

# Modeling and Control of a lumped parameter flexible beam with Port Hamiltonian approach<sup>\*</sup>

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**Abstract:** The purpose of this paper is to show how can be derived a simple finite dimension port Hamiltonian model for a flexible beam. The Port Hamiltonian approach to this problem gives the possibility of developing a control starting from energy considerations. On this purpose, a control law will be designed on the found model by using IDA-PBC method. This work has been done looking at the possible application on a endoscope actuated with multiple electroactive polymers attached directly in different places of the beam. The lumped parameters model offers the possibility of having input/output ports in every joint between two successive element in which the beam has been discretized. In such a way, the action of actuators is easily modeled as an input force in the joints where they are actually presented.

*Keywords:* Medical endoscope; port Hamiltonian system; IDA-PBC.

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## 1. INTRODUCTION

The modeling of medical endoscope based on the continuum robots have drawn many researchers attention since the last century (Anderson et al., 1967; Robert J. Webster and Jones, 2010). The modeling of medical endoscope has been considered in (Chikhaoui et al., 2014) by a kinematic approach. The dynamic modeling of continuum robots has been proposed in (Falkenhahn et al., 2015). The principle of the medical endoscope (Fig. 1) is to bending the main body with the Ionic polymer metal composites (IPMC) actuators. The IPMC is one class of most important electroactive polymer (EAP) actuators. The main body of the endoscope is a flexible structure and the IPMC consists of a poly-electrolyte gel and metal electrodes plated by a chemical process. Hence we need a powerful modeling approach to model this complex multi physical system.

In this paper, we propose to use the framework of port Hamiltonian system for the modeling and control a class of medical endoscope. This framework is very power for modeling and control of large class of mechanical, electro-mechanical and multi physical systems (Duindam et al., 2009). The modeling by port Hamiltonian approach is based on the energy exchanges between the components

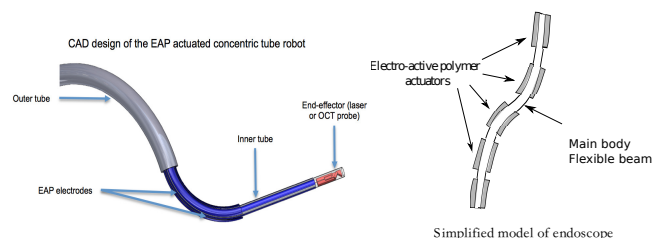


Fig. 1. Medical endoscope and its simplified model

of the systems. This framework proposes a very simple way to interconnect the different parts of system through the energy exchange ports. It is very adapted to model the medical endoscope interconnect with the IPMC actuators.

On the other hand, the port Hamiltonian approach also provides some useful control laws with the clear physical interpretation via energy shaping of the closed-loop system, damping injections, for example the Control by Interconnection (Ortega et al., 2001) and Interconnection and damping assignment passivity-based control (IDA-PBC) (Ortega et al., 2002). In this paper, we propose to an IDA-PBC design method for the endoscope model in order to control its position.

This paper is organized as follows. The Section 2, a port Hamiltonian model for medical endoscope system is presented. This model is based on the force actuated lumped parameter flexible beam. A position control design is proposed in Section 3 by using the IDA-PBC method.

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Some simulation results are shown in the Section 4 to illustrate the effectiveness of proposed methods. At last, the Section 5 gives some final remarks and perspectives in the future work.

## 2. MODELING OF A $N$ -DEGREE OF FREEDOM BEAM

The beam can be thought as composed of  $n$  different elements connected by  $n$  joints, where in every joint are present both a angular spring and a damper. The framework is planar and so all the links are allowed to move only in the  $x$ - $y$  plane.

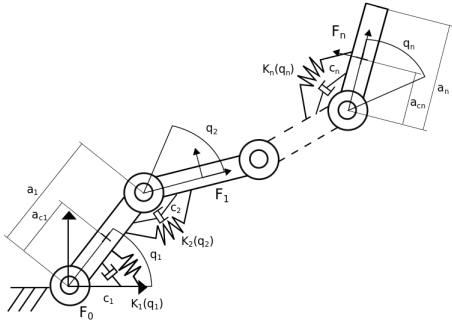


Fig. 2. Lumped parameters Beam

The parameters of the  $n$ -degree of freedom mechanism, with  $i = 1, 2, \dots, n$ , are:

- $q_i$  the  $i$ -th joint angular displacement;
- $m_i$  the  $i$ -th link's mass;
- $I_i$  the moment of inertia about the axe passing through the Center of Mass (CoM) of the  $i$ -th link;
- $a_i$  length of the  $i$ -th link;
- $a_{Ci}$  distance between the  $i$ -th Joint and the CoM of the  $i$ -th link;
- $\tau_i$  applied torque on the  $i$ -th joint;
- $K_i(q_i)$  non-linear stiffness of the  $i$ -th joint;
- $c_i$  viscous damping at the  $i$ -th joint;
- $P_i E_i$  Potential and Kinetic energy of the  $i$ -th link.
- $F_0$  is the inertial frame.
- $F_i$  is the reference frame attached to the CoM and with axes parallel to principal axes of inertia of the  $i$ -th link .

To find the total energy of the system, it's first necessary to find the kinetic and the potential energy associated to every single link and then sum everything together.

### 2.1 Total kinetic energy computation

The Kinetic energy of the  $i$ -th link has the form:

$$E_i = \frac{1}{2} m_i v_{Ci}^T v_{Ci} + \frac{1}{2} w_i^T R_i \tilde{I}_i R_i^T w_i, \quad (1)$$

where  $v_{Ci}$  is the speed of the CoM of the  $i$ -th link,  $w_i$  is the angular speed of the  $i$ -th link with respect to  $F_0$ ,  $\tilde{I}_i$  is the Inertia matrix of the  $i$ -th link with respect to  $F_i$ ,  $R_i$  is the Rotational matrix between  $F_i$  and  $F_0$ .

Now, the goal is to express the Kinetic energy of every link in respect only to the derivatives of the angular displacements. Thanks to the rigidity of the links, it is

possible to relate both the speed of Centers of mass and the angular speeds to the derivative of the angular displacement of every joint. The relation that links angular displacements derivative to angular speeds is trivial:

$$w_i = \dot{q}_1 + \dot{q}_2 + \dots + \dot{q}_i. \quad (2)$$

Then, this relation can be expressed through the use of the so called angular Jacobian,

$$w_i = J_w^i \dot{q}, \quad (3)$$

where  $q$  is the vector containing all the angular displacement and  $\dot{q}$  is the one containing all the derived angular displacement. In this case it can be seen that the angular velocity Jacobian isn't dependent on the angular displacements. This is not the case for the Jacobian related to velocities of the center of mass. The velocity jacobian of the  $i$ -th link can be founded differentiating with respect to time the equation that express the position of the  $i$ -th center of mass in the  $F_0$  frame.

$$q_{Ci} = \begin{bmatrix} x_{Ci} \\ y_{Ci} \end{bmatrix} = \begin{bmatrix} f_{xi}(q) \\ f_{yi}(q) \end{bmatrix} = f_i(q), \quad (4)$$

where,

$$f_{xi}(q) = \sum_{k=1}^{i-1} a_k \cos\left(\sum_{j=1}^k q_j\right) + a_{Ci} \cos\left(\sum_{k=1}^i q_k\right), \quad (5)$$

$$f_{yi}(q) = \sum_{k=1}^{i-1} a_k \sin\left(\sum_{j=1}^k q_j\right) + a_{Ci} \sin\left(\sum_{k=1}^i q_k\right). \quad (6)$$

Differentiating  $q_{Ci}$  with respect to time, we obtain:

$$\dot{q}_{Ci} = v_{Ci} = \frac{df_i(q)}{dq} \dot{q}. \quad (7)$$

And thus we obtain the velocity Jacobian,

$$J_v^i = \frac{df_i(q)}{dq}. \quad (8)$$

Now it is possible to express the Kinetic energy of every link with respect to the derivative of the displacement vector

$$E_i = \frac{1}{2} \dot{q}^T (m_i J_v^{iT}(q) J_v^i(q) + J_w^{iT}(q) R_i \tilde{I}_i R_i^T J_w^i(q)) \dot{q}. \quad (9)$$

And then the total Kinetic energy of the beam can be expressed as:

$$E = \frac{1}{2} \dot{q}^T \sum_{i=1}^n [m_i J_v^{iT}(q) J_v^i(q) + J_w^{iT}(q) R_i \tilde{I}_i R_i^T J_w^i(q)] \dot{q}. \quad (10)$$

Since the Kinetic energy of every mechanical system can be expressed as:

$$E = \frac{1}{2} \dot{q}^T M(q) \dot{q}, \quad (11)$$

where  $M(q)$  is the Mass Matrix of the system. By simply comparing the two previous equations it is possible to write,

$$M(q) = \sum_{i=1}^n [m_i J_v^{iT}(q) J_v^i(q) + J_w^{iT}(q) R_i \tilde{I}_i R_i^T J_w^i(q)]. \quad (12)$$

The Mass Matrix allows to relate the generalized speed with momentum of the mechanical system

$$p = M(q) \dot{q}, \quad (13)$$

where  $p = [p_1 \ p_2 \ \dots \ p_n]^T$ . In the end we obtain the Kinetic energy depending only on momentum:

$$E(q, p) = p^T M^{-1}(q) p. \quad (14)$$

This relation is crucial because the Hamiltonian of the system (i.e. the total energy of the system) must depend only to energy variables and not to derivatives of energy variables.

## 2.2 Total potential energy computation

In our framework we are supposing that the work plane is parallel to the ground, therefore we ignore the effect of the gravity to the dynamic of our system. Then, the potential energy is only due to the springs' deformation. To find the potential energy we first have to define the stiffness matrix of the system

$$K(q) = \text{diag}[K_1, K_2, \dots, K_n]. \quad (15)$$

Then, the constitutive relation between elastic torques and springs' deformation is given by

$$\tau_e = K(q)q. \quad (16)$$

Hence, the total potential energy can be trivially founded as

$$P(q) = \frac{1}{2}q^T K(q)q. \quad (17)$$

Also in this case we have that the potential energy depends only on energy variables (i.e. vector of displacements  $q$ ).

## 2.3 Port Hamiltonian model of the lumped parameter flexible beam

The Hamiltonian (i.e. the total energy of the system) is given by

$$H(q, p) = E(q, p) + P(q) = p^T M^{-1}(q)p + \frac{1}{2}q^T K(q)q. \quad (18)$$

To define the port Hamiltonian model of the system it is necessary to define the state vector:

$$x = \begin{bmatrix} q \\ p \end{bmatrix}. \quad (19)$$

The port Hamiltonian model of the system is of the form

$$\begin{cases} \dot{x} = [J(x) - R(x)] \frac{\partial H(x)}{\partial x} + g(x)u \\ y = g(x)^T \frac{\partial H(x)}{\partial x} \end{cases}, \quad (20)$$

where  $J(x) = -J^T(x)$  is the structural matrix,  $R(x) = R^T(x) \geq 0$  is the damping matrix,  $g(x)$  is the input matrix.

To completely determine the port Hamiltonian model it is necessary to define the above matrices. The structural matrix represent how the energy is exchanged within the system, and for all the physical system this matrix is skew symmetric. The system can be also expressed as,

$$\begin{cases} \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = (J(x) - R(x)) \begin{bmatrix} \frac{\partial H(x)}{\partial q} \\ \frac{\partial H(x)}{\partial p} \end{bmatrix} + g(x)u \\ y = g(x)^T \begin{bmatrix} \frac{\partial H(x)}{\partial q} \\ \frac{\partial H(x)}{\partial p} \end{bmatrix} \end{cases}. \quad (21)$$

The structural matrix  $J(x)$  and the damping matrix  $R(x)$  can be rewritten as

$$J(x) = \begin{bmatrix} J_{11}(x) & J_{12}(x) \\ J_{21}(x) & J_{22}(x) \end{bmatrix}, R(x) = \begin{bmatrix} R_{11}(x) & R_{12}(x) \\ R_{21}(x) & R_{22}(x) \end{bmatrix}. \quad (22)$$

Also the input matrix  $g(x)$  can be seen as

$$g(x) = \begin{bmatrix} g_1(x) \\ g_2(x) \end{bmatrix}. \quad (23)$$

The first part of the dynamic equation of (20) can be rewritten as

$$\dot{q} = J_{11} \frac{\partial H(x)}{\partial q} + J_{12} \frac{\partial H(x)}{\partial p} - R_{11} \frac{\partial H(x)}{\partial q} - R_{12} \frac{\partial H(x)}{\partial p} + g_1(x)u. \quad (24)$$

At the same time, from (11)

$$\dot{q} = M^{-1}(q)p. \quad (25)$$

From (17) it can be computed the Hamiltonian's derivative with respect to momentum

$$\frac{\partial H(x)}{\partial p} = M^{-1}(q)p. \quad (26)$$

Then, comparing (23) and (25) with (24) it can be deduced that

$$\begin{matrix} J_{11}(x) = 0_{n \times n} & J_{12}(x) = I_{n \times n} \\ R_{11} = 0_{n \times n} & R_{12} = 0_{n \times n} \end{matrix}. \quad (27)$$

This means that physically, the energy derived by the momentum is a velocity. From skew symmetry of matrix  $J(x)$  it is possible to determine that

$$J_{21} = -J_{12} = -I_{n \times n}. \quad (28)$$

Starting from the consideration that the matrix  $J(x)$  describes only the exchange of energy between the storing element of the system, it doesn't take into account dissipative terms

$$J_{22} = 0_{n \times n}. \quad (29)$$

The dissipative matrix  $R(x)$  will be also found with some physical considerations. Matrix  $R(x)$  is the one responsible of dissipation in the model. The friction that we suppose to have in our model is of viscous type; it is well known that the viscous friction enter in the dynamic with terms that depends on velocities. From this consideration and that  $\frac{\partial H(x)}{\partial p}$  represent the vector of velocities of every joint, it is possible to conclude

$$R_{21}(x) = 0_{n \times n} \quad R_{22}(x) = \text{diag}[c_1, c_2, \dots, c_n]. \quad (30)$$

In this case of study, the beam is force actuated. Then, it's necessary to find the input matrix  $g(x)$ . Let consider  $u$  the input forces applied on the beam:

$$u : R \rightarrow R^m, u \rightarrow u(t), \quad (31)$$

where  $m$  is the number of actuated joints. This input function collect the magnitude of every input force applied to the beam.

The conversion from forces to joint toques will lead to the matrix  $g(x)$ . First of all it is worth remembering that we can map the velocity of every point of the lumped beam to the joint's velocity using the Jacobian matrix as follow:

$$\dot{p}_i = J_v^{p_i} \dot{q}, \quad (32)$$

where  $J_v^{p_i}$  is the Jacobian matrix related to any point position  $p_i$  belonging to the lumped beam. By conservation of power, we obtain:

$$F^T \dot{p}_i = \tau^T \dot{q}, \quad (33)$$

where  $F$  is the vector containing the  $x$  and  $y$  components of every applied force, in particular

$$F = [F_{1x} \ F_{1y} \ \dots \ F_{mx} \ F_{my}]^T. \quad (34)$$

And the velocity Jacobian matrix is constructed as the composition of all the Jacobian of every force application point:

$$J_v = [J_v^{p_1} \ J_v^{p_2} \ \dots \ J_v^{p_m}]^T. \quad (35)$$

Then, it is easy to show that

$$\tau = J_v^T F(q), \quad (36)$$

where  $\tau$  is the vector of torque applied to the  $n$  different joints. Since the direction of every application force is assumed to be always perpendicular to the joint itself, the  $F_{ix}$  and  $F_{iy}$  components of every force can be written with respect to the force magnitude and the joint configuration. For this reason we define

$$f_{xi}(q) = -\sin\left(\sum_{k=1}^{l_{app}(i)} q_k\right), \quad (37)$$

$$f_{yi}(q) = +\cos\left(\sum_{k=1}^{l_{app}(i)} q_k\right) \quad (38)$$

such that

$$\begin{aligned} F_{xi}(q) &= -|F_i(t)|f_{xi}(q) \\ F_{yi}(q) &= +|F_i(t)|f_{yi}(q), \end{aligned} \quad (39)$$

where  $l_{app}(i)$  gives the link in which is applied the  $i$ -th force. Since the vector of input is composed by the magnitude of every applied force  $u = |F| = [|F_1(t)| \ \dots \ |F_m(t)|]^T$ , can be written that:

$$F(q) = L_F(q)u \quad (40)$$

where,

$$L_F(q) = \text{diag} \left[ \begin{bmatrix} F_{x1}(q) \\ F_{y1}(q) \end{bmatrix}, \begin{bmatrix} F_{x2}(q) \\ F_{y2}(q) \end{bmatrix}, \dots, \begin{bmatrix} F_{xm}(q) \\ F_{ym}(q) \end{bmatrix} \right]. \quad (41)$$

Finally, the input matrix of the port Hamiltonian system is:

$$g(x) = \begin{bmatrix} 0_{n \times m} \\ J_v^T(q) L_F(q) \end{bmatrix}. \quad (42)$$

The upper part of the  $g(x)$  matrix is null because the torques can't affect the first  $n$  equation, *i.e.* the displacement dynamics.

### 3. IDA-PBC CONTROL DESIGN

In this section, we will show how to design a control by IDA-PBC method. To this end, we will consider the beam contains 3 links. The input of the beam are magnitude of two forces, applied to the first and third links at a distance of  $a_f$  from respectively the first and third joint. The direction of the forces is supposed to be always perpendicular to the respective link as can be seen in the figure (3). It is important to notice that the springs and the dampers present in the picture are intended as torsional. The components of the port Hamiltonian model are:

- The interconnection matrix

$$J = \begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} \\ -I_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}; \quad (43)$$

- The damping matrix

$$R = \begin{bmatrix} 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & C \end{bmatrix} \quad (44)$$

with  $C = \text{diag}[c_1, c_2, c_3]$ ;

- The input matrix

$$g(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ a_f & a_f + a \cos(q_3) + a \cos(q_2 + q_3) \\ 0 & a_f + a \cos(q_3) \\ 0 & a_f \end{bmatrix}; \quad (45)$$

- The input vector of force magnitudes

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}. \quad (46)$$

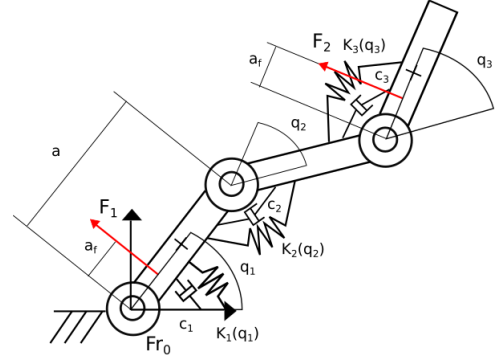


Fig. 3. Description of the three elements force actuated beam

The aim of the IDA-PBC is to find a state feedback control law such that the closed loop system became equal to the desired one:

$$\dot{x} = (J_d - R_d) \frac{\partial H_d}{\partial x}. \quad (47)$$

In this example the only thing that will be modified is the Hamiltonian. The IDA-PBC matching condition is such that

$$g^\perp(x)(J_d - R_d) \frac{\partial H}{\partial x} = g^\perp(x)((J + J_a) - (R + R_a)) \frac{\partial H_c}{\partial x} \quad (48)$$

where  $g^\perp$  is the annihilator ( $g^\perp(x)g(x) = 0$ ) and  $H_c$  is the Hamiltonian of the controller. The desired Hamiltonian  $H_d$  is defined as

$$H_d = H + H_c \quad (49)$$

It must be such that

$$\begin{cases} \frac{\partial H_d}{\partial x}(x^*)|_{x^*} = 0 \\ \frac{\partial^2 H_d}{\partial x^2}|_{x^*} = 0 \end{cases}. \quad (50)$$

Since the interconnection and the damping matrix won't be modified, *i.e.*  $J_a = 0$  and  $R_a = 0$ . Then the matching condition became

$$g^\perp(J - R) \frac{\partial H_c}{\partial x} = 0 \quad (51)$$

The annihilator, that represents the kernel of the matrix  $g(x)$ , is given by

$$g^\perp(x) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -d & c \end{bmatrix} \quad (52)$$

Exploiting the matching condition it is possible to get four conditions on the controller's Hamiltonian

$$\begin{cases} \frac{\partial H_c}{\partial p_1} = \frac{\partial H_c}{\partial p_2} = \frac{\partial H_c}{\partial p_3} = 0 \\ a_f \frac{\partial H_c}{\partial q_2} - (a_f + a \cos(q_3)) \frac{\partial H_c}{\partial q_3} = 0 \end{cases} \quad (53)$$

The first line of matching equation (53) means that  $H_c$  can't depend on the momentum variables, while the last is a partial differential equation that impose a relation between the controller's Hamiltonian derivatives. One can prove that the solution of this partial differential equation must be such that

$$H_c = H_c(-q_2 - \frac{2a_f \tanh^{-1}(\frac{(a-a_f)\tan(\frac{q_3}{2})}{\sqrt{a^2-a_f^2}})}{\sqrt{a^2-a_f^2}}, q_1) \quad (54)$$

In the most simple case the solution would be

$$\begin{cases} \frac{\partial H_c}{\partial q_2} = kq_2^* \\ \frac{\partial H_c}{\partial q_3} = \frac{a_f}{a_f + a \cos(q_3)} kq_2^* \end{cases} \quad (55)$$

The part of the Hamiltonian dependents on  $q_1$  can be chosen as wanted. Then, the controller's Hamiltonian can be found as the sum of the integral of these two partial derivatives plus the part depending on  $q_1$ . Now that the controller's Hamiltonian is completely defined, it is necessary that the condition on the first and the second derivative of the total Hamiltonian in the equilibrium position are satisfied. From the dynamic equation of the system it is possible to find the forced equilibrium position and the input vector at the equilibrium, imposing the state's derivatives equal to zero. In particular

$$0 = (J - R) \frac{\partial H}{\partial x} |_{x^*} + g(x^*)u^* \quad (56)$$

where  $p_1^*, p_2^*, p_3^*$  are mandatory imposed equal to zero, while  $q_1^*$  and  $q_2^*$  are arbitrary. The unknowns are  $q_3^*$  and the vector of inputs  $u^*$ . With the chosen  $H_c$  and  $k = K_2$ , we obtain

$$\frac{\partial H_d}{\partial x} = \begin{bmatrix} K_1(q_1 - q_1^*) \\ K_2(q_2 - q_2^*) \\ K_3q_3 + \frac{a_f}{a_f + a \cos(q_3)} K_2q_2^* \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (57)$$

where, in the equilibrium positions, can be proved that

$$\frac{\partial H_d}{\partial x} |_{x^*} = 0 \quad \forall q_1^*, q_2^* \in [0, 2\pi]. \quad (58)$$

The condition on the second derivatives in the equilibrium can be proven that is verified only for some  $q_2^*$ . In particular

$$\frac{\partial^2 H_d}{\partial x^2} |_{x^*} = \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & \frac{\partial^2 H_d}{\partial q_3^2} |_{x^*} \end{bmatrix} \quad (59)$$

where,

$$\frac{\partial^2 H_d}{\partial q_3^2} |_{x^*} = K_3 - K_2 \frac{a_f q_2^* a \sin(q_3)}{(a_f + a \cos(q_3))^2} \quad (60)$$

From simulations results, it has been seen that:

$$\frac{\partial^2 H_d}{\partial q_3^2} |_{x^*} > 0 \quad \forall q_1^* \in \mathbb{R}, q_2^* \in [-2.27, +2.27]. \quad (61)$$

This means that the new equilibrium points of the  $q_2$  variable can be chosen only in a certain interval. According to the IDA-PBC theory, the applied control input is

$$u = (g^T(x)g(x))^{-1}g^T(x)[(J_d - R_d) \frac{\partial H_d}{\partial x} + (J - R) \frac{\partial H}{\partial x}], \quad (62)$$

thus, according to the done choices, became

$$u = (g^T(x)g(x))^{-1}g^T(x)[(J - R) \frac{\partial H_c}{\partial x}]. \quad (63)$$

## 4. SIMULATION RESULTS

In this section, the simulation results are shown using a Beam modeled with 3 elements. In particular, the parameters used in the simulation are:

- $m_1 = m_2 = m_3 = 0,0017$  [kg]
- $I_1 = I_2 = I_3 = 1,3801 \times 10^{-6}$  [kg \* m<sup>2</sup>]
- $a_1 = a_2 = a_3 = 0,1$  [m]
- $a_{C1} = a_{C2} = a_{C3} = 0,05$  [m]
- $K_i(q) = k_i q_i^2 + q_i + k_i \quad i = 1, 2, 3$
- $k_1 = k_2 = 1000 \quad k_3 = 100$  [ $\frac{N}{m}$ ]
- $c_1 = c_2 = c_3 = 0,03$  [Pa \* s]

### 4.1 Open loop response

It is first illustrated the free oscillations of the beam starting from initial condition  $q_0 = [+ \frac{\pi}{8} \quad - \frac{\pi}{8} \quad + \frac{\pi}{6}]$  and  $p_0 = [0 \quad 0 \quad 0]$

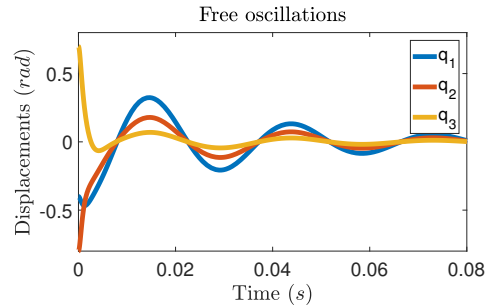


Fig. 4. Free Beam oscillations

### 4.2 Closed loop responses

Now we will show the closed loop system behavior by using IDA-PBC design method. The controller Hamiltonian is defined as following

$$H_c = +\frac{1}{2}K_1q_1^{*2} - q_1^*q_1K_1 + \frac{1}{2}K_2q_2^{*2} - q_2^*q_2K_2 - \frac{2a_fq_2^*K_2 \tanh^{-1}(\frac{(a-a_f)\tan(\frac{q_3}{2})}{\sqrt{a^2-a_f^2}})}{\sqrt{a^2-a_f^2}} \quad (64)$$

where one has the possibility of choosing  $q_1^*$  and  $q_2^*$ . In the case that the equilibrium that you want to choose is  $q_3^*$  it's necessary to find the forced  $q_2^*$  equilibrium position imposing the state's derivatives equal to zero in the dynamic equation, and solve this set of algebraic equation in the  $q_2^*$  unknown. Once the  $q_2^*$  equilibrium state is found, it is possible to use the same controller Hamiltonian above

defined. The resulting vector of derivatives of the controller Hamiltonian will be:

$$\frac{\partial H_c}{\partial x} = \begin{bmatrix} K_1 q_1^* \\ K_2 q_2^* \\ \frac{a_f}{a_f + a \cos(q_3)} K_2 q_2^* \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (65)$$

Below are shown different time response of the three element beam with different damping injection.

The initial condition of the system and the desired equilibrium position are given as following:

$$x = \begin{bmatrix} -0.4 \\ -0.8 \\ 0.7 \\ 0 \\ 0 \\ 0 \end{bmatrix}; q^* = \begin{bmatrix} q_1^* \\ q_2^* \\ q_3^* \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.52 \\ 0.15 \end{bmatrix} \quad (66)$$

The initial configuration and the final position are shown in the Fig. 5 and Fig. 6.

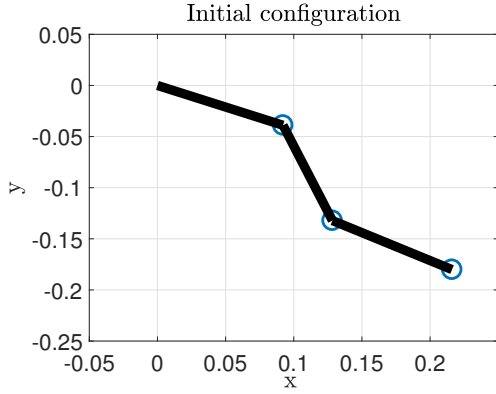


Fig. 5. Initial position

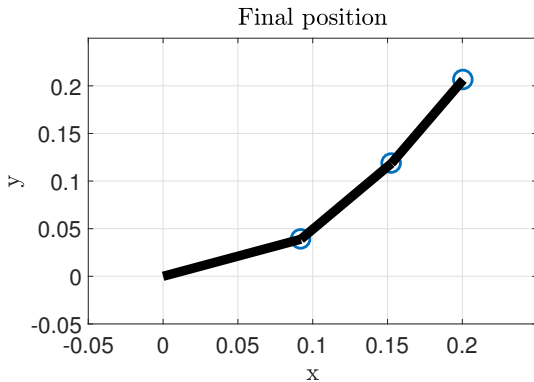


Fig. 6. Final position

The time responses of the angular of every link are shown in the Fig. 7-Fig. 9 with the different damping injections. The angular of every link converges to the desired equilibrium positions.

## 5. CONCLUSION

The port Hamiltonian framework has been used to model, control the medical endoscope system. The endoscope is

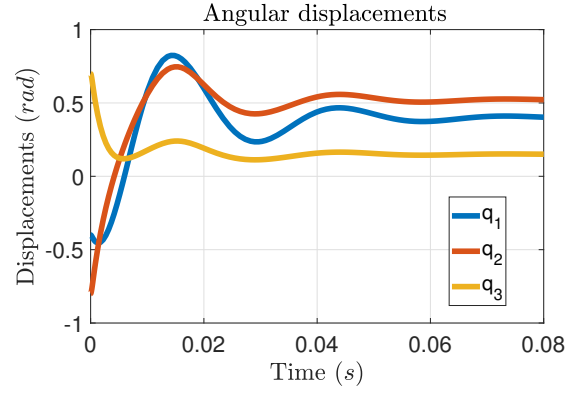


Fig. 7. Time response of beam,  $c = 0.025 \text{ Pa} \cdot \text{s}$

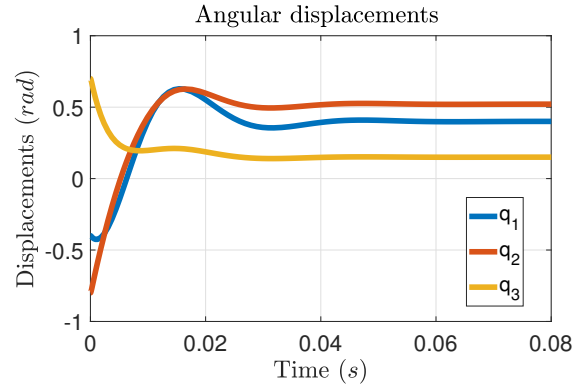


Fig. 8. Time response of beam,  $c = 0.04 \text{ Pa} \cdot \text{s}$

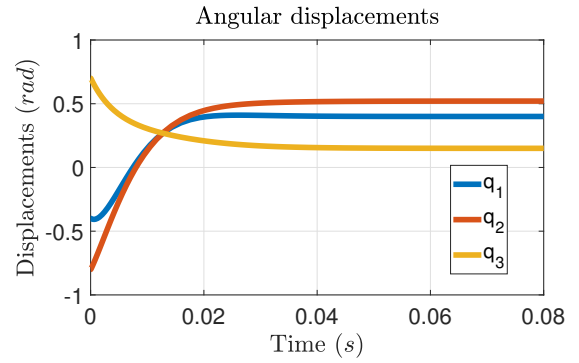


Fig. 9. Time response of beam,  $c = 0.08 \text{ Pa} \cdot \text{s}$

modeled as a lumped parameter flexible beam. A position control law has been proposed for this model by using the IDA-PBC method. The proposed model and control law have been illustrated by some simulation results.

The ongoing work is to deal with the experimental implantation of the proposed control design method on the experimental setup and with the non-linearity of the stiffness of the beam.

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