# Effective slowness surfaces for waves in anisotropic elastic composites

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The problem of the homogeneization of the wave properties of periodic elastic composites at low frequencies or long wavelengths is a rather old one. With the recent interest in phononic crystals and acoustic metamaterials, however, it has become important to predict accurately the effective wave properties for arbitrary materials, lattice types, and structures. Our motivation is to devise formulas for the estimation of the effective phase and group velocities in periodic composites containing sharp discontinuities of material distribution within a unit cell.

Basing on previous studies that used the plane wave expansion method [1,2], we constructed estimation formulas that are based on a variational formulation that is easily implemented with finite element analysis. Our derivation is based on second-order perturbation theory, with the propagation direction dependent zero-order solution being a simple uniform displacement and the first-order correction resulting from material discontinuities. In practice, in order to obtain an effective slowness surface it is only necessary to solve one periodic boundary value problem per propagation direction. The method is hence numerically very efficient.

The method is applied to sonic crystals (for pressure waves in fluids) and to phononic crystals (for elastic waves in solids). Formulas are obtained for all cases. Effective slowness surfaces are obtained and plotted. If a connected propagation matrix extends throughout the unit cell, it is usually found that the effective anisotropy is very small and is furthermore rather independent of the lattice type. If a solid skeleton is considered instead, the effective anisotropy can in contrast be quite large.

## References

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[2] Torrent, Daniel, Yan Pennec, and Bahram Djafari-Rouhani. "Resonant and nonlocal properties of phononic metasolids." Physical Review B **92** 174110 (2015).