Evanescent-wave tuning of a locally-resonant sonic crystal

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Locally-resonant sonic crystals can support band gaps at low frequencies defined by resonances internal to the unit cell. Band gap frequencies are dictated by the choice of resonators and their interaction with the medium supporting acoustic wave propagation. We show that locally-resonant band gaps can be tuned by engineering the dispersion of the evanescent waves appearing in the propagation medium at the resonator sites. Specifically, we consider experimentally a tubular waveguide filled with different levels of water and grafted with a periodic array of acoustic resonators. Water filling continuously tunes the dispersion of evanescent waves by changing the waveguide cross-section. Dispersion relations and transmission properties are obtained with a three-dimensional time-harmonic finite element model of wave propagation. Numerical and experimental results are found to be in good agreement. The present work is relevant to the practical design of tunable acoustic devices.

During the last 15 years, a great deal of efforts has been made to achieve control of waves propagating within phononic/sonic crystals exhibiting elastic/acoustic band gaps1. The propagation of waves is completely forbidden in the band gaps2. It has since then been observed that band gaps can be arise because of two different mechanisms, Bragg interference3 and local resonances of substructures in the unit cell4. Bragg band gaps can be generated when the wavelength has a definite relation to the periodicity. In contrast, locally resonant (LR) band gaps appear around local resonances and do almost not depend on periodicity. At resonance, the energy of waves propagating in the matrix can be efficiently stored and delayed. For a given periodicity, LR gaps can appear at wavelengths much larger than the lattice constant. Numerous applications have been researched based on locally resonant structures in the low-frequency range, such as filters5, waveguides6, negative effective parameters7, interior noise reduction8, or vibration control9.

Because of these features of LR band gaps, various LR structures are designed and fabricated10,11. However, topology or geometrical parameters12–14 of these systems are hardly tunable. In contrast, reconfigurability is easier to realize by using sonic crystals15. Elford et al.16 designed novel noise barriers by using Matryoshka sonic crystals with different number of C-shaped shells. Wang et al.17 investigated longitudinal near-field coupling between acoustic resonators grafted along a waveguide through altering the length of resonators. Arguably, the structures proposed in the above works are hardly continuously tunable. Selectively filling a fluid into resonators has been proposed as a way to control local resonators18, but it could be difficult to control precisely and to equalize the filling level of a large number of periodic resonators in practical applications. Consequently, changing wave propagation properties in the matrix rather than in the resonators could be an interesting alternative to obtain tunability of band gaps.

In this Letter, we consider a locally-resonant sonic crystal built using a tubular waveguide grafted with periodic acoustic resonators. Local resonance band gaps appear close to resonance frequencies and depend on the grafting conditions. Tunability is experimentally realized by filling the waveguide with water up to a controlled level, without acting on the properties of the resonators. The water level changes the waveguide cross-section and as a result tunes the dispersion of evanescent guided waves. In the following, we examine the modifications of spectral transmission that result and show tunability of locally-resonant band gaps.

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Figure 1 shows a sketch of the experiment. The sonic crystal sample is constructed on a cylindrical polyvinyl chloride (PVC) tube used as waveguide for acoustic waves in air. The tube has a length $L = 2\,\text{m}$, an inner diameter $d_1 = 10\,\text{cm}$ and a 2 mm thick wall. Two bent tubes are glued at the ends of the waveguide in order to hold water inside. The height of water, $h_w$, can be continuously adjusted by filling or removing water. A periodic array of resonators is introduced in the form of PVC tubes closed at one extremity and grafted onto the waveguide with a period $a = 8\,\text{cm}$. The radius of the resonators is $d_2 = 2.5\,\text{cm}$, and their length $b = 24\,\text{cm}$. A source and a receiver for sound in the audible range are used to measure acoustic transmission through the locally-resonant sonic crystal, following the method of Ref.\textsuperscript{19}.

In order to analyze experimental results, we use a time-harmonic finite element model (FEM) of pressure wave propagation\textsuperscript{19}. The sound velocity inside the tube, $c = 333\,\text{m/s}^{-1}$, was determined experimentally and is used without any adjustment in numerical simulations. The FEM model allows us for evaluating the transmission coefficient through the waveguide. The surface separating water and air is considered as imposing a perfectly rigid boundary condition, i.e. acoustic wave propagation in water is neglected. In addition, the dispersion of propagating and evanescent guided waves is determined by computing the complex band structure\textsuperscript{19–21}.

We first investigate numerically the local resonance mechanism for a single grafted resonator as a function of water height. Figure 2 shows the transmission around the three resonances that appear below $1800\,\text{Hz}$. When the waveguide is empty ($h_w = 0$), the transmission dips appear around 327.3, 975.4 and 1601.3 Hz. When the waveguide is half filled ($h_w = d/2$), the dips are moved upward to 332.4, 995.2, and 1651.7 Hz. Figure 2 shows that the pressure distributions inside the resonator at the

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$$\Omega^0 = \Omega^2 + \frac{\kappa_{22}}{2\alpha}, \quad (1)$$

where $\Omega$ is the natural frequency of the resonator, $\alpha$ is the imaginary part of the wavenumber for the least evanescent guided mode (the wave with the smallest imaginary part of the propagation constant), and $\kappa_{22}$ is a coupling coefficient proportional to the overlap integral of the modal distributions of the least evanescent guided wave and the fundamental propagating guided wave. Loosely speaking, Equation (1) implies that energy at resonance is the sum of the energy in the resonator plus the energy of the evanescent wave. As the local resonance frequency increases, the energy in the evanescent wave must increase, since $\Omega$ remains fixed.

These observations point at the change of the dispersion relation for guided waves as the water level changes. Fig. 3(a) show the complex dispersion relation in the case that $h_w = d/2$. At low frequencies, there is a single propagating guided wave with linear dispersion and speed of sound $c$. All other guided waves are evanescent, as indicated by their pure imaginary wavenumbers.
The least evanescent of these waves reaches a cut-off at \( \omega_c/(2\pi) \), above which it becomes propagating and thus propagation is not monomodal anymore. As a note, the dispersion relation is

\[
\omega^2 = c^2 k^2 + \omega_c^2
\]

for all waves, with \( k \) the complex wavenumber, and only the value of \( \omega_c \) is different between different waves. Fig. 3(b) shows the variation of the cut off frequency of the least evanescent guided wave as a function of the height of water. As the height of water increases, \( \omega_c \) first decreases until \( h_w = 0.3d \) and then increases again. Given that \( \alpha = \sqrt{\omega^2 - \omega_c^2}/c \), this number at a fixed frequency first decreases then increases following \( \omega_c \). The value of the coupling coefficient \( \kappa_{22} \) also changes with the water height. Indeed, the pressure distributions shown as insets of Fig. 3(b) show that the modal shape remains relatively stable though the cross-section of the waveguide changes significantly. Owing to the circular symmetry of the waveguide for \( h_w = 0 \), the two independent modes \( \omega_{00} \) and \( \omega_{00} \) are degenerate. In contrast, the symmetry of the waveguide is broken as soon as \( h_w \neq 0 \), leaving a single mode defining the least evanescent guided wave. Overall, it is obtained numerically that both changes in \( \alpha \) and \( \kappa_{22} \) combine to lead to a steady increase in the first three locally resonant frequencies \( \omega_0/(2\pi) \) as a function of water level, as shown by Fig. 3(c). The locally-resonant frequencies can thus be tuned continuously by adjusting the water level in the waveguide.

Next, we consider a sonic crystal composed of a periodic sequence of five grafted tubes and we observe the change in locally-resonant band gaps as the water level is changed. Locally-resonant band gaps are expected to appear around the resonant frequencies for the single resonator and their width is known to depend on the reflection coefficient on a single resonator\(^{19} \). We first compare in Fig. 4 band structures and transmission spectra for \( h_w = 0 \) and \( h_w = d/2 \), obtained both experimentally and numerically. Normalized pressure distribution at labeled points taken at the 0th, 1st, and 2nd local resonance frequencies respectively are shown in Fig. 4(a1) - (b1). The insets in Fig. 4(a1) and (b1) show the eigenmodes at the frequencies of the lower edge (L) and the upper edge (U) of the 0-th local resonance band gap. It is seen that the pressure distributions at the upper edge extend both in the resonator and in the waveguide. In contrast, the pressure distributions at the lower edge only concentrate inside the resonator. It was checked that the situation is similar for the other two band gaps. As a result, the lower edges of the band gaps remain almost unchanged, since they are not affected by the waveguide characteristics, while the upper edges shift to higher frequencies as water is filled into the waveguide. Overall, band gaps become wider. Experimental transmission curves appear more rounded than those obtained from numerical simulation, which we attribute to losses that are present in the experiment but are not taken into account numerically. Overall, both numerical and experimental transmissions are consistent and indicate that locally-resonant band gaps enlarge when \( h_w \) is changed from 0 to \( d/2 \). The third band gap that was hardly opened for \( h_w = 0 \) especially deepens for \( h_w = d/2 \). This could be attributed to the
but instead works by playing with the grafting conditions ing the propagation velocity of the supporting medium, any parameter of the resonators, and neither on chang-guided waves. Tunability thus does not rely on changing the cross-section and thereby the dispersion of evanescent level of water inside the waveguide, effectively changing Band gaps can be tuned continuously by changing the grafting with a periodic array of acoustic resonators. Since the level of water in the waveguide can be changed accordingly. (c) (d) Experimental transmission spectra as a function of the normal- level. Moreover, the transmission generally gets smaller. As a result, band gaps become gradually wider as the cross-section of the waveguide decreases with increasing water level. Moreover, the transmission generally gets smaller. Since the level of water in the waveguide can be changed continuously, the width of the band gaps can be tuned accordingly.

In summary, we have demonstrated the tunability of a locally-resonant sonic crystal built using a waveguide grafted with a periodic array of acoustic resonators. Band gaps can be tuned continuously by changing the level of water inside the waveguide, effectively changing the cross-section and thereby the dispersion of evanescent guided waves. Tunability thus does not rely on changing any parameter of the resonators, and neither on changing the propagation velocity of the supporting medium, but instead works by playing with the grafting conditions of the resonators to the propagation matrix. Both the central resonance frequency and the width of the band gaps were shown to depend on the water level. An application could be the acoustic monitoring of liquid level inside tubes and pipes. The present work gives insights for the practical design of tunable acoustic devices. The ideas in this letter can be directly extended to 2D acoustic metamaterials.

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