Modeling and control of an IPMC actuated flexible beam under the port-Hamiltonian framework ⋆

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Abstract: This paper deals with the modeling and control problem of an ionic polymer metal composites (IPMC) actuated flexible beam. The mechanical dynamic of the flexible beam and the electrical dynamic of the IPMC actuators have been taken into account in the modeling approach. Furthermore, in order to achieve the desired configuration of this IPMC actuated flexible beam, a control strategy is proposed based on the Linear quadratic Gaussian (LQG) control and damping injection. Finally, the proposed model is validated on a real experimental set-up. The effectiveness of the proposed control strategy is shown by the simulation results based on the real physical parameters of the experimental set-up.

Keywords: Port-Hamiltonian systems, Distributed control, LQG method, IPMC, Flexible beam

1. INTRODUCTION

The medical use of endoscope for the minimally invasive surgery becomes more and more common due to the advantage of the suffering alleviation of patients. The research on this topic has drawn the attention of researchers since the last century. In the recent years, because of the development of smart materials and manufacturing techniques, particular interest is to use the embedded actuators on endoscopic robotics for providing additional degrees of freedom. A micro endoscope model for endonasal skull base surgery has been proposed in (Chikhaoui et al., 2014). The bending of the endoscope is performed by electro-active polymer (EAP) actuators as shown in Fig. 1. The

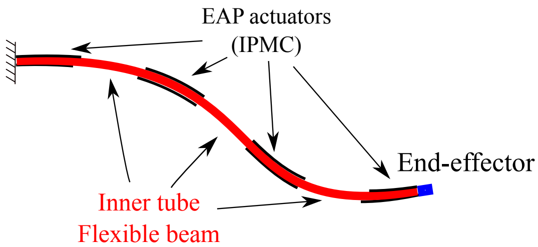

Fig. 1. Simplified EAP actuated compliant endoscope

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EAP used to bend the endoscope in this model is Ionic Polymer Metal Composites (IPMC). This actuator is one of the most important EAP actuators which has attractive properties such as: low actuation voltage, ease of fabrication and relatively high strain (Shahinpoor and Kim, 2001). The main part of the endoscope is a compliant inner tube. From the modeling point of view, this compliant inner tube can be regarded as a flexible beam.

Due to the flexibility of the inner tube and physical proprieties of the IPMC actuator, this IPMC actuated endoscope naturally leads to a complex multi-physical modeling and control problem. For this reason, the port-Hamiltonian framework shall be investigated in this paper to deal with this multi-physical modeling and control problem. This approach has been proven to be powerful for the modeling and control of complex physical systems (Maschke and van der Schaft, 1992, 1994). It has been generalized to distributed parameter systems described by partial differential equations (van der Schaft and Maschke, 2002; Le Gorrec et al., 2005). The port-Hamiltonian approach is based on the characterization of the energy exchanges between components of the system. It allows to interconnect different parts of the system through the energy change ports in a straight and clear way. Thanks to this interconnection advantage, the port-Hamiltonian approach is very adapted for modeling this IPMC actuated endoscope described as a couple system of partial differential equation (PDE) interconnected with an ordinary differential equation (ODE). Hence, the port-Hamiltonian approach is very suitable for modeling of the IPMC actuated endoscope taking both the flexibility of the inner tube and the IPMC physical propriety into account. On the other hand, the port-Hamiltonian framework provides also various passivity based control design strategies for the infinite-dimensional systems(Macchelli, 2011; Ramírez et al., 2014). Furthermore, an appropriate coordinate approximation and a lower order controller are proposed in (Wu et al., 2018). These results are based on the LQG control design method (Jonckheere and Silverman, 1983; King et al., 2006), but provide a passivity and Hamiltonian structure preserving reduction schema which can design a reduced order control for the infinite-dimensional port-Hamiltonian system.

The main contributions of this paper are to propose a reliable model for the IPMC actuated flexible beam which reproduces the basic properties of the medical endoscope and to design a control strategy for the proposed model based on the structure preserving LQG method. This paper is organized as follow: a PDE-ODE interconnected model for the 1-D IPMC actuated flexible beam is proposed in Section 2. In Section 3, an LQG and damping injection based finite-dimensional controller has been investigated in order to improve the dynamics of the system. An experimental set-up is used to validate the proposed model in Section 4 and the simulation results show the effectiveness of the proposed control law are shown by using physical parameters of this experimental set-up. Eventually, final remarks and perspectives of this work are discussed in Section 5.

2. MODELING OF IPMC ACTUATED FLEXIBLE BEAM

The Timoshenko beam model is used to reproduce the flexible behavior of the endoscope. The IPMC actuator patches are glued on the flexible beam in order to control the configuration of the beam. In this section, a complete model of an IPMC actuated flexible beam is proposed.

2.1 Port-Hamiltonian formulation for flexible beam with distributed control

Let first consider a Timoshenko beam described as a port-Hamiltonian system (Macchelli and Melchiorri, 2004; Jacob and Zwart, 2012):

$$\dot{x} = (\mathcal{J} - \mathcal{R})\,\mathcal{L}x \tag{1}$$

with the operator $\mathcal{J} = \left(P_1 \frac{\partial}{\partial z} + P_0\right)$ and matrices:

$$\mathcal{L} = \begin{bmatrix} K & 0 & 0 & 0 \\ 0 & \frac{1}{\rho} & 0 & 0 \\ 0 & 0 & EI & 0 \\ 0 & 0 & 0 & \frac{1}{I_\rho} \end{bmatrix}, \qquad \mathcal{R} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & R_t & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_r \end{bmatrix}, \tag{2}$$

$$P_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \qquad P_0 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

with the state (energy) variables: the shear displacement $x_1 = \frac{\partial w}{\partial z}(z,t) - \phi(z,t)$, the transverse momentum distribution $x_2 = \rho(z)\frac{\partial w}{\partial t}(z,t)$, the angular displacement $x_3 = \frac{\partial \phi}{\partial z}(z,t)$ and the angular momentum distribution $x_4 = I_\rho \frac{\partial \phi}{\partial t}(z,t)$ for $z \in (a,b)$, $t \geq 0$, where $w(z,t)$ is the transverse displacement and $\phi(z,t)$ is the rotation angle of the beam. The coefficients ρ, I_ρ, E, I and K are the mass per unit length, the angular moment of inertia of a cross section, Young's modulus of elasticity, the moment of inertia of a cross section, and the shear modulus respectively, and the state space $X = L^2([a,b];\mathbb{R}^4)$. The operator $\mathcal{J} = P_1 \frac{\partial}{\partial z} + P_0$ defined by the matrices $P_1 = P_1^T$ and $P_0 = -P_0^T$ is a first order skew symmetric differential operator acting on the state space X. The matrix $\mathcal{R}$ is the dissipation matrix contains the translation and angular viscous fraction constants R_t and Rr. The energy of the beam is expressed in terms of the energy variables,

$$\begin{aligned} H_b &= \frac{1}{2}\int_a^b (Kx_1^2 + \frac{1}{\rho}x_2^2 + EIx_3^2 + \frac{1}{I_\rho}x_4^2)dz \\ &= \frac{1}{2}\int_a^b x(z)^T(\mathcal{L}x)(z)dz = \frac{1}{2}\parallel x \parallel_{\mathcal{L}}^2 \end{aligned} \tag{4}$$

In order to define an extended Dirac structure including the boundary (Le Gorrec et al., 2005), we introduce the following boundary port-variables:

$$\begin{bmatrix} f_{\partial,\mathcal{L}x} \\ e_{\partial,\mathcal{L}x} \end{bmatrix} = \begin{bmatrix} (\rho^{-1}x_2)(b) - (\rho^{-1}x_2)(a) \\ (Kx_1)(b) - (Kx_1)(a) \\ (I_\rho^{-1}x_4)(b) - (I_\rho^{-1}x_4)(a) \\ (EIx_3)(b) - (EIx_3)(a) \\ (\rho^{-1}x_2)(b) + (\rho^{-1}x_2)(a) \\ (Kx_1)(b) + (Kx_1)(a) \\ (I_\rho^{-1}x_4)(b) + (I_\rho^{-1}x_4)(a) \\ (EIx_3)(b) + (EIx_3)(a) \end{bmatrix} = \begin{bmatrix} v(b) - v(a) \\ F(b) - F(a) \\ w(b) - w(a) \\ T(b) - T(a) \\ v(b) + v(a) \\ F(b) + F(a) \\ w(b) + w(a) \\ T(b) + T(a) \end{bmatrix} \tag{5}$$

where $F(z), T(z), v(z), w(z)$ are the force, moment, velocity and angular velocity at z point respectively.

The flexible beam is clamped at the a side and free at the other side b. In order to define the boundary condition of the Timoshenko beam (1), we define the input and output variables by the boundary ports as follows:

$$u_b = W \begin{bmatrix} f_{\partial,\mathcal{L}x} \\ e_{\partial,\mathcal{L}x} \end{bmatrix}, \; y_b = \tilde{W} \begin{bmatrix} f_{\partial,\mathcal{L}x} \\ e_{\partial,\mathcal{L}x} \end{bmatrix}, \tag{6}$$

where

$$\begin{aligned} W &= \frac{1}{2}\begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}, \\ \tilde{W} &= \frac{1}{2}\begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}. \end{aligned} \tag{7}$$

Using this partition the input and output boundary port variables are explicitly given as follows:

$$\begin{aligned} u_b &= \left[\, v(a) \;\; w(a) \;\; F(b) \;\; T(b) \,\right]^T, \\ y_b &= \left[\, F(a) \;\; T(a) \;\; -v(b) \;\; -w(b) \,\right]^T. \end{aligned} \tag{8}$$

Notice that $\mathcal{A}x = (\mathcal{J} - \mathcal{R})\,\mathcal{L}x$ with the domain

$$D(\mathcal{A}) = \left\{ \mathcal{L}x \in H^1\left([a,b];\mathbb{R}^4\right) \middle| \begin{bmatrix} f_{\partial,\mathcal{L}x} \\ e_{\partial,\mathcal{L}x} \end{bmatrix} \in \ker W \right\}$$

generates a contraction semi-group on X.

As the beam is clamped at the side a, and free at other side b. Thus the boundary conditions are: on the side a, the velocity and the angular velocity are zeros and on the side b, the force and the moment are zeros. This implies the $u_b = 0$. The outputs are the power conjugated of inputs. According to the boundary conditions, they are the reaction forces at the side a, $F(a)$ and $T(a)$, the velocity and the angular velocity are free at the side b respectively.

The control objective is to modify the configuration of the flexible beam by the IPMC actuator patches. The actuation of this kind of actuator is the bending moment due to the voltage applied on it. Hence, we consider some distributed port defined by distributed moment acting on the beam. With the distributed port $(\, y_d, u_d\,)^T$, the system becomes:

$$\begin{cases} \dot{x} \,=\, (\mathcal{J} - \mathcal{R})\,\mathcal{L}x + \mathcal{B}u_d \\ y_d \,=\, \mathcal{B}^* \mathcal{L}x \end{cases} \tag{9}$$

where the $\mathcal{B} : \mathbb{C}^i \mapsto X$ is the distributed input map, $u_d \in \mathbb{C}^i$ are the distributed moment applied on the beam, $y_d \in \mathbb{C}^i$ are the power conjugated variables of u_d, *i.e.* the angular velocities.

The configuration of the beam is controlled by the distributed moment generated by IPMC actuators over the domain of the beam. The distributed input variables are the distributed moment: $b_i(z)u_{di}(t)$ on the $i-$th small intervals $I_{bi} = [\alpha_i, \beta_i]$ of the spatial space $[a, b]$, i.e. $b_i(z) = 1$ if $z \in I_{bi}$ and $b_i(z) = 0$ elsewhere and $i \in \{1, 2, \cdots, m\}$ if there are m actuators glued on the beam. As output, we consider the angular velocity mean values in the same intervals $f_{di} = y_{di} = \int_a^b b_i(z) \frac{1}{I_\rho} x_4 dz$. As a consequence the distributed input is given by:

$$\mathcal{B}u_d = \sum_i \begin{bmatrix} 0 \\ 0 \\ 0 \\ b_i(z) \end{bmatrix} u_{di}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ b(z) \end{bmatrix} u_d(t) \tag{10}$$

where $\mathcal{B} : \mathbb{C}^m \mapsto X$, $b(z) = [b_1(z), \cdots, b_m(z)]$ and $u_d(z) = [u_{d1}(z), \cdots, u_{dm}(z)]^T$. The output is the power conjugated variable of the input, *i.e.*, $y_d = \mathcal{B}^*\mathcal{L}x$. The energy balance equation is defined as $\frac{\partial H_b}{\partial t} \leq y_d^T u_d$. The inputs u_d are bending moments generated by IPMC actuators. The dynamic of the IPMC actuator and the interconnection between the IPMC actuator and the flexible beam will be explicated in the next subsection.

2.2 The IPMC actuator model

The bending of the IPMC with respect to the applied voltage is mainly attributed to the cations flux and polar solvents in the polymer membrane diffusion between the electrodes (see left side in Fig 2) (Shahinpoor and Kim, 2001).

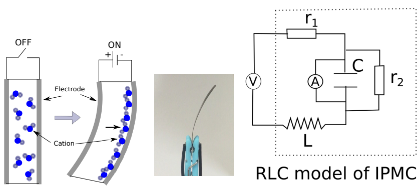

Fig. 2. IPMC bending principle and its electrical model

The dynamics of IPMC is composed of three parts: the electric part from the electrodes, the diffusion in the polymer and the mechanical part of the actuator. In this work, since we assume a perfect interconnection between the actuator and the beam, the mechanical contribution of the IPMC actuator is considered as part of the flexible structure. Hence, we shall consider only the electric interface/polymer diffusion components of the IPMC. A lumped RLC equivalent circuit model of the IPMC has been proposed in (Gutta et al., 2009) and is shown on the right side of Fig 2. The output torque of the IPMC is proportional to the voltage across the capacitor. The interconnection ports are placed across the capacitor.

The electric model of one IPMC actuator can be written as the following PHS

$$\begin{cases} \begin{bmatrix} \dot{\varphi} \\ \dot{Q} \end{bmatrix} = \begin{bmatrix} -r_1 & -1 \\ +1 & -\dfrac{1}{r_2} \end{bmatrix} \begin{bmatrix} \dfrac{\partial H_a}{\partial \varphi} \\ \dfrac{\partial H_a}{\partial Q} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_a, \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{\partial H_a}{\partial \varphi} \\ \dfrac{\partial H_a}{\partial Q} \end{bmatrix}, \quad y_a = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \dfrac{\partial H_a}{\partial \varphi} \\ \dfrac{\partial H_a}{\partial Q} \end{bmatrix} \end{cases},$$

where the total energy of the system is defined as the sum of the magnetic and electric energies $H_a = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}\frac{\varphi^2}{L}$ and the state vector is $x_a = [\varphi, Q]^T$ with φ the flux and Q the charge of the capacitor, r_1 and r_2 are the resistances, u is the applied voltage on the IPMC actuator and y is the current in the inductance and y_a is the voltage across the capacitor. The above system is only for one actuator. We assume that the beam is driven by m IPMC actuators, thus we can write their electrical dynamics together as follows:

$$\begin{cases} \begin{bmatrix} \dot{\varphi} \\ \dot{Q} \end{bmatrix} = \begin{bmatrix} -R_1 & -I_m \\ I_m & -R_2 \end{bmatrix} \begin{bmatrix} \dfrac{\partial H_a}{\partial \varphi} \\ \dfrac{\partial H_a}{\partial Q} \end{bmatrix} + \begin{bmatrix} I_m \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ I_m \end{bmatrix} u_a \\ y = \begin{bmatrix} I_m & 0 \end{bmatrix} \begin{bmatrix} \dfrac{\partial H_a}{\partial \varphi} \\ \dfrac{\partial H_a}{\partial Q} \end{bmatrix}, \quad y_a = \begin{bmatrix} 0 & I_m \end{bmatrix} \begin{bmatrix} \dfrac{\partial H_a}{\partial \varphi} \\ \dfrac{\partial H_a}{\partial Q} \end{bmatrix} \end{cases} \tag{11}$$

with state variables

$$\varphi = [\varphi_1 \cdots \varphi_m]^T \in \mathbb{R}^m,$$
$$Q = [Q_1 \cdots Q_m]^T \in \mathbb{R}^m,$$

input output variables

$$u, u_a, y, y_a \in \mathbb{R}^m$$

dissipation matrices $R_1 = \text{diag}[r_1, r_1, \cdots, r_1] \in \mathbb{R}^{m\times m}$, and $R_2 = \text{diag}[1/r_2, 1/r_2, \cdots, 1/r_2] \in \mathbb{R}^{m\times m}$.

Furthermore, the bending moments applied on the flexible structure are generated by y_a with constant coefficients $k_i, i \in \{1, 2, \ldots, m\}$. From the power conserving interconnection, u_a is the current applied on the capacitor due to the mechanical movement of the structure. The interconnection relation is defined by

$$\begin{bmatrix} u_d \\ u_a \end{bmatrix} = \begin{bmatrix} 0 & +k \\ -k & 0 \end{bmatrix} \begin{bmatrix} y_d \\ y_a \end{bmatrix}.$$

with $k = \text{diag}\,[k_1, \cdots, k_m] \in \mathbb{R}^m$. The interconnected model of the flexible beam and the IPMC actuators can be written as

$$\begin{aligned} \dot{\mathbf{x}} \,&=\, \underbrace{\begin{bmatrix} \mathcal{J} - \mathcal{R} & \mathcal{B}k \\ -k^T\mathcal{B}^* & J - R \end{bmatrix}}_{\mathbf{J}-\mathbf{R}} \frac{\partial \mathbf{H}}{\partial \mathbf{x}} + \begin{bmatrix} 0 \\ I_m \\ 0 \end{bmatrix} u \\ y \,&=\, [0 \;\; I_m \;\; 0]\, \frac{\partial \mathbf{H}}{\partial \mathbf{x}}, \end{aligned} \tag{12}$$

where $u, y \in \mathbb{R}^m$, $\mathbf{x} = [x, \varphi, Q]^T$ and 0 are zero matrices of appropriate dimensions. The state space of the complete system is $X = L^2([a, b]; \mathbb{R}^4) \times \mathbb{R}^{2m}$. Notice that

$$J - R = \begin{bmatrix} -R_1 & -I_m \\ I_m & -R_2 \end{bmatrix}.$$

The total Hamiltonian of the interconnected system is:

$$\mathbf{H} = H_a + H_b = \frac{1}{2}\mathbf{x}^T \mathbf{Q} \mathbf{x} = \frac{1}{2} Q^T C^{-1} Q + \frac{1}{2}\varphi^T L^{-1}\varphi + \frac{1}{2} \parallel x \parallel^2_{\mathcal{L}}$$

with the capacitance matrix $C = \text{diag}[C_1, C_2, \cdots, C_m] \in \mathbb{R}^m$ and the inductance matrix $L = \text{diag}[L_1, L_2, \cdots, L_m] \in \mathbb{R}^m$. The energy matrix of the whole system is given by $\mathbf{Q} = \text{diag}[\mathcal{L}, L, C]$.

3. LQG CONTROL DESIGN AND DAMPING INJECTION

In this section, we shall consider the control problem of this IPMC actuated flexible beam model (12). The control objective is to achieve the desired configuration of the flexible beam using the appropriate electrical tension applied on the IPMC patches. To this end, we proposed an LQG control design plus a damping injection method shown in Fig. 3.

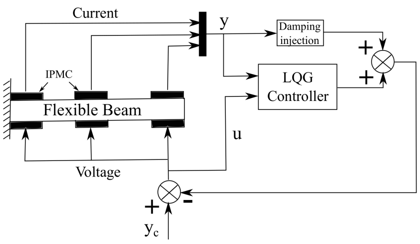

Fig. 3. Experimental set-up

This control design strategy contains two parts. Firstly, the LQG controller is used to reduce the vibration of the flexible beam. However, the LQG controller has the disadvantage of having the same order as the system itself, and the LQG controller can not preserve the passivity in general case. Hence, the Hamiltonian LQG control design and reduction method shall be used to get a reduced order controller which is easier to implement on real physical systems. Secondly, the damping injection is used to ameliorate the setting time of the system. The LQG controller contains a Kalman filter and an optimal state feedback problem. It can be reformulated as:

$$\begin{cases} \dot{\hat{\mathbf{x}}} = ((\mathbf{J} - \mathbf{R})\,\mathbf{Q} - \mathbf{B}K - F\mathbf{B}^*\mathbf{Q})\,\hat{\mathbf{x}} + F u_c \\ y_c = K\hat{\mathbf{x}} \end{cases} \tag{13}$$

where $\hat{\mathbf{x}}$ is the state variable of the LQG controller, F and K are the filter and optimal state feedback gains. These gains can be computed as:

$$K = \bar{R}^{-1} B^* P_c \quad \text{and} \quad F = P_f \mathbf{Q} B R_w^{-1}$$

where the P_c and P_f are the solutions of the following filter and control Riccati equations:

$$(\mathbf{J} - \mathbf{R})\,\mathbf{Q} P_f + P_f \mathbf{Q}\,(\mathbf{J} - \mathbf{R})^* - P_f \mathbf{Q}\mathbf{B} R_w^{-1} \mathbf{B}^* \mathbf{Q} P_f + Q_v = 0 \tag{14}$$

$$\mathbf{Q}\,(\mathbf{J} - \mathbf{R})^* P_c + P_c\,(\mathbf{J} - \mathbf{R})\,\mathbf{Q} - P_c \mathbf{B} \bar{R}^{-1} \mathbf{B}^* P_c + \bar{Q} = 0 \tag{15}$$

where Q_v and R_w are the covariance operators of the state and the output measurement white noises, $P_f = P_f^* > 0$ is the unique solution of the Riccati equation (14). $\bar{Q}$ and $\bar{R}$ are the weighting operators of the optimal control problem consisting the following cost function:

$$J_c = \int_0^{+\infty} \left(\parallel \mathbf{x} \parallel^2_{\bar{Q}} + \parallel u \parallel^2_{\bar{R}} \right) dt. \tag{16}$$

In general case, the LQG controller (13) is not passive and the Hamiltonian structure can not be preserved in the closed-loop system. Nevertheless, the LQG controller can be reformulated as the port-Hamiltonian system if the weighting operators $\bar{Q}$ and $\bar{R}$ and the covariance operators Q_v and R_w are chosen as in the following theorem (Wu et al., 2018):

Theorem 1. (Hamiltonian LQG method). The LQG controller (13) with the associated weighting operators and covariance operators

$$\bar{R} = R_w. \tag{17}$$

and $\bar{Q}$ and Q_v such that:

$$Q_v z = \mathbf{Q}^{-1}\left(2\mathbf{Q}\mathbf{J}^* P_c + 2P_c \mathbf{J}\mathbf{Q} + \bar{Q}\right)\mathbf{Q}^{-1} z, \tag{18}$$

with $z \in X$, is passive and has a port-Hamiltonian realization. Furthermore the operator equations (15) and (14) admit a unique solution, P_c and P_f respectively. These two solutions are related by:

$$\mathbf{Q}^{-1} P_c = P_f \mathbf{Q} \tag{19}$$

The above theorem provides a passive LQG control design method and derives a port-Hamiltonian closed-loop system. Another advantage of this LQG control design that it provides a balanced reduction coordinate because $P_c P_f \neq I$ implies the state space is separable due to their different contribution for the controller design. Thus, the control design and reduction problem can be considered in the same time. The balanced reduction coordinate for the port-Hamiltonian system (12) as:

Definition 2. The port-Hamiltonian system is called *Hamiltonian LQG balanced* if there exist positive and non-increasing sequence $(\sigma_n)_{n\in\mathbb{N}}$ such that the Riccati equation solutions P_f and P_c are both equal to the diagonal operator:

$$P_f = P_c = \Sigma = \text{diag}(\sigma_n)_{n\in\mathbb{N}} \in \mathcal{L}(\ell_2). \tag{20}$$

Let T be the transformation operator that diagonalizes P_c and P_f such that:

$$T P_f T^* = T^{+} P_c T^{+*} = \Sigma. \tag{21}$$

Then the Hamiltonian LQG balanced realization of the port-Hamiltonian system (12) can be denoted as follow:

$$\begin{cases} \dot{\mathbf{x}}_b = (\mathbf{J}_b - \mathbf{R}_b)\,\mathbf{Q}_b \mathbf{x}_b + \mathbf{B}_b u \\ y = \mathbf{B}_b^* \mathbf{Q}_b \mathbf{x}_b \end{cases}. \tag{22}$$

Then the Petrov-Galerkin projection method shall be used to reduce the balanced realization (22) and its Hamiltonian LQG controller with preserving the passivity and the Hamiltonian structure. Interested readers can find further details in (Harkort and Deutscher, 2012). By using the structure preserving method, the reduced port-Hamiltonian system can be written as:

$$\begin{cases} \dot{x}_r = (J_r - R_r)\, Q_r x_r + B_r u \\ y = B_r^T Q_r x_r \end{cases}. \tag{23}$$

The reduced order LQG controller can be designed by the above reduced order system and Theorem 1. This controller can be applied to the complete system (12) in order to compensate the vibration of the flexible beam.

In order to ameliorate the response time of the system, we shall employ the damping injection method proposed for the port-Hamiltonian framework (van der Schaft, 2000). In this case, we consider the following control law:

$$u = -r_c y, \tag{24}$$

where output y of the system is the current, which can be measured very easily. The main objective of the control is to improve the response performance which is, in this case, to reduce the response time. So we can use a positive damping injection to accelerate the dynamic of the system, *i.e.* the control parameter $r_c < 0$. However, in order to guarantee the stability of the system, this parameter is lower bounded by the nature damping coefficient r_1, *i.e.* $r_c > -r_1$.

However, the variation of the flexible beam becomes more important when the response time gets faster. Hence, we combine the LQG controller and the damping injection together in order to find a compromise between the flexible beam vibration and the time response.

4. VALIDATION OF THE MODEL AND THE CONTROL

In this section, we will validate our proposed model on an experimental set-up and show the effectiveness of the proposed control law using the physical parameters of this set-up.

4.1 Experimental validation of the model

The validation will be done on a experimental set-up which reproduces the basic mechanical property of the endoscope. A dSPACE board and a computer (with Matlab Simulink) is used to generate the control signals $U \in [0, 7V]$ on the IPMC, to get the measurements and to further implement the controller. The measurements are the displacement of the flexible structure, the applied voltage and the current of the IPMC actuator. The displacement is measured by a laser displacement sensor from KEYENCE company (LK-G152).

The flexible beam is clamped on one side and let free on the other side. Since we consider a linear Timoshenko beam model with a rectangular cross section, the moment of inertia can be computed by the width and the thickness of the beam. The physical parameters of the flexible beam and the IPMC actuator can be found in the data sheet and the literature shown in Table 1:

L	Length	$1.6 \times 10^{-1}\ m$
W	Width	$7 \times 10^{-3}\ m$
T	Thickness	$2.2 \times 10^{-4}\ mm$
ρ	Mass density	$936\ kg/m^3$
I	Inertia moment of area	$4.7 \times 10^{-15}\ m^4$
I_ρ	Angular moment of inertia	$4.34 \times 10^{-12}\ kg.m$
L_a	Length of IPMC patch	$3 \times 10^{-2}\ m$
C	Capacitance	$5.8 \times 10^{-2}\ F$
r_1	Resistance r_1	$29.75\ \Omega$
r_2	Resistance r_2	$700\ \Omega$

Table 1. Physical parameters of the flexible beam and IPMC actuator

The unknown parameters are the Young's modulus E, the shear modulus K and two dissipation constants R_t, R_r and the beam-actuator coupling constant k_i. In order to identify the parameters of the flexible beam, we measure the displacement with the laser sensor. The positioning of the laser sensor is at $5mm$ from the tip of the flexible structure in equilibrium position. The beam-actuator coupling constant k_i can be identified by measuring the blocking force of the IPMC.

For the identification procedure under Matlab® and for the control design and implementation afterword, we use the method proposed in (Golo et al., 2004) to discretize the system with preserving the Hamiltonian structure. The identification result is shown in the left figure of Fig 4. The curve fitting of the model simulation with optimally identified parameters (black solid line) and the experimental data (red dashed line) is satisfying, with a fitting percentage of 89.67%.

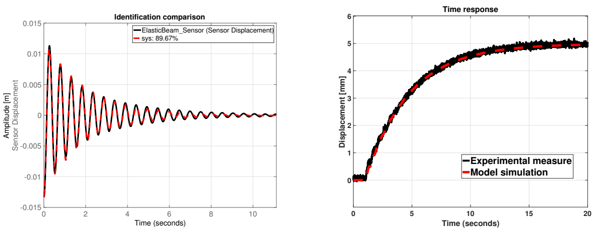

Fig. 4. Left: Parameter estimation. Right: IPMC actuated beam model validation

The identification results are shown in the Table 2.

E	Young's modulus	$4.14 \times 10^9\ Pa$
K	shear modulus	$1.418 \times 10^9\ Pa$
R_t	Traversal viscous fraction	$2 \times 10^{-5}\ kg.m^3/s$
R_r	Angular viscous fraction	$1 \times 10^{-5}\ kg.m/s$
k_i	Coupling constant	$3 \times 10^{-5}\ N.m/V$

Table 2. Identified parameters of the flexible beam and IPMC actuator

In order to validate the proposed model, we use the experimental set-up In this set-up, the flexible beam is actuated by one IPMC patch on the clamped side. In the right figure of Fig. 4, we compare the proposed model with the experimental measurement. The red dashed line is the simulation result with the proposed model and the black solid line is the experimental measure with applying $1.5V$ on the actuator.

4.2 Control of IPMC actuated flexible beam

The open-loop response of the IPMC actuated flexible beam has shown in the right figure of Fig. 4. The response time of this system is very slow ($T_r = 11s$). In order to improve the performance of this system, the control design strategy proposed in Section 3 will be used. We first consider the damping injection control law $u = -r_c y$ where output y of the system is the current which can be measured very easily. We use a positive damping injection to accelerate the dynamic of the system, *i.e.* the control parameter $r_c < 0$. However, in order to guarantee the stability of the system, this parameter is lower bounded by the nature damping coefficient r_1, *i.e.* $r_c > -r_1$. The oscillation of the beam on the free tip becomes important with using this simple positive damping injection. In order to reduce the vibration of the beam, we use the LQG based controller proposed in Theorem 1.

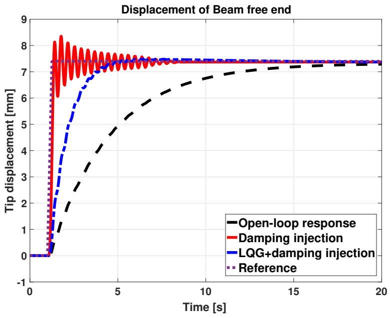

Fig. 5. Positive damping injection control and LQG+positive damping injection control

The comparison of different control laws for equilibrium position assignment has shown in Fig. 5. The Hamiltonian LQG method allows to get a lower order controller for the infinite-dimensional system. We recall the discretisation elements of the infinite-dimensional beam is 100, *i.e.*, the state variables of the full order LQG controller should be the same *i.e.* $\mathbf{x} \in \mathbb{R}^{402}$. We show the closed loop response using a reduced order LQG controller with only 2 state variables $x_c \in \mathbb{R}^2$. The purple dotted line is the reference of the tip displacement. The black dashed line is the open-loop response and the red solid line shows the positive damping injection closed-loop response. The blue dashed-dotted line is the closed-loop response with the Hamiltonian LQG controller plus the positive damping injection. By using this control law, the response time is significantly improved compared to the open-loop system. At the same time, this response has less vibration compared than only using the positive damping injection. Finally, a good compromise between oscillations and time response (around 2 second) can be found.

5. CONCLUSION AND FUTURE WORK

The problem of the modeling and the control design for an IPMC actuated flexible beam has been studied in this paper by using the port-Hamiltonian approach. The mechanical dynamic of the flexible beam has been modeled by the Timoshenko beam model. The IPMC actuator dynamic is considered as a RLC circuit. Two subsystems are interconnected through a power preserving way. Furthermore, a control law has been proposed based on the Hamiltonian LQG control method and damping injection. The damping injection is used to accelerate the response time of the system and the LQG controller is used to compensate the oscillation of the flexible beam. The proposed model has been validated by an experimental set-up. Then the simulation results show the effectiveness of the proposed control law by using real physical parameters of this experimental set-up. The future work will deal with the control law implantation on the experimental set-up. In the application of this paper, only one IPMC actuator patch has been studied. How to modify the shape of the flexible beam shall be investigated by using more actuator patches in the future.

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