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Complex Resolvent Band Structure of Phononic Crystals

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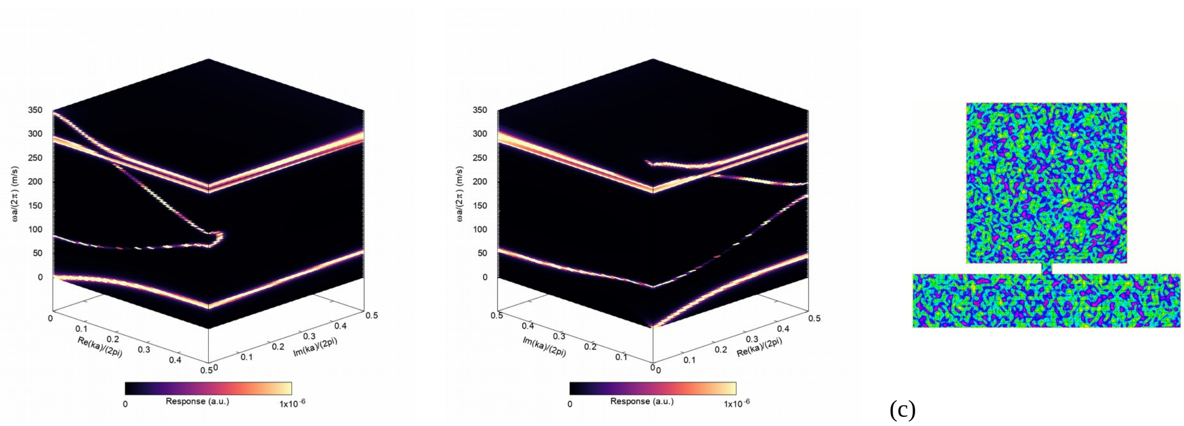
Abstract: We examine the extension of the definition of the resolvant band structure (RBS) to complex wavenumbers and complex frequencies. The RBS is obtained by applying a stochastic excitation to the unit-cell of a crystal and monitoring the response in dispersion space. It is found that the poles of the RBS appear in complex dispersion space around the complex band structure.

Obtaining the band structure (BS) of a particular phononic or sonic crystal, including all its peculiarities, has been a central problem in the field of phononics from the start[1-2]. Indeed, the band structure is a representation of the dispersion relation for Bloch waves, giving possible combinations of the wavevector k and of the angular frequency ω. For a closed and bounded domain, and in the absence of loss, ω is a real function of real k and the (real) band structure is easily defined and obtained by solving an eigenvalue problem. When loss is added, complex-valued $\omega(k)$[3] or $k(\omega)$[4] band structures (CBS) can be defined by extending the domain of definition of the wavevector and/or of the frequency to the complex plane. The question of which quantity should be made complex - or even if both should be made complex at the same time - remains largely open[5].

Recently, the concept of the resolvent band structure (RBS) was introduced[6]. With the RBS, one abandons the idea of obtaining a functional relationship in the form of bands and instead proceeds to map the resolvent set over the full dual (k,ω) space. The resolvent set is by definition the complement of the spectrum of eigenvalues: it is composed of the whole complex plane with the isolated eigenvalues composing the spectrum removed. In practice, the RBS can be obtained easily by considering a spatially random source distributed in the phononic crystal. Indeed, the random source will excite all modes of vibration and it then suffices to map the response of the crystal as a function of w and k.

The RBS was previously obtained for real k and ω and it was shown that it could adequately take into account material loss and radiation at infinity in open structures such as waveguides[6]. Nothing, however, prevents against letting k and w venture freely in the complex plane. This is in sharp contrast with the classical BS and the CBS. In this paper, this freedom is exploited to verify that the RBS gives dispersion information fully consistent with the complex $k(\omega)$ and $\omega(k)$ band structures, in a case where they all can be directly compared.

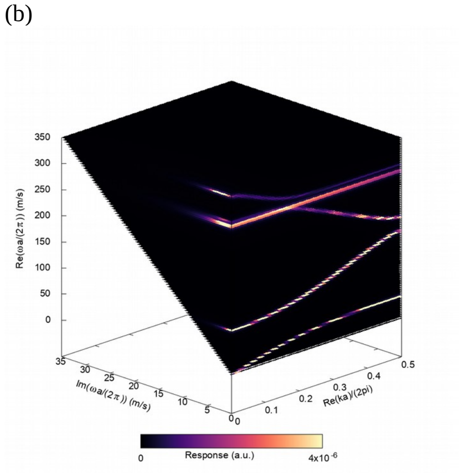


(a) (b) (c)

Figure 1: (a-b) Complex resolvant band structure shown for complex wavenumber and real frequency in the case of a lossless sonic crystal of rectangular resonators grafted along an air tube. The model is 2D. *a* is the lattice constant. (c) Stochastic excitation of the sonic crystal.

We consider a 2D sonic crystal composed of a periodic repetition of square resonators grafted along an air tube (as in chapter 2 of Ref. [2]). The unit-cell of the sonic crystal is closed and bounded: it thus admits well-defined bands. It also supports both Bragg and locally-resonant band gaps. Frequency-dependent material loss is added by making the bulk modulus of air complex. Figure 1 shows the RBS in case the wavenumber is allowed to live in the complex plane while the frequency is kept real, first in the absence of loss. For display reasons, a 3D representation of the RBS is shown on a cube. It is observed that the stochastic response concentrates exactly along the complex $k(\omega)$ bands. Next, we consider real k and let ω *wander* in the complex plane; such a choice is particularly desirable in case material loss is considered, since one is interested in the attenuation measured by the imaginary part of the frequency. It is found again in figure 2 that the stochastic response concentrates exactly along the complex $\omega(k)$ bands also presented in the figure.

(b)



(a)

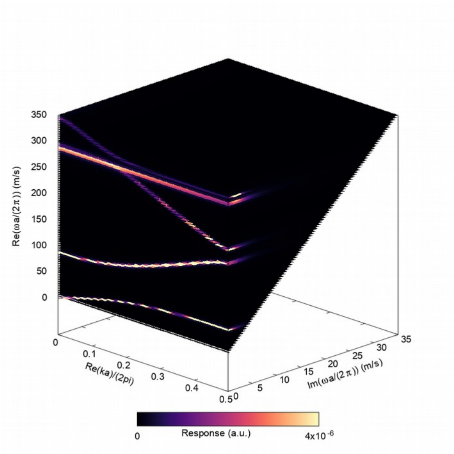


Figure 2: (a-b) Complex resolvant band structure shown for real wavenumber and complex frequency in the case of a lossy sonic crystal of rectangular resonators grafted along an air tube.

As a conclusion, it is found on the example of simple sonic crystal that the complex resolvant band structure identifies with both the $k(\omega)$ and $\omega(k)$ complex band structures. The latter two CBS can be obtained exactly in the case considered, which is not necessarily true for all phononic and sonic crystals, especially if their unit cell is infinite and thus radiation at infinity is involved. The complex RBS is a useful tool to obtain the full complex dispersion relation of Bloch waves and it applies to a much larger range of periodic structures than band structure methods based on eigenvalue solvers.

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