Irreversible port-Hamiltonian formulation of non-isothermal electromechanical systems with hysteresis

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Abstract: An irreversible port-Hamiltonian system (IPHS) representation of non-isothermal electromechanical systems with hysteresis is proposed. By representing the hysterisis through hysterons interconnected with the mechanical and electrical components, it is shown that the hysteresis behaves as an irreversible process. This is elegantly captured by the IPHS structure and makes it possible to isolate the different irreversible phenomena of the overall system. Furthermore, it is shown that in general an electromechanical system with hysteresis corresponds to a reversible-IPHS, i.e., the combination of a conservative Hamiltonian system with an irreversible one defined with respect to the same Hamiltonian. A micro-mechatronic example is used to illustrate the approach.

Keywords: Port-Hamiltonian system, irreversible thermodynamics, hysteresis, micro-mechatronics

1. INTRODUCTION

Recent technological progresses in material science make possible the use of new active materials, such as piezo material, electroactive polymers, magnetic memory shape alloys, etc., for actuators design. The advantage of such actuators is their good dynamic performances and their efficiency in some specific working ranges. Their main drawback is their hysteretic behaviour associated with the modification of their christallographic organisation. This hysteretic behaviour is very problematic from a control point of view. Usually hysteresis is modelled as an input/output map through nonlinear equations such as in Bouc-Wen (Wen, 1976) or Duhem (Duhem, 1902) models. This view is not consistent with energy formulation as discussed in Goldfarb and Celanovic (1997). In Karnopp (1983) a passive formulation of hysteretic systems is proposed. It accounts for both memory and internal dissipation, and that has been formulated as dissipative port Hamiltonian system in Calchand (2014). Unfortunately such kind of representation does not allow to take the thermal domain into account, which is very important when a precise description of the material is needed. In this paper we propose a new model of hysteretic material using the irreversible port-Hamiltonian framework. Irreversible port-Hamiltonian system (IPHS) were proposed in Ramirez et al. (2013a) as an extension of port-Hamiltonian systems (PHS) (Maschke and van der Schaft, 1992; Maschke et al., 1992) towards the structural representation of irreversible thermodynamics. For processes described by lumped and distributed models (Ramirez and Le Gorrec, 2016) it has been shown that the formalism encompasses a large and general class of irreversible thermodynamic systems, such

as heat-exchangers, chemical reactions, chemical reaction networks and coupled mechanic-thermodynamic systems (Ramirez et al., 2013b). Different to representations which seek to encode the dynamics of irreversible thermodynamic systems by a differential geometric structure, such as GENERIC (Grmela and Öttinger, 1997; Öttinger and Grmela, 1997) or contact systems (Mrugala et al., 1991; Eberard et al., 2007), the aim of the IPH formulation is to encode the dynamics with a pseudo-PH control structure. Indeed, using the definition of the availability function (Keenan, 1951; Alonso and Ydstie, 2001) the IPHS structure has recently been employed to exploit the thermodynamic properties of irreversible processes to derive nonlinear passivity based controllers (Ramirez et al., 2016). The paper is organized as follows. Section 2 presents the basics on IPHS. Section 3 shows how simple electrical and mechanical system with non-isothermal behaviours can be formulated as IPHS. Section 4 presents how the irreversible entropy production of the hysteresis in mechanical and electrical systems can be precisely characterised and used as domain coupling mechanism. In Section 5 a micro-mechatronic actuators is used to illustrate the formalism. Finally, Section 6 presents some conclusions and lines of future work.

2. IRREVERSIBLE PORT-HAMILTONIAN SYSTEMS

Irreversible port-Hamiltonian systems (IPHS) have been defined in Ramirez et al. (2013a) as an extension of port-Hamiltonian systems (PHS) for the purpose of representing not only the energy balance but also the entropy balance, essential in thermodynamic systems.

Definition 1. (Ramirez et al., 2013a) An input affine IPHS is defined by the dynamic equation and output relation

$$\dot{x} = R\left(x, \tfrac{\partial U}{\partial x}\right) J \frac{\partial U}{\partial x}(x) + g\left(x, \tfrac{\partial U}{\partial x}\right) v,$$
$$y = g^{\top}\left(x, \tfrac{\partial U}{\partial x}\right) \frac{\partial U}{\partial x}(x) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, the smooth functions $U(x) : \mathbb{R}^n \to \mathbb{R}$ and $S(x) : \mathbb{R}^n \to \mathbb{R}$ represent, respectively, the internal energy (the Hamiltonian) and the entropy functions, $J \in \mathbb{R}^{n \times n}$ is a constant skew-symmetric structure (interconnection) matrix of the Poisson bracket (Maschke et al., 1992) acting on any two smooth functions Z and G as:

$$\{Z, G\}_J = \frac{\partial Z}{\partial x}^{\top}(x) J \frac{\partial G}{\partial x}(x). \tag{2}$$

The real function $R = R\left(x, \tfrac{\partial U}{\partial x}\right)$ is composed by the product of a positive definite function γ and the Poisson bracket between the entropy and the energy functions:

$$R\left(x, \tfrac{\partial U}{\partial x}\right) = \gamma\left(x, \tfrac{\partial U}{\partial x}\right) \{S, U\}_J,$$

with $\gamma\left(x, \tfrac{\partial U}{\partial x}\right) : \mathbb{R}^n \to \mathbb{R}$, $\gamma \geq 0$, a non-linear positive function. The input map is defined by $g\left(x, \tfrac{\partial U}{\partial x}\right) \in \mathbb{R}^{n \times m}$ with the input $v(t) \in \mathbb{R}^m$ a time dependent function.

The drift dynamic in (1) is defined by a non-linear relation between the time derivative $\dot{x}$ of the state (extensive) variables and $\frac{\partial U}{\partial x}$, characterized by the modulating function $R\left(x, \tfrac{\partial U}{\partial x}\right)$, which explicitly depends on the co-energy (intensive) variables $\frac{\partial U}{\partial x}$. The balance equations of the total energy and entropy functions of IPHS express the first and second principles of irreversible Thermodynamics: the conservation of energy and the irreversible creation of entropy due to irreversible phenomena. By skew-symmetry of J, the balance equation of the internal energy, which is a convex function,

$$\frac{dU}{dt} = y^{\top} v, \tag{3}$$

expresses that the system (1) is a lossless dissipative systems with (energy) supply rate $y^{\top} v$ (Willems, 1972). The balance equation of the entropy function is given by

$$\frac{dS}{dt} = R\left(x, \tfrac{\partial U}{\partial x}, \tfrac{\partial S}{\partial x}\right) \frac{\partial S}{\partial x}^{\top} J(x) \frac{\partial U}{\partial x} + \frac{\partial S}{\partial x}^{\top} g\left(x, \tfrac{\partial U}{\partial x}\right) v$$
$$= \gamma\left(x, \tfrac{\partial U}{\partial x}\right) \{S, U\}_J^2 + \left(g^{\top}\left(x, \tfrac{\partial U}{\partial x}\right) \frac{\partial S}{\partial U} \frac{\partial U}{\partial x}\right)^{\top} v. \tag{4}$$
$$= \gamma\left(x, \tfrac{\partial U}{\partial x}\right) \{S, U\}_J^2 + y_s^{\top} v.$$

where $y_s = \frac{\partial S}{\partial U} y$ is an entropy conjugated output. By Definition 1 the first term is positive: $\gamma\left(x, \tfrac{\partial U}{\partial x}\right) \{S, U\}_J^2 = \sigma\left(x, \tfrac{\partial U}{\partial x}\right) \geq 0$. For irreversible thermodynamic systems, this term represents the *internal entropy production* and its positivity expresses the second principle of Thermodynamics. The second term in (4) corresponds to the definition of an entropy supply rate. For further details on IPHS and its thermodynamic interpretation we refer the reader to Ramirez et al. (2013a). An extension of purely IPHS has been given un Ramirez et al. (2013b), where reversible-IPHS have been introduced in order to cope with systems which simultaneously exhibit a reversible, i.e., conservative, behaviour in some components and irreversible behaviour in others. One of the interesting aspects

of reversible-IPHS is that the entropy function is a Casimir (van der Schaft, 2000) of the reversible structure matrix.

Definition 2. A Reversible-IPHS is defined by the dynamical equation

$$\dot{x} = J_{ir}\left(x, \tfrac{\partial U}{\partial x}, \tfrac{\partial S}{\partial x}\right) \frac{\partial U}{\partial x}(x) + g\left(x, \tfrac{\partial U}{\partial x}\right) u, \tag{5}$$

where the skew symmetric matrix J_{ir} is defined as the sum:

$$J_{ir}\left(x, \tfrac{\partial U}{\partial x}, \tfrac{\partial S}{\partial x}\right) = J_0(x) + R\left(x, \tfrac{\partial U}{\partial x}, \tfrac{\partial S}{\partial x}\right) J \tag{6}$$

where $J_0(x)$ is the structure matrix of a Poisson bracket and $R\left(x, \tfrac{\partial U}{\partial x}, \tfrac{\partial S}{\partial x}\right)$ and J are defined according to Definition 1 of an IPHS. Furthermore the entropy function $S(x)$ is a Casimir function of the Poisson structure matrix $J_0(x)$.

The reversible-IPHS is the composition of a PHS and an IPHS with structure matrices being the sum of a Poisson structure matrix and a quasi-Poisson structure matrix in the sense of Definition 1 and with common Hamiltonian function. Computing the time derivative of the Hamiltonian $U(x)$, by skew-symmetry of $J_{ir}\left(x, \tfrac{\partial U}{\partial x}, \tfrac{\partial S}{\partial x}\right)$, the Hamiltonian obeys the same balance equation as for IPHS, depending only on the power product at the port of the system. Computing the time derivative of the total entropy for an isolated system one obtains

$$\frac{dS}{dt} = \frac{\partial S}{\partial x}^{\top} J_0 \frac{\partial U}{\partial x} + R \frac{\partial S}{\partial x}^{\top} J \frac{\partial U}{\partial x}$$
$$= \{S, U\}_{J_0} + \gamma\left(x, \tfrac{\partial U}{\partial x}\right) \{S, U\}_J^2$$
$$= \gamma\left(x, \tfrac{\partial U}{\partial x}\right) \{S, U\}_J^2$$

using that $S(x)$ is a Casimir function of the Poisson structure matrix $J_0(x)$, that is it satisfies $\{S, U\}_{J_0} = 0$ for any Hamiltonian $U(x)$. In consequence the entropy balance equation of the reversible-IPHS is identical with the entropy balance equation (4) of the IPHS. The benefit of the energy based formulation of IPHS (Hamiltonian given by the internal energy) is clearly emphasised in this case, since it allows to naturally perform the interconnection with conventional PHS. This is not the case for quasi-Hamiltonian formulations of thermodynamic systems where for instance the entropy (or some function of the entropy) is used as Hamiltonian.

3. IPHS FORMULATION OF ELECTRICAL AND MECHANICAL SYSTEMS

In this section we shall represent general electrical and mechanical systems as IPHS. We shall make very general assumptions, such as that all parameters may depend on the temperature. Even thought this may be to general in practice, we are interested in developing the mathematical framework of the model.

3.1 The RLC circuit

Consider the PHS formulation of a simple RLC circuit

$$\begin{bmatrix} \dot{Q} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -r \end{bmatrix} \begin{bmatrix} \frac{Q}{C} \\ \frac{\lambda}{L} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_e, \quad y_e = [0 \; 1] \begin{bmatrix} \frac{Q}{C} \\ \frac{\lambda}{L} \end{bmatrix} \tag{7}$$

The electrical energy and its derivative in time are given by

$$H_e(Q,\lambda) = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}\frac{\lambda^2}{L},$$

$$\dot{H}_e = -r\left(\frac{\lambda}{L}\right)^2 + y_e^\top u_e.$$

Consider the case when the thermal effects cannot be neglected, i.e., the electrical dissipation induces some microscopic Brownian motion which alters the dynamic behaviour of the system. We can for instance assume that all the electrical components are temperature dependent

$$C = C(T(S)), \qquad L = L(T(S)), \qquad r = r(T(S)),$$

where $T(S)$ is the temperature of the system which is a function of the entropy S. Consider now the internal energy of the system, which is the sum of the electrical energy and some smooth function of the entropy

$$U_e(Q,\lambda,S) = \frac{1}{2}\frac{Q^2}{C(S)} + \frac{1}{2}\frac{\lambda^2}{L(S)} + \mathcal{S}_e(S) \qquad (8)$$

The variation in time of the internal energy is given by

$$\dot{U}_e = \frac{\partial U_e}{\partial Q}\dot{Q} + \frac{\partial U_e}{\partial \lambda}\dot{\lambda} + \frac{\partial U_e}{\partial S}\dot{S}$$

$$\dot{U}_e = -r(s)\left(\frac{\lambda}{L(s)}\right)^2 + \frac{\partial U_e}{\partial S}\frac{dS}{dt} + y_e^\top u_e. \qquad (9)$$

From Gibb's relation (Callen, 1985; Kondepudi and Prigogine, 1998) we have that $\frac{\partial U_e}{\partial S} = T(S)$. Hence, and considering $u_e = 0$ for simplicity, it is obtained from (9) that

$$\frac{dS}{dt} = \frac{r(S)}{T(S)}\left(\frac{\lambda}{L(S)}\right)^2 = \sigma_r \geq 0 \qquad (10)$$

where σ_r corresponds to the internal entropy production and which is indeed related to the electrical dissipation. Hence an IPHS formulation of the thermodynamic model of the RLC circuit is

$$\begin{bmatrix}\dot{Q}\\\dot{\lambda}\\\dot{S}\end{bmatrix} = \left(\begin{bmatrix}0 & 1 & 0\\-1 & 0 & 0\\0 & 0 & 0\end{bmatrix} + \frac{r}{T}\frac{\lambda}{L}\begin{bmatrix}0 & 0 & 0\\0 & 0 & -1\\0 & 1 & 0\end{bmatrix}\right)\begin{bmatrix}\frac{Q}{C}\\\frac{\lambda}{L}\\\frac{\lambda}{T}\end{bmatrix} + \begin{bmatrix}0\\1\\0\end{bmatrix}u_e$$
$$(11)$$

where we have skipped the arguments for simplicity. Notice that (11) actually corresponds to the combination of a reversible PHS and an IPHS. Indeed, we identify two matrices, related to the reversible and irreversible phenomena, respectively,

$$J_e = \begin{bmatrix}0 & 1 & 0\\-1 & 0 & 0\\0 & 0 & 0\end{bmatrix}, \qquad J_r = \begin{bmatrix}0 & 0 & 0\\0 & 0 & -1\\0 & 1 & 0\end{bmatrix}$$

The bracket defined by J_r evaluated with the entropy and the internal energy defines the irreversible driving force of the system

$$\{S, U_e\}_{J_r} = \frac{\lambda}{L} \qquad (12)$$

equals the current which is the driving force of the irreversibility in the system. Hence we have that the modulating function of the irreversibility is

$$R_r = \frac{r}{T}\frac{\lambda}{L}.$$

Hence the RLC circuit can be written as the reversible-irreversible PHS (Ramirez et al., 2013b)

$$\dot{x}_e = (J_e - R_r J_r)\frac{\partial U_e}{\partial x} + g_e u_e \qquad (13)$$

with $x_e = [Q, \lambda, S]^\top$ and $g_e = [0\ 1\ 0]^\top$.

3.2 The mass-spring-damper (MSD) system

Consider a MSD system

$$\begin{bmatrix}\dot{q}\\\dot{p}\end{bmatrix} = \begin{bmatrix}0 & 1\\-1 & -b\end{bmatrix}\begin{bmatrix}kq\\\frac{p}{m}\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}u_m, \quad y_m = [0\ 1]\begin{bmatrix}kq\\\frac{p}{m}\end{bmatrix}. \quad (14)$$

The mechanical energy and its derivative in time are given by

$$H_m(q,p) = \frac{1}{2}kq^2 + \frac{1}{2}\frac{p^2}{m},$$

$$\dot{H}_m = -b\left(\frac{p}{m}\right)^2 + y_m^\top u_m.$$

If the thermal effects are taken into account, the mechanical friction induces heating of the system. Assume that the mechanical parameters are temperature dependent as it is the case for micro-mechatronic systems,

$$k = k(T(S)), \qquad m = m(T(S)), \qquad b = b(T(S)),$$

Consider now the internal energy of the system U_m is composed of the mechanical energy and an entropy dependent function.

$$U_m(q,p,S) = \frac{1}{2}k(S)q^2 + \frac{1}{2}\frac{p^2}{m(S)} + \mathcal{S}_m(S) \qquad (15)$$

In the same way as for the RLC circuit we deduce that the internal entropy creation is given by

$$\frac{dS}{dt} = \frac{b(s)}{T(s)}\left(\frac{p}{m}\right)^2 = \sigma_b \geq 0 \qquad (16)$$

where σ_b corresponds to the internal entropy production. An IPHS formulation of the thermodynamic model of the MSD system is then given by

$$\begin{bmatrix}\dot{q}\\\dot{p}\\\dot{S}\end{bmatrix} = \left(\begin{bmatrix}0 & 1 & 0\\-1 & 0 & 0\\0 & 0 & 0\end{bmatrix} + \frac{b}{T}\frac{p}{m}\begin{bmatrix}0 & 0 & 0\\0 & 0 & -1\\0 & 1 & 0\end{bmatrix}\right)\begin{bmatrix}kq\\\frac{p}{m}\\\frac{p}{T}\end{bmatrix} + \begin{bmatrix}0\\1\\0\end{bmatrix}u_m$$
$$(17)$$

where we have skipped the arguments for simplicity. As for the RLC circuit we can identify matrices related to the reversible and irreversible phenomena, respectively,

$$J_m = \begin{bmatrix}0 & 1 & 0\\-1 & 0 & 0\\0 & 0 & 0\end{bmatrix}, \qquad J_b = \begin{bmatrix}0 & 0 & 0\\0 & 0 & -1\\0 & 1 & 0\end{bmatrix}.$$

The irreversible driving force of the system is computed using the bracket defined by J_b,

$$\{S, U\}_{J_b} = \frac{p}{m}, \qquad (18)$$

i.e., the velocity which is indeed the driving force of the irreversibility in the system. The modulating function is in this case

$$R_b = \frac{b}{T}\frac{p}{m}.$$

Hence the micro MSD can be written as the reversible-irreversible PHS (Ramirez et al., 2013b)

$$\dot{x}_m = (J_m - R_b J_b)\frac{\partial U_m}{\partial x} + g_m u_m \qquad (19)$$

with $x_m = [q, p, S]^\top$ and $g_m = [0\ 1\ 0]^\top$.

4. HYSTERESIS AS A DOMAIN COUPLING MECHANISM

In this section we shall focus in the case when hysteresis is the predominant coupling mechanism, and shall for simplicity not consider the reversible coupling, between
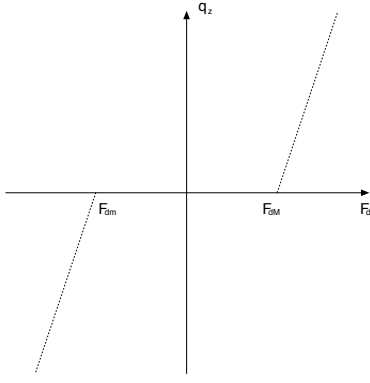
Fig. 1. Non-linear damping relation between $\dot{q}_z$ and F_d.

Fig. 2. Hysteretic curve.

different physical domains. We shall model the hysteresis of the system by means of an idealised component, namely an hysteron (Karnopp, 1983). The hysteresis is assumed to have non-isothermal behaviour, hence it will produce entropy in an irreversible fashion. We shall extend the concepts developed in Karnopp (1983); Calchand (2014) to incorporate hysteresis and its non-isothermal contribution in mechanical, electrical and electromechanical systems.

4.1 Hysteresis in the mechanical domain

An hysteron (Karnopp, 1983) is an element composed of a (possible non-linear) spring and a non-linear, non-smooth damper interconnected in parallel. The state variable associated to the spring accounts for the memory of the hysteron and the damping coefficient accounts for the dissipation. Hence, during one cycle of the hysteron, part of its energy is stored while another part is transformed into irreversible entropy production in a non-linear fashion. Define the total force applied to the hysteron as

$$F_h = F_k + F_d \tag{20}$$

where $F_k = k_h q_z$ corresponds to the force associated to the conservative element, with k_h the possible non-linear spring coefficient and q_z the displacement of the spring. The force F_d is produced by the non-linear dissipative element. The dynamic of the hysteron is derived from the constitutive law of the dissipative element

$$\dot{q}_z = d(F_d) = d(F_h - F_k) = d(F_h - k_h q_z), \tag{21}$$

where d is a positive non-linear non-smooth operator. Physically it corresponds to a mechanical conductance type of element, since it maps forces to velocities. The simplest form of this dissipative element can be observed in Figure 1. From the second law the energy dissipated through the dissipative element produces entropy irreversibly. Hence,

$$\dot{q}_z F_d = d(F_d) F_d = \sigma_d \geq 0$$

where σ_d corresponds to the irreversible entropy production of the hysteron of the mechanical component. This irreversible entropy production corresponds to the surface in between the hysteresis curve of a system (cf. Figure 2). In order to include hysteresis in the MSD system the hysterion is interconnected in series with the spring with elastic coefficient k in (17). The hysteron will hence act as a non-linear state dependent velocity source (cf. Figure 3). By defining the vector of states variables $x = [q_t, p, q_z, S]^\top$, where q_t is the total displacement of the mass m, i.e.,
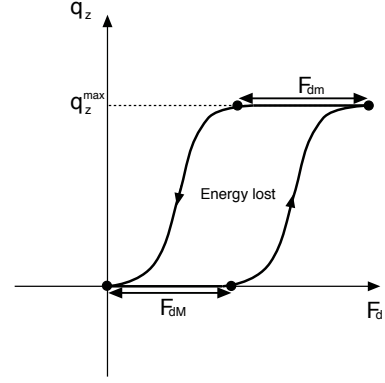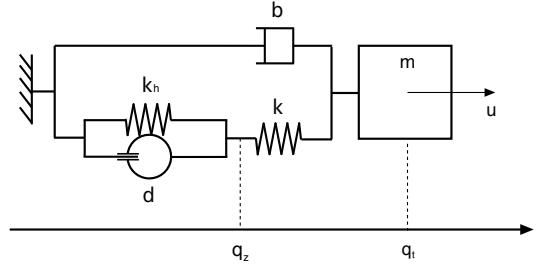

Fig. 3. Equivalent representation of an hysteron interconnected with a MSD system.

$q_t = q + q_z$, we obtain the following IPHS of the mechanical system with hysteresis

$$
\begin{bmatrix} \dot{q}_t \\ \dot{p} \\ \dot{q}_z \\ \dot{S} \end{bmatrix} = \left(\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \frac{b}{T}\frac{p}{m} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} + \right.
$$
$$
\left. \frac{1}{T}d(F_d) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \right) \begin{bmatrix} k(q_t - q_z) \\ \frac{p}{m} \\ k_h q_z - k(q_t - q_z) \\ T \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u_m
$$
$$\tag{22}$$

with $F_d = k(q_t - q_z) - k_h q_z$ and internal energy function

$$U_m = \frac{1}{2}k(q_t - q_z)^2 + \frac{1}{2}\frac{p^2}{m} + \frac{1}{2}k_h q_z^2 + \mathcal{S}_m(S). \tag{23}$$

We observe that the modulating function of the irreversible entropy production of the hysteron is $R_d = \gamma_d\{S,U\}_{J_d} = \frac{1}{T}d(F_d)$ with $\gamma_d = \frac{1}{T}d(\cdot) > 0$ and $\{S,U\}_{J_d} = k(q_t - q_z) - k_h q_z = F_d$, which is indeed the dissipative force produced by the hysteron and which defines the thermodynamic driving force of the irreversible dissipation, and where

$$J_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}.$$

Consider for simplicity $u_m = 0$, then the entropy balance of (22) is given by

$$\dot{S} = \frac{b}{T}\left(\frac{p}{m}\right)^2 + \frac{1}{T}d(F_d)F_d$$
$$= \sigma_b + \sigma_d \geq 0,$$

the total entropy production is thus the sum of the mechanical dissipation and the hysteresis. As explained in Karnopp (1983); Calchand (2014), one hysteron is

usually not enough to precisely represent the hysteresis of a system. Hence a collection of hysterons has to be employed to gain in accuracy. It is straightforward to include a number n of hysterions in the previous model. Define as state vector $x = [q_t \ p \ q_{z_1} \ \cdots \ q_{z_n} \ S]^\top$, then an IPHS representation for a model including n hysterons is

$$\dot{x} = \left(J_m - R_b J_b - \sum_{i=1}^{n} R_{d_i} J_{d_i} \right) \frac{\partial U_m}{\partial x} + g_m u_m \qquad (24)$$

where

$$J_m = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, \quad J_b = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & -1 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \end{bmatrix},$$

$$J_{d_1} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ \vdots & \ddots & \ddots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & -1 & \cdots & 0 & 0 \end{bmatrix}, \quad J_{d_n} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & -1 & 0 \end{bmatrix}$$

with

$$R_b = \frac{b}{T} \left(\frac{p}{m} \right)^2,$$

$$R_{d_1} = \frac{1}{T} d_1 \left(k \left(q_t - \sum_{i=1}^{n} q_{z_i} \right) - k_{h_1} q_{z_1} \right),$$

$$\vdots \quad \vdots$$

$$R_{d_n} = \frac{1}{T} d_n \left(k \left(q_t - \sum_{i=1}^{n} q_{z_i} \right) - k_{h_n} q_{z_n} \right),$$

and internal energy function

$$U_m = \frac{1}{2} k \left(q_t - \sum_{i=1}^{n} q_{z_i} \right)^2 + \frac{1}{2} \frac{p^2}{m} + \frac{1}{2} \sum_{i=1}^{n} k_{h_i} q_{z_i}^2 + \mathcal{S}_m(S).$$

It is straightforward to verify that for the i-th hysterion $\gamma_{d_i} = \frac{1}{T} d_i(\cdot) > 0$ and $\{S, U\}_{J_{d_i}} = k(q_t - \sum_{i=1}^{n} q_{z_i}) - k_h q_{z_i} = F_{d_i}$, the dissipative force, which defines indeed the thermodynamic driving force of the irreversible dissipation of the i-th hysterion. Consider for simplicity $u_m = 0$, then the entropy balance of (24) is given by

$$\dot{S} = \frac{b}{T} \left(\frac{p}{m} \right)^2 + \frac{1}{T} \sum_{i=1}^{n} d_i \left(F_{d_i} \right) F_{d_i}$$

$$= \sigma_b + \sum_{i=1}^{n} \sigma_{d_i} \geq 0,$$

As expected, the total entropy production is the sum of the entropy production of the mechanical dissipation and each individual hysteron.

4.2 Hysteresis in the electrical domain

Hysteresis can be introduced in a similar manner in the electrical domain. Consider an hysteron composed of a (possible non-linear) capacitor and a non-linear, non-smooth resistance. Define the total voltage applied to the hysteron as

$$V_h = V_C + V_w \qquad (25)$$

where $V_C = \frac{Q_z}{C_h}$ corresponds to the voltage associated to the conservative element and V_w the voltage across the dissipative element. The dynamic of the hysteron is derived from the constitutive law of the resistive element

$$\dot{Q}_z = w(V_w) = w(V_h - V_C) = w \left(V_h - \frac{Q_z}{C_h} \right),$$

where $w(\cdot)$ is a non-linear, non-smooth operator. Physically it corresponds to a conductance. From the second law the energy dissipated through the dissipative element produces entropy irreversibly. Hence,

$$\dot{Q}_z V_w = w \left(V_w \right) V_w = \sigma_w \geq 0$$

where σ_w corresponds to the irreversible entropy production of the hysteron. Just as for the MSD system, we shall represent the hysteresis in the RLC system by an hysterion interconnected in series with the capacitor with capacitive coefficient C in (11). The hysterion will hence act as a non-linear state dependent current source. The deduction of the model is equivalent as for the MSD system by defining as state vector $x = [Q_t \ \lambda \ Q_{z_1} \ \cdots \ Q_{z_n} \ S]^\top$,

$$\dot{x} = \left(J_e - R_r J_r - \sum_{i=1}^{n} R_{w_i} J_{w_i} \right) \frac{\partial U_e}{\partial x} + g_e u_e \qquad (26)$$

with $J_e = J_m$, $J_r = J_b$, $J_{w_1} = J_{d_1}$, $J_{w_n} = J_{d_n}$,

$$R_r = \frac{r}{T} \left(\frac{\lambda}{L} \right)^2,$$

$$R_{w_1} = \frac{1}{T} w \left(\frac{1}{C} \left(Q_t - \sum_{i=1}^{n} Q_{z_i} \right) - \frac{1}{C_{h_1}} Q_{z_1} \right),$$

$$\vdots \quad \vdots$$

$$R_{w_n} = \frac{1}{T} w \left(\frac{1}{C} \left(Q_t - \sum_{i=1}^{n} Q_{z_i} \right) - \frac{1}{C_{h_n}} Q_{z_n} \right),$$

and internal energy function

$$U_e = \frac{1}{2C} \left(Q_t - \sum_{i=1}^{n} Q_{z_i} \right)^2 + \frac{\lambda^2}{2L} + \sum_{i=1}^{n} \frac{Q_{z_i}^2}{2C_{h_i}} + \mathcal{S}_e(S).$$

Just as for the mechanical domain, it is straightforward to see that for the i-th hysterion $\gamma_{w_i} = \frac{1}{T} w_i(\cdot) > 0$, $\{S, U\}_{J_{w_i}} = \frac{1}{C} (Q_t - \sum_{i=1}^{n} Q_{z_i}) - \frac{1}{C_{h_i}} q_{z_i} = V_{w_i}$, the voltage across dissipative element, which defines indeed the thermodynamic driving force of the irreversible dissipation of the i-th hysterion. Consider for simplicity $u_e = 0$, then the entropy balance of (26) is given by

$$\dot{S} = \frac{r}{T} \left(\frac{\lambda}{L} \right)^2 + \frac{1}{T} \sum_{i=1}^{n} w_i \left(V_{w_i} \right) V_{w_i}$$

$$= \sigma_r + \sum_{i=1}^{n} \sigma_{w_i} \geq 0.$$

The total entropy production is the sum of the entropy production of the electrical dissipation and each individual hysteron.

4.3 Hysteresis as a multidomain coupling term

Lets us consider an electromechanical system coupled through shared hysteresis. This is for instance the case of piezo-electric actuators or magnetic shape memory alloys

(Calchand, 2014). Since the coupling relation is given through the hysteresis, the hysterions of the previous models can be used to formally induce the multidomain interaction. Define the following IPHS

$$\dot{x} = \left(J_0 + R_r J_R + R_b J_B + \sum_{i=1}^{n} R_{w_i} J_{W_i} + \sum_{i=1}^{n} R_{d_i} J_{D_i}\right) \frac{\partial U}{\partial x} + gu$$

where $U = U_e + U_m$, $g = [g_e \ g_m]^\top$ and

$$J_0 = \begin{bmatrix} J_e & 0 \\ 0 & J_m \end{bmatrix}, \qquad J_R = \begin{bmatrix} J_r & 0 \\ 0 & 0 \end{bmatrix}, \qquad J_B = \begin{bmatrix} 0 & 0 \\ 0 & J_b \end{bmatrix}$$

$$J_{W_i} = \begin{bmatrix} J_{w_i} & 0 \\ 0 & 0 \end{bmatrix}, \qquad J_{D_i} = \begin{bmatrix} 0 \\ 0 & J_{d_i} \end{bmatrix}$$

The state variables of the hysterons are used to define the multidomain coupling, hence they will appear in the internal energy function as "cross terms" between the different physical domains. Indeed, in order to explicitly make appear the coupling the state variable of an hysteron defined in for instance the mechanical domain will appear in the electrical energy function, inducing a non-linear voltage on the dynamic of the magnetic flux. This will be illustrated in next section.

5. EXAMPLE: A MAGNETIC SHAPE MEMORY ALLOY

In Calchand (2014) a control model for an isothermal Magnetic Shape Memory Alloy (MSMA) has been proposed. The simplest version of this model includes only one hysteron and is given by

$$\begin{bmatrix} \dot{\lambda} \\ \dot{q}_z \\ \dot{q}_t \\ \dot{p} \end{bmatrix} = \begin{bmatrix} -r & 0 & 0 & 0 \\ 0 & -d(\cdot) & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -b \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial \lambda} \\ \frac{\partial H}{\partial q_z} \\ \frac{\partial H}{\partial q_t} \\ \frac{\partial H}{\partial p} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u$$

with input $u = [V, F]^\top$, an external input voltage and force respectively, and energy function

$$H(\lambda, q_z, q_t, p) = H_e(\lambda, q_z) + H_m(q_t, p, q_z)$$
$$= H_e(\lambda, q_z) + \frac{1}{2}\frac{p^2}{m} + \frac{1}{2}k(q_t - q_z)^2 \quad (27)$$

Where $H_e(\lambda, q_z)$ correspond to the magnetic energy of the electrical part of the system and which depends on q_z. In Calchand (2014) the magnetic energy has been identified as a non-linear sector function which takes into account the hysterisis of the mechanical domain. We shall not give the explicit definition of $H_e(\lambda, q_z)$ since it is not relevant for the present study and the reader is refereed to Calchand (2014) for further details. In order to include the non-isothermal phenomena we extend the state space and include the entropy as state variable. This leads to the following IPHS

$$\begin{bmatrix} \dot{\lambda} \\ \dot{q}_z \\ \dot{q}_t \\ \dot{p} \\ \dot{S} \end{bmatrix} = \left(\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \frac{r}{T}\frac{\partial H}{\partial \lambda} \begin{bmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \right.$$

$$\left. \frac{b}{T}\frac{p}{m} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} + \frac{1}{T}d(F_d) \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix} \right) \begin{bmatrix} \frac{\partial H}{\partial \lambda} \\ \frac{\partial H}{\partial q_z} \\ \frac{\partial H}{\partial q_t} \\ \frac{\partial H}{\partial p} \\ \frac{\partial H}{\partial S} \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^\top u$$

The IPHS has one reversible and three irreversible structure matrices, hence there are three sources of internal irreversible entropy production. Assume $u = [0 \ 0]^\top$ for simplicity, then

$$\dot{S} = \frac{r}{T}\left(\frac{\partial H}{\partial \lambda}\right)^2 + \frac{b}{T}\left(\frac{p}{m}\right)^2 - \frac{1}{T}d(F_d)\frac{\partial H}{\partial q_z}$$
$$= \sigma_r + \sigma_b + \sigma_d$$

which corresponds indeed to the irreversible internal entropy creation of the system since $-\frac{\partial H}{\partial q_z} = F_d$. It is straightforward to extend this model to a model with n hysterons, as illustrated in the preceding section.

6. CONCLUSION

An irreversible port-Hamiltonian system (IPHS) representation of non-isothermal electromechanical systems with hysteresis has been proposed. By representing the hysterisis through hysterons interconnected with the mechanical and electrical components, it has been shown that the hysteresis behaves as an irreversible process. This is elegantly captured by the IPHS structure and makes it possible to isolate the different irreversible phenomena of the overall system. Furthermore, it has been shown that in general an electromechanical system with hysteresis corresponds to a reversible-IPHS, i.e., the combination of a conservative Hamiltonian system with an irreversible one defined with respect to the same Hamiltonian. This is very interesting since it implies geometric properties between the different structure matrices, which can then be exploited for control design. Future work will deal with the control and the experimental validation of the proposed models.

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