventive tasks are realized periodically. These tasks are planned for each machine. They require that machine should be stopped. Then, a scheduling problem appears when production tasks are also forecast. Since maintenance have to be on time, it is often considered as a constraint for the production scheduling problem.

The objective for maintenance department is to optimize the availability of machines by proposing the optimal policy for maintain machines. The periodicity of preventive maintenance limits the risk of breakdown. The cost of maintenance then will increase if preventive maintenance is delayed since the risk of failures increases. Then, the objective for maintenance plan is to be on time. early maintenance task will increase the cost of maintenance and late task will lead to unexpected cost for corrective maintenance.

C. Prognostic problem

We suppose that each machine is subject to predictive maintenance policy. That means machines are monitored and prognostic systems evaluate the remaining useful life continuously. We suppose that each machine can be controlled using two running modes. The first mode, denoted "nominal mode", corresponds to the normal use of the machine. Jobs are then processed normally, all tasks are performed at the nominal time. In the second mode, denoted "degraded mode", machine is slowed down to avoid early failures. As a consequence, the production tasks will be longer than expected but in counter part remaining useful life is increasing.

For instance, consider a machine that use cutting tool. Such kind of machine can be monitored by a system that predict the useful life of the tool. Nevertheless, machine can be used in a degraded mode, for example by slowing down the cutting speed. Then, for the same machining, maybe the number of passes have to be greater and will spent more time than expected. But, this will allow to increase the life of the tool and consequently the useful life of the machine.

By this way, some machines would be able to fulfil the whole production tasks before the maintenance tasks. The possibility offers by prognostics systems leads to a new decision problem. What is the best maintenance policy for a given scheduling problem ?

D. A decision support problem

The problem can be set as follow:

considering a set of n jobs to be scheduled on m machines. each job have to be processed on the m machines in the same order. each machine can process only one tasks at a time. Machine are monitored be a prognostic system that provides the remaining useful life for a nominal running mode and for a degraded running mode. Predictive maintenance task have to be processed on machine before its remaining useful life.

Under these hypothesis, what is the schedule that minimizes the total production time (makespan) and guarantees the availability of all machines during the whole schedule. We will consider that the availability of a machine is at its maximum value if predictive maintenance operation are realized on time, i.e. before the remaining useful life of the machine. A second objective is to minimize the maintenance cost. This can be achieved if the maintenance tasks are programmed the latest as possible.

III. Mixed integer linear programming model

We are modelling the flow-shop scheduling problem that optimize the makespan objective where machines are subjected to predictive maintenance operations.

The decision will have two different issues : first, the production tasks have to be scheduled. In this part only the entry order will be defined. Then, predictive maintenance tasks have to be planned. This could also leads to decide the control mode of each machine.

A. Hypothesis

We make the following assumptions:

- the scheduling problem is a flow-shop with m machines;
- n jobs have to be scheduled while respecting their operating order;

- only the solution of permutation flow-shop will be consider;
- for each machine, a prognostic system provides the remaining useful life *RUL*. It is based on the use profile of the machine. Machines can be used either in "nominal mode" or in "degraded mode". Then prognostic system furnishes for each machine i two RUL values RUL_i^n and RUL_i^d that corresponds respectively to remaining useful life in nominal mode and remaining useful life in degraded mode;
- Machines are subject to at most one maintenance operation. This task cannot be planned later than the RUL time. We consider that beyond this date the failure risk is too important.
- we also suppose that at least one machine should be maintained during the horizon of the schedule. That means, it exists one machine i where the end of the last production task is greater than RUL_i^n;
- when maintenance operation is performed on a machine, the latter is running again as if it was new and we consider that no failure can occurred during the schedule horizon.

B. Notations

In the following we will used the notation defined here:

- J_j : job number j;
- $p_{i,j}$: production task processing time of job j on machine i when machine runs in nominal mode;
- $p'_{i,j}$: production task processing time of job j on machine i when machine runs in degraded mode;
- RUL_i^n : remaining useful life provide by the prognostic system at the beginning of the schedule when machine is running in nominal mode;
- RUL_i^d : remaining useful life provide by the prognostic system at the beginning of the schedule when machine is running in degraded mode;
- t_i : duration of maintenance operation on machine i.

C. Mixed integer linear program

We propose to model the problem with mixed integer programming.

1) Variables:

- $c_{i,j}$: completion time of task for job j on machine i;
- c_i^m : completion time of maintenance operation on machine i;
- $P_{i,j}$: real processing time of job j on machine i. It is either equals to $p_{i,j}$ if machine is running in nominal mode, or equals to $p\prime_{i,j}$ if machine is running in degraded mode.
- $X_{j,k}$: bivalent variable. $X_{j,k} = 1$ if $J_j \prec J_k$, 0 otherwise;
- $Y_{i,j}$: bivalent variable. $Y_{i,j} = 1$ if maintenance task is planned before job j on machine i, 0 otherwise;
- Z_i : bivalent variable. $Z_i = 1$ if machine i is set in degraded mode at the beginning of the schedule.

2) Constraints:

The problem is subjected to various constraints.

- The processing time of production tasks are not known in advance since, machine can switch between nominal mode and degraded mode.
- task order in each job is the second constraint.
- the machine are able to perform only one task at a time.
- maintenance task can be programmed on each machine. Then machine is stopped. Maintenance task should also be planned before the RUL of the machine.

We will now describe all the constraint by linear inequalities.

a) processing time: The processing time depends on the running mode of the machine. Three cases are possible :

- machine is running in nominal mode since the beginning of the schedule. It corresponds to $Z_i = 0$ then $P_{i,j} = p_{i,j}$;
- machine is running in degraded mode ($Z_i = 1$). In this case, two sub cases can occur :
  - $P_{i,j} = p'_{i,j}$ if J_j is planned before maintenance on machine i
  - $P_{i,j} = p_{i,j}$ si J_j is planned after maintenance on machine i

Then, we can deduce the following expression for $P_{i,j}$:

$$P_{i,j} = p_{i,j} * (1 - Z_i) + (p_{i,j} * Y_{i,j} + p'_{i,j} * (1 - Y_{i,j})) * Z_i \quad (1)$$

This equation have to be linearized. One possibility is to use linearization methods like those proposed in [1]. The simplest model consists in adding new bivalent variables. product of variables $Y_{i,j} * Z_i$ can be replaced by new variables $Z_{i,j}$ with additionnal constraints as follow :

$$P_{i,j} = p_{i,j} + (p'_{i,j} - p_{i,j}) * Z_i + (p_{i,j} - p'_{i,j}) * Z_{i,j} \quad (2)$$

$$\begin{aligned} & Z_{i,j} \leq Y_{i,j} \\ & Z_{i,j} \leq Z_i \\ & 1 - Y_{i,j} - Z_i + Z_{i,j} \geq 0 \\ & Z_{i,j} \geq 0 \end{aligned} \quad (3)$$

b) job constraints: All jobs have to follow the production line. Then, tasks of a job must satisfied precedence constraints that can be expressed by the inequalities 4.

$$\begin{aligned} & c_{i+1,j} - P_{i,j+1} \geq c_{i,j}, \forall j = 1 \ldots n, i = 1 \ldots m-1 \\ & c_{1,j} \geq P_{1,j}, \forall j = 1 \ldots n \end{aligned} \quad (4)$$

c) machine constraints: Each machine can perform only one task at a time. This constraint can be expressed by a disjunction of inequalities. For a couple of tasks $J_{i,j}$ and $J_{i,k}$ being hold on the same machine i, either $J_{i,j}$ holds before $J_{i,k}$, which is guaranteed by first inequalities of relation 5 or $J_{i,j}$ holds after $J_{i,k}$ which is expressed by the second part of relation 5.

$$\forall j = 1 \dots n-1; k = j+1 \dots n \text{ and } i = 1 \dots m$$
$$\begin{cases} c_{i,j} & \geq c_{i,k} + P_{i,j} \\ \text{or } c_{i,k} & \geq c_{i,j} + P_{i,k} \end{cases} \quad (5)$$

The relation 5 can be linearized by adding bivalent variables $X_{j,k}$ for all couple of tasks involved in the relation.

$$\begin{cases} c_{i,j} \geq c_{i,k} + P_{i,j} + M.X_{j,k} \\ c_{i,k} \geq c_{i,j} + P_{i,k} - M.(1 - X_{j,k}) \end{cases} \quad (6)$$

where M is a great number

d) Maintenance constraints: The objective of this constraint is to guarantee that maintenance tasks are not performed simultaneously to job tasks on a given machine. Thus, production task $J_{i,j}$ is placed either before maintenance operation on machine i (first inequality of relation 7), or after (second inequality of relation 7).

$$\forall j = 1 \dots n \text{ and } i = 1 \dots m$$
$$\begin{cases} c_{i,j} \leq c_i^m - t_i \\ \text{or } c_{i,j} - P_{i,j} \geq c_i^m \end{cases} \quad (7)$$

As we already done for machine constraints, it is possible to linearize the relation 7 by adding bivalent variables. Let $Y_{i,j}$ be this variable. $Y_{i,j} = 1$ if maintenance task is planned before job j on machine i, 0 otherwise. Then, relation 7 can be modified as follow :

$$\begin{cases} c_{i,j} \leq c_i^m - t_i + M.Y_{i,j} \\ c_i^m \leq c_{i,j} - P_{i,j} + M.(1 - Y_{i,j}) \end{cases} \quad (8)$$

e) remaining useful life constraints: Last constraints are due to prognostic system that provide remaining useful life values.

$$\forall i = 1 \dots m$$
$$c_i^m - t_i \leq RUL_i^n * (1 - Z_i) + RUL_i^d * Z_i \quad (9)$$

f) Optimisation constraint: Since one objective is to minimize the makespan, one more constraint should be added to the mathematical model.

$$\forall j = 1 \dots n$$
$$c_{m,j} \leq C_{max} \quad (10)$$

3) Objective function:

Two objectives have to be optimize:

- the makespan : it corresponds to the minimization of variable C_{max};
- the maintenance advance : each machine should be maintained before their remaining useful life, but it seems to be too expensive to maintain machine early. Then the main goal for maintenance placement is to minimize the earliness of this kind of operations. This can be expressed with the sum of earliness.

Finally, these two objectives can be combined in the same linear expression.

$$min(\alpha.C_{max} + \beta.\sum_{i=1}^{m}(RUL_i^n * (1 - Z_i) + RUL_i^d * Z_i - c_i^m)) \quad (11)$$

where α and β are two coefficients that allow to give a priority either on makespan, i.e. preference is given to the production, or on the maintenance delay.

The model is now complete. We will see in the next section how it has been implemented and what are the main results obtained.

IV. Model implementation

The previous model was implemented with *Gurobi library* in Java language.

A. Gurobi library with java

The Gurobi Optimizer is a solver for linear programming, quadratic programming and mixed-integer programming. For the latter models, it incorporates the latest methods including cutting planes and powerful solution heuristics.

The Gurobi Optimizer is written in C and is accessible from various languages, such as C, C++ python or Java. In Java it is a library (Java archive resource), available for all operating systems.

For our purpose, the main class used `GRBModel` which allows to create a mixed-integer model. Then, solving a given model is done by `optimize` method.

B. Data generation

In order to test our mixed integer model, we propose to generate random problem, for various sizes. The generation was completed as follows:

- size of problems:
  - $n \in \{3, \dots, 8\}$;
  - $m \in \{3, \dots, 10\}$;
- processing time of jobs is selected from a uniform distribution (U) over [20;50]: $p_{i,j} \in U[20, 50]$;
- processing time of jobs in degraded mode is obtain by extended $p_{i,j}$. $p'_{i,j} = p_{i,j} * \gamma_i$, where γ_i is generated for each machine i from U[1;1.5];
- processing time of maintenance operation is selected from a uniform distribution (U) over [10;30]: $t_i \in U[10, 30]$;
- RUL_i^n is generated from U[0,$\sum p_{i,j}$];
- RUL_i^d is equals to $RUL_i^n * \lambda$ where λ is generated from U[1.5;2];

Then, for all combination of n and m values, we have generated 10 instances of problem.

TABLE I
COMPUTATION TIME (IN MS)

n	m	cpu FS	cpu FSM	cpu FSP
4	3	12.4	18.7	55.3
	4	15.7	24.2	61.0
	5	17.3	28.9	78.7
	6	21.4	40.0	90.5
	7	20.7	41.1	127.6
	8	22.2	39.3	146.3
	9	27.4	47.4	140.0
	10	27.9	52.4	216.0
6	3	43.6	60.3	314.8
	4	49.3	85.4	661.1
	5	68.0	109.7	842.8
	6	72.1	115.4	975.4
	7	86.0	126.8	989.0
	8	91.0	211.3	1261.8
	9	105.6	217.2	1928.3
	10	111.0	224.7	1775.5
8	3	761.7	698.5	4559.6
	4	937.3	1280.6	7069.8
	5	1176.4	1182.2	9986.9
	6	1191.8	1275.7	12318.6
	7	1237.7	1493.3	22451.5
	8	1435.0	1917.5	23996.3
	9	1573.2	2829.8	31202.4
	10	1689.4	2310.5	39226.0

TABLE II
COMPARISON OF MAKESPAN FOR THE 3 MODELS

n	m	C_{max} (FS)	C_{max} (FSM)	$\% \nearrow C_{max}$	C_{max} (FSP)	$\% \searrow C_{max}$
3	4	205	299	45.85	262	12.37
		195	238	22.05	221	7.14
		200	233	16.50	232	0.43
		234	286	22.22	286	0.00
		200	271	35.50	249	8.12
5	5	319	359	12.54	359	0.00
		337	427	26.71	380	11.01
		324	375	15.74	368	1.87
		331	427	29.00	423	0.94
		320	364	13.75	364	0.00
8	5	428	454	6.07	454	0.00
		429	471	9.79	471	0.00
		395	432	9.37	425	1.62
		426	510	19.72	490	3.92
		456	510	11.84	504	1.18
8	10	599	634	5.84	634	0.00
		651	702	7.83	702	0.00
		624	734	17.63	734	0.00
		622	733	17.85	706	3.68
		609	660	8.37	650	1.52

TABLE III
RESULTS WITH $\alpha = 1$ AND $\beta = 1$

n	m	C_{max} (FS)	C_{max} (FSM)	Maint. (FSM)	$\% \nearrow C_{max}$	C_{max} (FSP)	Maint. (FSP)	$\% \searrow C_{max}$
5	3	242.3	278.5	6.8	15.0	275.9	5.8	0.8
	5	324.2	368.2	3.7	13.6	364.8	4.3	0.9
	7	405.2	451.5	3.4	11.5	451.0	2.9	0.1
	9	462.6	509.2	4.0	10.2	509.8	0.7	-0.1
8	3	348.0	378.9	1.0	8.9	375.3	4.0	0.9
	5	423.8	463.4	7.3	9.4	463.9	6.3	-0.1
	7	496.2	535.8	3.9	8.0	535.4	3.5	0.1
	9	578.3	623.0	1.7	7.7	622.0	1.8	0.2

C. Experimental results

This part proposes the results of the mixed integer model optimization. For all instances, three models were solved. The first one is those presented previously. In the second one, we try to solve exactly the problem where no degraded mode are proposed. Thus, it corresponds to the problem called flow-shop with availability constraints. On each machine one maintenance should be planned before a given time. Finally, the third model solved is the classical flow-shop without maintenance tasks. In the rest of this section we called FS (flow-shop) the latter model, FSM (flow-shop with maintenance) the second model and FSP (flow-shop with predictive maintenance) our new model.

In table I one can see a comparison of the cpu time for solving all cases. The time given here is the average time of 10 instances for each combination of n and m values. In this first test our objective is to give the priority to maintenance tasks. Then, coefficients α and β were fixed to 0.01 and 1 in relation 10.

It is not a surprise, our model is much more greater than the two others. Then, with more constraints and more variables it is the slowest to solve. Second result, shows in table II is the effect of maintenance tasks on the makespan objective when maintenance objective has priority. Makespan obtain are increased by about 13%. Moreover, in several cases (about 51%) solutions are better with use of degraded mode on at least one machine. Table II shows also that when degraded mode is proposed the makespan reduction can reached 13% compared with the model FSM.

Another experiment were done where coefficient α and β were modified. More importance were given to the makespan objective. We fixed, for this last experiment $\alpha = 1$ and $\beta = 1$. Table III shows the results obtain if the priority is given to the production tasks. Indeed, makespan is better optimize. Deviation of C_{max} compared to previous tables is less important (about 10%). Nevertheless, the second part of the criteria is not so bad, since maintenance objective is not so great (column Maint. in table III).

V. CONCLUSION AND PERSPECTIVES

We proposed in this paper an original approach for solving scheduling problem with predictive maintenance constraint. We have presented a mixed integer linear model that is able to consider flow-shop scheduling problem, where maintenance operation can be planned on each production machine. if machine are able to be controlled with two production modes and prognostic systems are able to give the remaining useful life in the both cases, then, our approach propose the best schedule of n jobs. The model presented is able to optimize a combination of two criteria. One for the production tasks and the other for maintenance operations that have to be placed before the RUL of the machines.

We show, considering the results obtained, that for several cases the best solution is reached when some machine are switched in degraded mode. This allows to delay the maintenance operation which minimizes the preventive maintenance cost.

With regard to these first results, further works would be continued. One can manage other typologies of scheduling problems. For instance, parallel machines were often used either in industry or in academic scheduling approach. Thus, it is an interesting context were our approach can be extended.

Another way to pursue these works, is to develop dedicated heuristics. It may then be possible to solve large size problems.

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