Observer-Based State Feedback Controller for a class of Distributed Parameter Systems

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Abstract: This paper aims to propose a finite-dimensional observer-based state feedback controller to stabilize a class of boundary controlled system. To this end, we propose to use an early-lumping approach, where the infinite-dimensional port-Hamiltonian system is first discretized using a structure-preserving method. Then, we build a passive observed-based controller using a Linear Matrix Inequality (LMI) and finally, the controller is interconnected with the infinite-dimensional system in a passive way. Due to its passivity and Hamiltonian structure, this observer-based controller can stabilize not only the discretized lumped parameter system but also the original distributed parameter system. This approach avoids the intrinsic drawback of early lumping approach and spillover effects. Finally, the boundary controlled undamped wave equation is used to illustrate the effectiveness of the proposed controller.

Keywords: Port-Hamiltonian Systems (PHS), Boundary Control Systems (BCS), Linear Matrix Inequalities (LMI).

1. INTRODUCTION

The stabilization and the control of Boundary Control Systems (BCS) *i.e.* systems driven by Partial Differential Equations (PDE) with boundary sensing and control, has raised major attention among the control system community in the last decades. Recently, the control of BCS has been addressed by using the port-Hamiltonian framework (Le Gorrec et al., 2005). Port-Hamiltonian formulations are an extension of Hamiltonian formulations derived for mechanical to open multi-physic systems *i.e.* multi-physic systems with inputs and outputs (Maschke and van der Schaft, 1992; Duindam et al., 2009; van der Schaft, 2006). This formalism has proven to be particularly suitable for the modeling and control of complex systems such as infinite-dimensional and non-linear systems. The port-Hamiltonian formulations of distributed parameter systems (DPSs) have been investigated in (van der Schaft and Maschke, 2002; Le Gorrec et al., 2005; Villegas, 2007; Jacob and Zwart, 2012). Different stability results and control strategies have been proposed based on the structure and the passivity properties of these systems (Villegas et al., 2009; Ramírez et al., 2014; Macchelli et al., 2017; Ramírez et al., 2017). In particular, interesting results on the stability of boundary controlled PHS connected to dynamic controllers have been proposed in (Ramírez et al., 2017).

In the finite-dimensional setting, the observer-based state feedback control has shown to be a very efficient and popular control design technic, due to a large number of degrees of freedom that can be used for assigning the closed-loop performances of the system. The non controllability of the observer poles allows getting rid of the dynamic extension focusing on the original plant dynamics assignment. Many extensions to nonlinear and distributed parameter systems have been proposed since the primary works of Luenberger and Kalman (Kalman et al., 1960; Luenberger, 1964, 1966, 1971). Recently, the port-Hamiltonian representation has been drawing the attention because its easy way to deal with complex systems. But dealing with linear systems, not many works have been developed until now with this formulation. In fact, the only work that achieves equivalent results to the pole placement or Linear Quadratic Regulation (LQR) has been done in (Prajna et al., 2002). Using this same approach, it was developed a reduced order observer-based satate feedback controller in (Kotyczka and Wang, 2015).

On the other hand, it has been shown that the port-Hamiltonian system and the passivity are useful for the observer design of nonlinear systems in (Shim et al., 2003; Venkatraman and van der Schaft, 2010). The Interconnection and Damping assignment Passivity-Basd Control (IDA-PBC) method (Ortega et al., 2002) have been ex-

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tended to the observer design of the port-Hamiltonian system in (Biedermann et al., 2018; Vincent et al., 2016).

In the infinite-dimensional case, two approaches are possible. The first one is the late lumping approach in which the observer is designed from the infinite-dimensional systems. The main problem comes from the infinite-dimensional aspect of the controller structure that needs to be reduced for the practical and real-time implementation. The second one is the early lumping approach. In this case, the system is first discretized and then, the finite-dimensional controller is designed from the reduced order system. The main drawback is the spillover effect induced by the use of a reduced order controller on the infinite-dimensional system, leading to high-frequency mode destabilization.

The aim of this paper is to propose a new reduced order observer-based control design technics, for boundary controlled systems, that guarantees the passivity property of the resulting dynamic controller. This controller will allow assigning the low-frequency modes and will guarantee that when applied to the infinite-dimensional system it, will not destabilize the high-frequency modes, avoiding spillover effects.

This paper is organized as follows: Section 2 presents the control problem considered in this work. Then, Section 3 contains the main contribution of this work, where we propose a passive observer-based controller. After that, the effectiveness of the proposed control design is illustrated in Section 4 using the undamped wave equation. Finally, the conclusion of this work and the perspectives are given in the last section.

2. PROBLEM FORMULATION

Let first consider linear finite-dimensional port-Hamiltonian systems of the form:

$$P\begin{cases}\dot{x}(t)=(J-R)Qx(t)+gu(t)\\ y(t)=g^\top Qx\end{cases}\tag{1}$$

where $x\in\mathbb{R}^n$, $u,y\in\mathbb{R}^m$ are the state, input and output variables, $J=-J^\top\in\mathbb{R}^{n\times n}$, $\mathbb{R}^{n\times n}\ni R=R^\top\geqslant 0$, $\mathbb{R}^{n\times n}\ni Q=Q^\top>0$ and $g\in\mathbb{R}^{n\times m}$ are the structure matrix, dissipation matrix, the energy matrix of the system and the input matrix respectively. The total energy of the system is given as

$$H(x)=\frac{1}{2}x^\top Qx.\tag{2}$$

In order to set a desired behavior for this class of systems we consider an observer-based state feedback of the form

$$u=-K\hat{x}$$

where $\hat{x}$ is driven by the following Ordinary Differential Equation (ODE)

$$\hat{P}\begin{cases}\dot{\hat{x}}(t)=(J-R)Q\hat{x}(t)+gu(t)+g_c\left(y-\hat{y}\right)\\ \hat{y}(t)=g^\top Q\hat{x}\end{cases}\tag{3}$$

The state feedback gain matrix K and the Luenberger observer gain matrix g_c are designed separately by using pole placement, optimal control (LQR) or the LMI method in (Prajna et al., 2002). Combining the observer dynamics and the state feedback, the observer-based controller can be written as follows:

$$C\begin{cases}\dot{\hat{x}}(t)=\left((J-R)Q-g_cg^\top Q-gK\right)\hat{x}(t)+g_cu_c\\ y_c(t)=K\hat{x}\end{cases}\tag{4}$$

where $\hat{x}\in\mathbb{R}^n$, u_c and y_c are the observer state, input and output of the controller respectively. The closed-loop system can be written as the interconnection of the system (1) and the dynamic controller (4) with the following power preserving interconnection law:

$$\begin{aligned}u_c(t)&=\ \ y(t)\\ u(t)&=-y_c(t).\end{aligned}\tag{5}$$

Even if the closed loop performances are guaranteed by the state feedback, the resulting controller (4) loses the passive and port-Hamiltonian representation, because the matrix R is not necessarily semi-positive definite and the input-output pair are not conjugated. Then, when the finite-dimensional system (1) is obtained from the approximation of an infinite-dimensional system, there is no guarantee that the observer-based controller applied to the infinite-dimensional system will lead to satisfactory performances and even worst, the stability of the closed-loop system can not be guaranteed.

In this paper, we consider boundary controlled port-Hamiltonian systems of the form:

$$\mathcal{P}\begin{cases}\dfrac{\partial x}{\partial t}(t,z)=P_1\dfrac{\partial}{\partial z}(\mathcal{L}x)+P_0(\mathcal{L}x), & x(0,z)=x_0(z)\\ u(t)=\mathcal{B}x(t,z), & z\in[a,b]\\ y(t)=\mathcal{C}x(t,z), & t\geq 0\end{cases}\tag{6}$$

where $x(t,z)\in X=L_2([a,b];\mathbb{R}^n)$, $u(t)$ and $y(t)\in\mathbb{R}^n$ are the state variables of the system, the inputs used for control and the measured outputs respectively, $z\in[a,b]$ and $P_1=P_1^\top$, $P_0=-P_0^\top$ and $\mathcal{L}$ is a coercive operator in $X=L_2([a,b];\mathbb{R}^n)$. Finally, $\mathcal{B}$ and $\mathcal{C}$ are boundary operators. The system $\mathcal{P}$ in (6) is discretized in order to design a finite-dimensional controller (early lumping approach). To avoid the loss of structure and passivity of the infinite dimensional system (6), structure preserving discretization methods (Golo et al., 2004; Trenchant et al., 2017) are used. Then, the aproximation of the infinite-dimensional system (6) results in a finite-dimensional system with the same structure of (1)

$$P\begin{cases}\dot{x}_d(t)=(J_d-R_d)Q_dx_d(t)+g_du(t)\\ y_d(t)=g_d^\top Q_dx_d\end{cases}\tag{7}$$

where $x_d(t)\in\mathbb{R}^{n_dn}$ is the estimation of $x(t,z)$ in some specific points of the spatial domain. The dimension of this finite dimensional system is n_dn, where n is the number of state variables of the infinite-dimensional system (6) and n_d is the number of variables desired for the discretization of each sate variable in $x(t,z)$. Then, the dimension of the controller will be the same of (7), i.e. $n_c=n_dn$. Note that, one can change the size n_c depending on the type of discretization used. However, this does not make any difference in the controller design

In this article, we propose a new design methods that allows to assign the close-loop dynamic from an observer already designed and guarantees the passivity of the controller (4). This passivity property will be used to prove that when the passive finite-dimensional controller is applied to the infinite-dimensional system, the closed-loop system is asymptotically stability.

The main idea of this approach can be summarized in the following steps: discretize the infinite-dimensional system $\mathcal{P}$ in (6) choosing an n_d small enough to facilitate the design and large enough to describe the dynamics of $\mathcal{P}$ correctly. Then, design the controller as it is presented in this work considering the plant P in (7) and finally, use the controller on the real plant $\mathcal{P}$ in (6).

3. PASSIVE OBSERVER-BASED CONTROL DESIGN

In order to get rid of the reference signal we consider a general formulation of the observer-based controller (4) of the form:

$$C \begin{cases} \dot{\hat{x}}(t) = (J_c - R_c)Q_c\hat{x}(t) + g_c u_c(t) + g_d r(t) \\ y_c(t) = g_c^\top Q_c \hat{x} \\ y_p(t) = g^\top Q_c \hat{x} \end{cases} \tag{8}$$

with $\hat{x}(t) \in \mathbb{R}^{n_c}$ the state of the controller, $u_c(t)$ and $y_c(t) \in \mathbb{R}^n$ respectively, the input and output used to control the infinite-dimensional plant $\mathcal{P}$ in (6), $J_c = -J_c^\top$, $R_c = R_c^\top$, $Q_c = Q_c^\top$ and g_c some matrices to design, with J_c, R_c, $Q_c \in \mathbb{R}^{n_c \times n_c}$ and $g_c \in \mathbb{R}^{n_c \times n}$. $r(t)$ and $y_p(t) \in \mathbb{R}^n$ other ports used for observer purposes. We consider the interconnection of Fig. 1

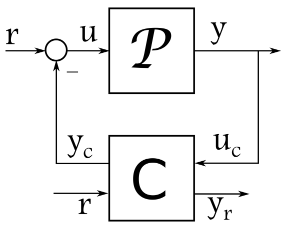

Fig. 1. Block Diagram of Control by Interconnection.

that correspond to the power preserving interconnection :

$$\begin{aligned} u_c(t) &= y(t) \\ u(t) &= r(t) - y_c(t) \end{aligned} \tag{9}$$

It has been shown in (Ramírez et al., 2017) that such interconnected system is stable as soon as the finite-dimensional system is passive. Hence, we aim at building a passive controller of the form (8). For this purpose we use the Theorem 1.

Theorem 1. Given the system (7) and the matrix g_c such that

$$A_o = (J_d - R_d - g_c g_d^\top)Q_d$$

is Hurwitz. If the following Linear Matrix Inequality (LMI) has a solution in the unknown symmetric matrix $\mathbf{X} = \mathbf{X}^\top$

$$2\alpha I_{n_c} - g_d g_c^\top - g_c g_d^\top + A_o\mathbf{X} + \mathbf{X}A_o^\top \leq 0 \tag{10a}$$
$$-2\beta I_{n_c} + g_d g_c^\top + g_c g_d^\top - A_o\mathbf{X} - \mathbf{X}A_o^\top \leq 0 \tag{10b}$$
$$-\tfrac{1}{\gamma} I_{n_c} + \mathbf{X} \leq 0 \tag{10c}$$
$$\tfrac{1}{\delta} I_{n_c} - \mathbf{X} \leq 0 \tag{10d}$$

with control design parameters α, β, γ and δ such that $0 \leq \alpha < \beta$ and $0 < \gamma < \delta$, then considering the following matrices

$$Q_c = \mathbf{X}^{-1} \tag{11}$$
$$S_c = A_o Q_c^{-1} - g_d g_c^\top, \tag{12}$$
$$J_c = \frac{1}{2}(S_c - S_c^\top) \tag{13}$$
$$R_c = -\frac{1}{2}(S_c + S_c^\top) \tag{14}$$

then, the following results hold

(i) $\lim_{t\to\infty} (x_d(t) - \hat{x}(t)) = 0$;
(ii) The matrices R_c and Q_c satisfy
  (a) $\alpha I_n \leq R_c \leq \beta I_n$;
  (b) $\gamma I_n \leq Q_c \leq \delta I_n$
  and the controller (8) is passive and has a port-Hamiltonian representation;
(iii) The closed-loop system can be written as the control by interconnection of the infinite-dimensional system (6) with the controller (8) and remains stable.

Proof. We consider the error signal

$$\tilde{x}(t) = x_d(t) - \hat{x}(t). \tag{15}$$

The result (i) in Theorem 1 is similar to prove that the error $\tilde{x}$ converges asymptotically to zero. Deriving the error (15) with respect to time, replacing $\dot{x}_d$ and $\dot{\hat{x}}$ from equations (7) and (8) respectively, and using the interconnection (9) the error dynamics is

$$\dot{\tilde{x}}(t) = (J_d - R_d - g_c g_d^\top)Q_d x_d - (J_c - R_c + g_d g_c^\top)Q_c\hat{x} \tag{16}$$

By the statement of the theorem 1, A_o is Hurwitz and is given by

$$A_o = (J_d - R_d - g_c g_d^\top)Q \tag{17}$$

Replacing (17), (13), (14) and (12) into (16) we show that

$$\dot{\tilde{x}} = A_o\tilde{x} \tag{18}$$

and the dynamics of the error is asymptotically stable.

For the result (ii), we check from the LMI (10) that

$$\begin{aligned} 2\alpha I_{n_c} \leq g g_c^\top + g_c g^\top - A_o Q_c^{-1} - Q_c^{-1} A_o^\top \leq 2\beta I_{n_c} \\ \tfrac{1}{\delta} I_{n_c} \leq Q_c^{-1} \leq \tfrac{1}{\gamma} I_{n_c} \end{aligned}$$

Replacing S_c and $S_c^\top$ from (12) and inverting the second inequality we obtain

$$\begin{aligned} 2\alpha I_{n_c} \leq -(S_c + S_c^\top) \leq 2\beta I_{n_c} \\ \gamma I_{n_c} \leq Q_c \leq \delta I_{n_c} \end{aligned}$$

then, replacing R_c by (14) we can conclude the result (ii), where $J_c = -J_c^\top$ from (13), $R_c = R_c^\top \geq 0$ because $\alpha \geq 0$ and $Q_c = Q_c^\top > 0$ because $\gamma > 0$.

Finally, the result (iii) is proved using Theorem 10 from (Ramírez et al., 2017). □

Remark 1. One condition of the theorem is that the matrix $A_o = (J_d - R_d - g_c g_d^\top)Q_d$ is Hurwitz. g_c is nothing else than the Luenberger observer gain. Then, A_o can be written as $A_o = A - LC$, where $A = (J_d - R_d)Q_d$, $L = g_c$ and $C = g_d^\top Q_d$. Finally, it is possible to check the observability of the system and design the observer with conventional methods as LQR, pole placement or the LMI method proposed in (Prajna et al., 2002).

Remark 2. One special case of Theorem 1 is the LQG controller design method proposed in (Wu et al., 2018), in which Q_c is chosen equal to Q_d.

4. NUMERICAL EXAMPLES

In the following, undamped wave equation is considered through the practical application case of boundary control of an elastic string. The observer-based controller is derived using the control by interconnection of Theorem 1. First, we recall the port-Hamiltonian formulation of the elastic string and its discretization. Then, the control design procedure is shown. At last, the simulation results of the closed-loop system are shown.

4.1 Wave Equation Model

Consider the elastic string model in the port-Hamiltonian form (6) *i.e.*

$$P_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad P_0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{19}$$

$$x(t,z) = \begin{pmatrix} p \\ q \end{pmatrix}(t,z), \quad \mathcal{L} = \begin{pmatrix} \frac{1}{\rho(z)} & 0 \\ 0 & T(z) \end{pmatrix} \tag{20}$$

with inputs and outputs

$$u(t) = \begin{pmatrix} \frac{1}{\rho(z)} p(t,a) \\ T(z) q(t,b) \end{pmatrix}, \quad y(t) = \begin{pmatrix} -T(z) q(t,a) \\ \frac{1}{\rho(z)} p(t,b) \end{pmatrix} \tag{21}$$

where $T(z)$ and $\rho(z)$ are Youngs' modulus and the mass density respectively, $p(t,z)$ and $q(t,z)$ are the momentum and strain respectively defined as

$$p(t,z) = \rho(z) \frac{\partial w}{\partial t}(t,z), \tag{22}$$

$$q(t,z) = \frac{\partial w}{\partial z}(t,z) \tag{23}$$

with $w(t,z)$ as the displacement of the string. For more details about the formulation of this Boundary Control System, the reader is refered to (Villegas, 2007). In order to design the finite dimensional controller, the staggered grids finite difference discretization method (Trenchant et al., 2017) is used to derive the finite dimensional approximation of the above BCS.

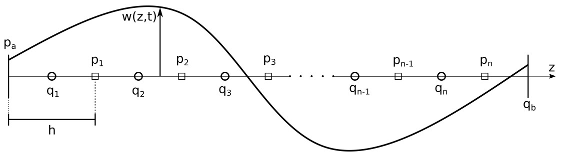

Fig. 2. Spatial discretization.

The spatial discretization scheme in Fig. 2 results in the finite-dimensional system

$$x_d = \begin{pmatrix} p_d \\ q_d \end{pmatrix}, \quad p_d = \begin{pmatrix} p_1 \\ \vdots \\ p_{n_d} \end{pmatrix}, \quad q_d = \begin{pmatrix} q_1 \\ \vdots \\ q_{n_d} \end{pmatrix} \tag{24}$$

where $x_d = x_d(t)$, $p_d = p_d(t)$, $q_d = q_d(t)$, $p_i = p_i(t)$ and $q_i = q_i(t)$ with $i = 1, \ldots, n_d$. The inputs of the systems are

$$u(t) = \begin{pmatrix} \frac{1}{\rho} p_a(t) \\ T q_b(t) \end{pmatrix} = \begin{pmatrix} \frac{1}{\rho} p(t,a) \\ T q(t,b) \end{pmatrix} \tag{25}$$

This input of the reduced order system is the same as the one defined for the infinite dimensional system (21). Unfortunately, it is not possible to get the same output with this kind of discretization scheme. In this case, the output of the finite dimensional system is chosen as close as possible to the one defined in (21) *i.e.*

$$y(t) = \begin{pmatrix} -T q_1(t) \\ \frac{1}{\rho} p_{n_d}(t) \end{pmatrix} \approx \begin{pmatrix} -T q(t,a) \\ \frac{1}{\rho} p(t,b) \end{pmatrix} \tag{26}$$

Finally, the matrices of the discretized system (7) are

$$J = \begin{pmatrix} 0 & D \\ -D^\top & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \tag{27}$$

$$Q = \begin{pmatrix} \frac{h}{\rho} I_{n_d} & 0 \\ 0 & ThI_{n_d} \end{pmatrix}, \quad g = \begin{pmatrix} 0 & g_b \\ g_a & 0 \end{pmatrix}, \tag{28}$$

with

$$D = \frac{1}{h^2} \begin{pmatrix} -1 & 1 & \ldots & 0 & 0 \\ 0 & -1 & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ldots & -1 & 1 \\ 0 & 0 & \ldots & 0 & -1 \end{pmatrix}, \tag{29}$$

$$g_a = \frac{1}{h} \begin{pmatrix} -1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}, \quad g_b = \frac{1}{h} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \tag{30}$$

where I_{n_d} is the identity matrix of dimension n_d, h is the distance between two consecutive variables as shown is Fig. 2, J, R, $Q \in \mathbb{R}^{2n_d \times 2n_d}$, $g \in \mathbb{R}^{2n_d \times 2}$, g_a, $g_b \in \mathbb{R}^{n_d}$ and $D \in \mathbb{R}^{n_d \times n_d}$.

4.2 Controller Design

In order to design the passive observer-based controller using Theorem 1, the first step is to design the matrix g_c in order to assign the observer performances. To this purpose, we use the Linear Quadratic Regulator (LQR) problem through the use of the Matlab function

$$g_c^\top = lqr(A_d^\top, C_d^\top, Q_o, R_o) \tag{31}$$

where $A_d = (J_d - R_d)Q_d$ and $C_d = g_d^\top Q_d$ are respectively, the state matrix and the output matrix of the plant, and $Q_o \in \mathbb{R}^{2n_d \times 2n_d}$ and $R_o \in \mathbb{R}^{2\times 2}$ are the state and input weighting functions used to design the observer. The numerical values used for the design are given in Table 1, where I_{2n_d} and I_2 are the identity matrices of size $2n_d$ and 2 respectively. The resulting matrix A_o defined by (17) is computed by using the resulting gain g_c derived from (31).

Then, we solve the LMI (10) by using the Matlab LMI toolbox. Then, we replace the solution of the LMI in (11), (12), (13) and (14) in order to obtain Qc, Sc, Jc and Rc respectively. In this example, the design parameters α, β and δ are chosen accordingly to the values of the Table 1 and we change the values of γ by γ_1, γ_2 and γ_3 as shown in the Table 1.

In order to compare closed-loop behavior, it was designed the controller for different eigenvalues of the matrix Qc. Because that, it was chosen different values for the parameter γ, with γ_1, γ_2 and γ_3 as Table 1 shows. Fig. 3 shows the eigenvalues of matrices R_c and Q_c obtained with the different values of γ. In this example, we fix α and β as it is shown in Table 1. Then, the eigenvalues of R_c remain between α and β as shown in the upper figure of Fig. 3. $\gamma_1 < \gamma_2 < \gamma_3$ implies that $Q_c(\gamma_1) < Q_c(\gamma_2) < Q_c(\gamma_3)$ as shown in the lower figure of Fig. 3.

The eigenvalues of the augmented closed-loop are given by the matrix

$$A_{cl} = \begin{pmatrix} (J-R)Q & -g g_c^\top Q_c \\ g_c g^\top Q & (J_c - R_c) Q_c \end{pmatrix} \tag{32}$$

which describe the dynamic of the vector $\left(x_d^\top(t) \ \hat{x}^\top(t)\right)^\top$.

Table 1. Parameters to tune and simulation

L	1	Length of the string
T	1	Youngs' modulus
ρ	1	mass density
Q_o	$40I_{2n_d}$	Observer Design
R_o	I_2	Observer Design
α	0	R_c parameter design
β	400	R_c parameter design
γ_1	0.0001	Q_c parameter design
γ_2	0.0030	Q_c parameter design
γ_3	0.0080	Q_c parameter design
δ	0.2	Q_c parameter design
n_d	10	Discretization for controller design
n_d	100	Discretization for the Simulation
t_0	0	Initial time $[s]$
t_f	2.5	Final time $[s]$
T_s	0.02	Time step $[s]$
	$ode15s$	Matlab function for time discretization
$r(t)$	0	New input

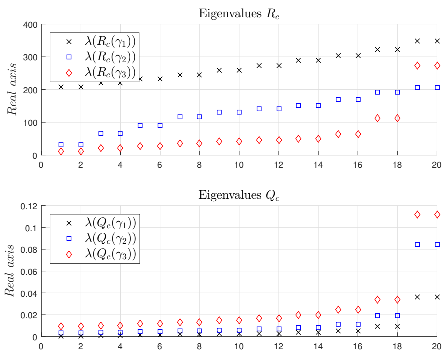

Fig. 3. Eigenvalues of R_c and Q_c for different tuning parameters

The eigenvalues of the matrix A_{cl} for different tuning parameters γ are shown in Fig. 4. In fact, the matrix A_{cl} contains the eigenvalues of the state feedback and the observer given by $A - BK = A - gg_c^\top Q_c$ and $A - CL = A - g_c g^\top Q$ respectively, with $A = (J - R)Q$, $B = g$ and $C = g^\top Q$. Remember that, in this work we focused in the controller design and we suppose that the matrix g_c is already design. And note that, the state feedback matrix $K = g_c^\top Q_c$ also depends on the observer matrix g_c. So, we are combining the design of the observer with the state feedback, instead of designing them separately as in the traditional way.

For the three different tuning parameters of the controller, a set of poles of the closed-loop system are the same (the ones superposed in Fig. 4). These poles correspond to those given by the observer. The rest of poles are the ones related to the state feedback and in this case, one can observe when we increase the eigenvalues of Q_c by tuning the design parameter γ (Fig. 3), the eigenvalues of the close loop system go to the left side of the complex plan as shown in Fig. 4. Hence, it is possible to conclude that, when we increase the eigenvalues of Q_c, then the dynamic of the closed-loop system is faster.

4.3 Closed-loop simulation

In the following, it is shown the dynamical simulation of the closed-loop system tuned with the parameters α, β, γ and δ shown in Table 1. In this case, we choose $\gamma = \gamma_2$.

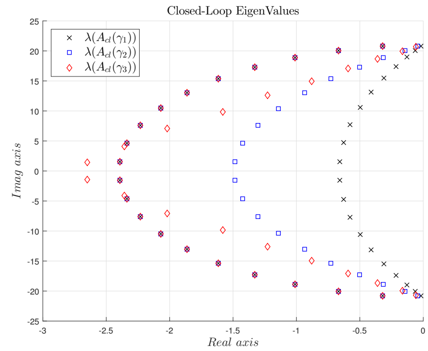

Fig. 4. Closed-Loop Eigenvalues for different tuning parameters

The initial conditions for the momentum it was chosen as zero, while for the strain, a sinusoidal initial condition was chosen. On the other hand, the controller is initialized at 0, *i.e.* $\hat{x}(0) = 0$.

Although the controller was designed for a specific discretization with $n_d = 10$, in the simulation we increase the order of the discretization with the values of $n_d = 100$ in order to make it close to the infinite-dimensional system.

In Fig. 5, we show the convergence of the observer estimations of the momentum and the strain to the real ones at $z \approx 0.81m$. One can observe that observer estimations converge to the state variables in $1.5s$ despite the initial condition of the strain and its estimation are different.

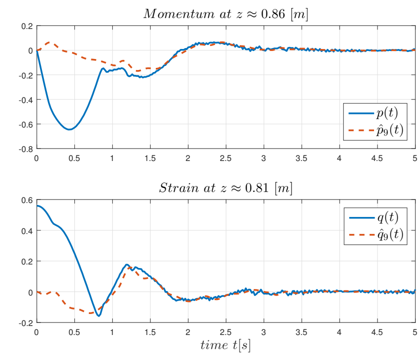

Fig. 5. Comparison of the momentum and the stain with their estimations at one point of the string $z \approx 0.8m$.

Fig. 6 shows the temporal and spatial response of the real strain of the closed-loop system from the non-zeros initial condition to the equilibrium position. Notice that, the higher order system ($n_d = 100$) is stabilized by using a reduced order observer-based controller with $n_c = 10$. Despite this, with this passive controller, the closed-loop system remains stable, because the property given by the interconnection of passive systems.

5. CONCLUSIONS AND PERSPECTIVES

In this paper, we propose a passive observer-based state feedback control design method for one class of Boundary Control Systems (BCS) under the port-Hamiltonian framework. Starting from the point that the system can be discretized by a structure preserve method and also that the observer it was already designed with some criteria.

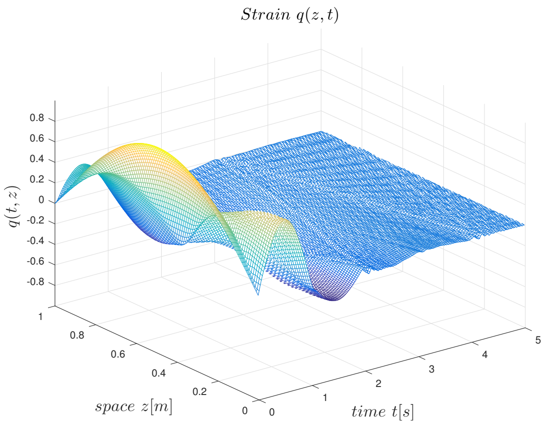

Fig. 6. The Strain of the high order closed-loop system with the reduced order controller.

Then, one can get Q_c by solving the LMI (10) and furthermore, the matrices J_c and R_c are obtained by substituting Q_c in the equations (13) and (14) respectively. The design parameters α, β, γ and δ can be used to set the closed-loop system performance. For instance, as shown in the simulation example, when we increase the parameter γ, the state feedback eigenvalues are increasing in the same time. The response time of the closed-loop system becomes faster.

The ongoing work is to analyze the influence of the control design parameters on the closed-loop performances. Secondly, the extension of the proposed passive observer-based control design method to the nonlinear case would be investigated.

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