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Reduced order optimal control of infinite dimensional port Hamiltonian systems

Yongxin Wu, Boussad Hamroun, Yann Le Gorrec, Bernhard Maschke

Abstract— **This paper deals with the reduced order controller design for infinite dimensional port Hamiltonian systems (IDPHS). Firstly, a structure preserving and passive LQG control design equivalent to *C*ontrol by Interconnection is proposed. Based on this LQG controller, a structure preserving reduction method is used to approximate both the closed loop IDPHS and the LQG controller. This closed loop reduction guarantees that the reduced order controller will ensure acceptable closed loop performances on the infinite dimensional system. The proposed methods is applied to the control of a vibro-acoustic system.**

I. INTRODUCTION

In the last decade, the modeling and control of infinite dimensional systems, *i.e.*, systems described by partial differential equations (PDEs), have drawn an increasing attention from both engineering and mathematician communities. Among all system oriented approaches, the port Hamiltonian approach [1], [2], [3] has shown to be of particular interest for analysis and control of distributed parameter systems as it allows to link in a physically meaningful way the boundary variables to the energy of the system through a specific geometrical structure. This additional information associated to the passivity properties of the system is very useful when control design is concerned, as the closed loop energy function can be chosen as target Lyapunov function and passivity properties used for stabilization.

The most common approach for control design is to first approximate the infinite dimensional system and then design a finite dimensional controller. Passivity and structure preserving approximation of infinite dimensional port Hamiltonian systems has been derived on the basis of spatial discretization through mixed finite elements method in [4], [5], pseudo-spectral approximation in [6] and Petrov-Galerkin approximation in [7] for *open-loop* systems. However, for a large class of infinite dimensional systems described by hyperbolic PDEs, this approach does not allow to select appropriately the suitable reduced order model for control design as all states have the same importance from input-output behaviour point of view. In this case all the poles are on the imaginary axis and control design using the reduced order system usually leads to spillover effects. Thus, this *open-loop* approximation/reduction then control design strategy is not satisfactory. In order to avoid the spillover effect, we propose to use the Linear Quadratic Gaussian (LQG) balanced method which takes the *closed-loop* behaviour into account in the reduction procedure. The LQG balanced method has been proposed in [8] for finite dimensional systems and has been generalized to infinite dimensional systems in [9]. The LQG balanced truncation has already been used to reduce infinite dimensional systems in [10] and [11].

However, the LQG method cannot preserve the passivity and the structure of the PHS in the closed loop system. Hence, we first propose to use the LQG control design method with a specific choice of the weighting functions such that the closed loop system is passive. Furthermore, the Petrov-Galerkin method is used to achieve a **structure/passivity preserving model and controller reduction method** for the PHS.

The paper is organized as follows. In Section II, we present the considered class of infinite dimensional port Hamiltonian systems and recall their properties. A structure preserving LQG control design method is proposed in Section III. Section IV introduces the model and controller reduction scheme based on the proposed LQG control design method. The proposed method is illustrated in Section V on a vibro-acoustic system. At last, some concluding remarks and comments on future works are given in Section VI.

II. A CLASS OF INFINITE DIMENSIONAL PORT HAMILTONIAN SYSTEMS

We consider the class of linear infinite-dimensional dissipative systems defined as follows:

Definition 1: [7] A linear infinite-dimensional system of the form:

$$\Sigma_{\text{PHS}} \left\{ \begin{array}{rcl} \dot{x}(t) & = & \mathcal{M}\mathcal{Q}x(t) + \mathcal{B}u(t) \\ y(t) & = & \mathcal{B}^*\mathcal{Q}x(t) \end{array} \right. \quad (1)$$

is called a linear infinite-dimensional dissipative port-Hamiltonian system (IDPHS) if it satisfies

- $x(t) \in X$, X is a Hilbert space with inner product $\langle \cdot, \cdot \rangle_X$ and norm $\| \cdot \|_X^2$;
- $\mathcal{M} : D(\mathcal{M}) \subset X$, the domain of the operator $\mathcal{M}$ is a densely definite maximal dissipative (m-dissipative) linear operator;
- $\mathcal{Q} : X \mapsto X$ is a bounded linear operator that is self-adjoint ($\mathcal{Q}^* = \mathcal{Q}$) and coercive ($\langle \mathcal{Q}h, h \rangle_X \geq \alpha ||h||_X^2$ $\forall h \in X$ with $\alpha > 0$);
- The input operator $\mathcal{B} : \mathbb{C}^p \mapsto X$ is bounded and $\{0\} \neq \text{Im}\,(\mathcal{B}) \subset X$.
- The inputs u and outputs y have the same dimension.

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The operator $\mathcal{MQ}$ is dissipative with respect to the inner product $\langle g,h\rangle_{\mathcal{Q}} = \langle g,\mathcal{Q}h\rangle_X,\ g,h \in X$. In addition, $\text{Ran}(\lambda I - \mathcal{MQ}) = X$ is satisfied for some $\lambda \in \mathbb{C}_0$, because $\mathcal{M}$ is m-dissipative and $\mathcal{Q}$ is bijective. Hence $\mathcal{MQ}$ is m-dissipative and therefore generates a contraction C_0-semigroup [12, Thm. 1.2.3].

The *total energy* of the system is defined through the *Hamiltonian* (Energy storage equation) as

$$H(x(t)) = \frac{1}{2}\langle \mathcal{Q}x(t), x(t)\rangle_X . \quad (2)$$

Assumption 2: Throughout this paper, we suppose the domain of operator $\mathcal{M}$ is equal to the domain of $\mathcal{M}^*$, i.e.,

$$D(\mathcal{M}^*) = D(\mathcal{M}) \quad (3)$$

By using Assumption 2, system (1) can be written as:

$$\begin{cases} \dot{x}(t) &= (\mathcal{J}-\mathcal{R})\mathcal{Q}x(t) + \mathcal{B}u(t) \\ y(t) &= \mathcal{B}^*\mathcal{Q}x(t) \end{cases} \quad (4)$$

where

$$\mathcal{J} = \frac{1}{2}(\mathcal{M}-\mathcal{M}^*) \quad \text{and} \quad \mathcal{R} = -\frac{1}{2}(\mathcal{M}+\mathcal{M}^*) \quad (5)$$

with $D(\mathcal{J}) = D(\mathcal{R}) = D(\mathcal{M}) \subset X$. Hence the system (1) can be regarded as an infinite dimensional port Hamiltonian system (IDPHS) defined in [2]. Here the operator $\mathcal{J} = -\mathcal{J}^* \in \mathcal{L}(X)$ is a skew-adjoint differential operator which presents the conservative energy exchanges in the domain, and the operator $0 \le \mathcal{R} = \mathcal{R}^* \in \mathcal{L}(X)$ is a self-adjoint and semi definite positive differential operator which represents the energy dissipation in the domain.

III. PASSIVE LQG CONTROL DESIGN OF IDPHS

Passive LQG control design has been applied to finite dimensional positive real systems in [13]. In [14] a similar approach equivalent to the *control by interconnection* [15], [16] has been proposed. In this section, we extend this approach to infinite dimensional port Hamiltonian systems.

A. LQG control of infinite dimensional port Hamiltonian systems

In order to apply the LQG control design to infinite dimensional port Hamiltonian systems we make the following assumptions.

Assumption 3: The IDPHS (1) with m-dissipative operator $\mathcal{MQ}$ is exponentially stabilizable, i.e., there exists an operator $K \in \mathcal{L}(X,\mathbb{C}^p)$ such that $\mathcal{MQ} - \mathcal{B}K$ generates an exponentially stable semigroup and is exponentially detectable, i.e., there exists an operator $F \in \mathcal{L}(\mathbb{C}^p, X)$ such that the operator $\mathcal{MQ} - F\mathcal{B}^*\mathcal{Q}$ generates an exponentially stable semigroup.

The LQG control problem of IDPHS (1) is then defined as follows:

Problem 4 (LQG control problem [17]): Let $\tilde{\mathbf{Q}}, \mathbf{Q}_v \in \mathcal{L}(X)$ be self-adjoint positive definite operators, $\tilde{\mathbf{R}}, \mathbf{R}_w \in \mathcal{L}(\mathbb{C}^p)$ also be self-adjoint strictly positive definite operators and $x \in D(\mathcal{M})$. Then the state feedback $K = \tilde{\mathbf{R}}^{-1}\mathcal{B}^*P_c$ with P_c the unique positive-definite solution to the operator Riccati equation:

$$\left(\mathcal{QM}^*P_c + P_c\mathcal{MQ} - P_c\mathcal{B}\tilde{\mathbf{R}}^{-1}\mathcal{B}^*P_c + \tilde{\mathbf{Q}}\right)x = 0 \quad (6)$$

is such that $\mathcal{MQ} - \mathcal{B}\tilde{\mathbf{R}}^{-1}\mathcal{B}^*P_c$ generates an exponentially stable semigroup. The filter gain is $F = P_f\mathcal{B}\mathbf{R}_w^{-1}$ where P_f is the unique positive definite solution to

$$\left(\mathcal{MQ}P_f + P_f\mathcal{QM}^* - P_f\mathcal{QB}\mathbf{R}_w^{-1}\mathcal{B}^*\mathcal{Q}P_f + \mathbf{Q}_v\right)x = 0 \quad (7)$$

and is such that $\mathcal{MQ} - P_f\mathcal{QB}\mathbf{R}_w^{-1}\mathcal{B}^*\mathcal{Q}$ generates an exponentially stable semigroup.

Thus, the control design problem consists to solve the Riccati equation (6) and (7) in order to minimize the control cost function:

$$J_{co} = \int_0^\infty \left\langle x, \tilde{\mathbf{Q}}x\right\rangle_X + \left\langle u, \tilde{\mathbf{R}}u\right\rangle_{\mathbb{C}^p} dt \quad (8)$$

and the estimation error:

$$e(t) = x(t) - x_c(t). \quad (9)$$

As consequence the dynamic controller (Fig. 1) associated with the LQG control problem (4) can be written as:

$$\begin{cases} \dot{x}_c &= \left(\mathcal{MQ} - \mathcal{B}\tilde{\mathbf{R}}^{-1}\mathcal{B}^*P_c - P_f\mathcal{QB}\mathbf{R}_w^{-1}\mathcal{B}^*\mathcal{Q}\right)x_c \\ & \quad +P_f\mathcal{QB}\mathbf{R}_w^{-1}u_c \\ y_c &= \tilde{\mathbf{R}}^{-1}\mathcal{B}^*P_cx_c \end{cases} \quad (10)$$

where x_c represents the state of the observer.

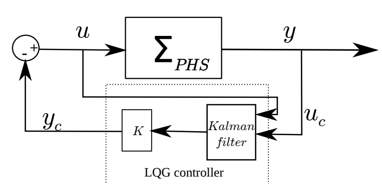

Fig. 1. LQG control design for port Hamiltonian system

The closed loop system with the above LQG controller is stable but not passive in general because the Hamiltonian structure is lost in the closed loop formulation.

B. LQG formulation of control by interconnection

In order to design a passive LQG controller which can be seen as control by interconnection [15], [18], [16], [19] one has to transform the control scheme of Fig. (1) into the one of Fig. 2 where the controller has a IDPHS structure. In this

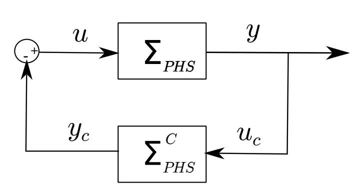

Fig. 2. Control by interconnection

perspective, (10) can be rewritten as:

$$\begin{array}{rcl} \dot{x}_c & = & \left(\mathcal{M} - \mathcal{B}\tilde{\mathbf{R}}^{-1}\mathcal{B}^* P_c \mathcal{Q}^{-1} - P_f \mathcal{Q}\mathcal{B}\mathbf{R}_w^{-1}\mathcal{B}^*\right)\mathcal{Q}x_c \\ & & + P_f \mathcal{Q}\mathcal{B}\mathbf{R}_w^{-1}u_c \\ & = & (\mathcal{J} - \mathcal{R}_c)\,\mathcal{Q}x_c + P_f\mathcal{Q}\mathcal{B}\mathbf{R}_w^{-1}u_c \\ y_c & = & \tilde{\mathbf{R}}^{-1}\mathcal{B}^* P_c \mathcal{Q}^{-1}\mathcal{Q}x_c \end{array} \quad (11)$$

with

$$\mathcal{R}_c = \mathcal{R} + \mathcal{B}\tilde{\mathbf{R}}^{-1}\mathcal{B}^* P_c \mathcal{Q}^{-1} + P_f \mathcal{Q}\mathcal{B}\mathbf{R}_w^{-1}\mathcal{B}^* \quad (12)$$

In this expression the state operator is decomposed into the product $(\mathcal{J} - \mathcal{R}_c)\mathcal{Q}$ with the energy operator $\mathcal{Q}$ defined in (1). The operator $\mathcal{R}_c = \mathcal{R} + \mathcal{B}\tilde{\mathbf{R}}^{-1}\mathcal{B}^* P_c \mathcal{Q}^{-1} + P_f \mathcal{Q}\mathcal{B}\mathbf{R}_w^{-1}\mathcal{B}^*$ is in general not self-adjoint nor positive. Next we derive the conditions on the LQG control Problem 4 such that the controller (11) has a port Hamiltonian realisation.

Theorem 5 (Hamiltonian LQG method): The LQG controller designed using Theorem 4 considering

$$\tilde{\mathbf{R}} = \mathbf{R}_w \quad (13)$$

and $\tilde{\mathbf{Q}}$ and $\mathbf{Q}_v$ such that:

$$\mathbf{Q}_v z = \mathcal{Q}^{-1}\left(2\mathcal{Q}\mathcal{J}^* P_c + 2P_c\mathcal{J}\mathcal{Q} + \tilde{\mathbf{Q}}\right)\mathcal{Q}^{-1}z, \quad (14)$$

with $z \in X$, is passive and has a port Hamiltonian realization. Furthermore the operator equations (6) and (7) admit a unique solution, P_c and P_f respectively. These two solutions are related by:

$$\mathcal{Q}^{-1}P_c = P_f\mathcal{Q} \quad (15)$$

Proof: To prove this Theorem, we have to show first the input and output of the LQG controller are power conjugate with the condition (14). Secondly, the operator $\mathcal{R}_c$ can been shown self-adjoint with the conditions (15) and (13). The positive definiteness of this operator can be proven by using the exponential stability of the closed loop system (Assumption 3). See the details in [20]. ■

The closed loop system by using *Hamiltonian LQG controller* can be regarded as control by interconnection of two port Hamiltonian systems. Hence the structure and passivity are preserved in closed loop.

In the next section, we discuss the passivity and structure preserving reduction method of port Hamiltonian systems through the *Hamiltonian LQG method*.

IV. REDUCED LQG CONTROL DESIGN

In this section, we use a balanced realisation of the IDPHS (1) with respect to the LQG Gramians associated with the control Problem 5 and the relations (13) and (14):

$$P_f P_c = P_f \mathcal{Q} P_f \mathcal{Q}$$

These Grammians being different from the identity, we can then reduce the system by state transformation and truncation.

A. Preliminary results

We first introduce the *Hamiltonian LQG Hankel operator* associated with the IDPHS (1).

Definition 6: Consider two operators $S \in \mathcal{L}(X_s; X)$ and $L \in \mathcal{L}(X_L; X)$ with X_S and X_L two Hilbert spaces such that the Hamiltonian LQG Gramians satisfy

$$P_c = SS^* \text{ and } P_f = LL^* \quad (16)$$

Then

$$\mathcal{H}_{LQG} = S^* L \in \mathcal{L}(X) \quad (17)$$

is called *Hamiltonian LQG Hankel operator* of the IDPHS (1).

We also define the balanced realisation of system (1) with respect to the Hamiltonian LQG Gramians P_f and P_c.

Definition 7: The IDPHS is called *Hamiltonian LQG balanced* if $X = \ell_2$ and there exists positive and non-increasing sequence $(\sigma_n)_{n\in\mathbb{N}}$ such that the Hamiltonian LQG Gramians P_f and P_c are both equal to the diagonal operator:

$$\Sigma = \text{diag}(\sigma_n)_{n\in\mathbb{N}} \in \mathcal{L}(\ell_2). \quad (18)$$

In other words:

$$P_f = P_c = \Sigma. \quad (19)$$

B. Hamiltonian LQG balanced realization of IDPHS

The Hamiltonian LQG Hankel operator $\mathcal{H}_{LQG} = S^*L \in \mathcal{L}(X)$ associated with the control Problem 5 and the relations (13) and (14) admits a singular value decomposition:

$$S^*L = V\Sigma U^* \quad (20)$$

where

$$\Sigma = \text{diag}(\sigma_n)_{n\in\mathbb{N}} \in \mathcal{L}(\ell_2)$$

with positive sequence of Hamiltonian LQG Hankel singular values (σ_n). $V, U \in \mathcal{L}(\ell_2; X)$ are isometrics onto their ranges, i.e.,

$$V^*V = I, \quad U^*U = I. \quad (21)$$

Theorem 8: The balanced realisation of the IDPHS (1) is given by

$$\left\{\begin{array}{rcl} \dot{x}_b(t) & = & M_b Q_b x_b(t) + B_b u(t) \\ y(t) & = & B_b^* Q_b x_b(t) \end{array}\right. \quad (22)$$

with

$$M_b = T\mathcal{M}T^* \quad Q_b = T^{+*}\mathcal{Q}T^{+} \quad B_b = T\mathcal{B} \quad (23)$$

and

$$\begin{array}{l} T := \Sigma^{-1/2}V^*S^* \subset X \mapsto \ell_2; \\ T^{+} := LU\Sigma^{-1/2} \subset \ell_2 \mapsto X. \end{array} \quad (24)$$

The state space of the balanced IDPHS is $x_b \in \ell_2$.

This balanced realization is defined on an ℓ_2 space, and the state variables are separated and arranged in decreasing order according to their importance in the closed-loop system defined from the Hamiltonian LQG singular values. In other words, the state variables associated with large singular values are more important for the Hamiltonian LQG control design than the other ones. Hence from the closed loop point of view, this balanced realization gives us the good choice of state space to reduce the IDPHS. This reduction method is derived in the next sub-section.

C. Approximation of IDPHS

In order to preserve the passivity and the Hamiltonian structure of the system after reduction, a direct truncation cannot be used and we propose to adapt the Petrov-Galerkin projection method [7].

1) Petrov-Galerkin projection method: In Petrov-Galerkin projection method the state variables are decomposed in $x(t) = x_n(t) + x_r(t)$ with $x(t) \in X$, $x_n(t) \in V$ and $x_r(t) \in W^\perp$, where $V = \text{span}\{v_1, \cdots, v_n\}$ and $W = \text{span}\{w_1, \cdots, w_n\}$ with v_i in the state operator domain ($v_i \in D(\mathcal{M}\mathcal{Q})$ in system (1)) and $w_i \in X$ the n-dimensional subspaces of X, and $W^\perp$ the orthogonal complement of W. This decomposition exists and is unique if $V \cap W^\perp = \{0\}$. The linear operators $\mathcal{V} : \mathbb{C}^n \mapsto X$ and $\mathcal{W} : X \mapsto \mathbb{C}^n$ defined by:

$$\mathcal{V}\alpha = \sum_{i=1}^{n} v_i \alpha_i \quad \mathcal{W}h = \begin{bmatrix} \langle h, w_1 \rangle_X \\ \vdots \\ \langle h, w_n \rangle_X \end{bmatrix} \tag{25}$$

for all $\alpha \in \mathbb{C}^n$, $h \in X$ are such that $V \cap W^\perp = \{0\}$. This property can be easily verified by using $\det(\mathcal{W}\mathcal{V}) \neq 0$ where $\mathcal{W}\mathcal{V} \in \mathbb{C}^{n \times n}$. Thus one can use the internal direct sum decomposition $X = V \oplus W^\perp$ if the choices of W and V are such that $\det(\mathcal{W}\mathcal{V}) \neq 0$. In order to determine a finite-dimensional model that describes the dynamics of x_n it is advantageous to introduce the projection $\mathcal{P} : X \mapsto V$ of X onto V along $W^\perp$, yielding to the relation $x_n(t) = \mathcal{P}X(t)$. This projection can be expressed as $\mathcal{P} = \mathcal{V}(\mathcal{W}\mathcal{V})^{-1}\mathcal{W}$ and satisfies $\mathcal{P} = \mathcal{P}^2$. Its range and null space satisfy:

$$\text{Ran}\mathcal{P} = V; \quad \text{Ker}\mathcal{P} = W^\perp$$

2) Passivity and structure preserving approach: To preserve the passivity of the port Hamiltonian system (1) by using the Petrov-Galerkin projection method, a special choice of operator $\mathcal{V}$ and $\mathcal{W}$ is given in [7]. In this method, the authors did not give the choice of vectors v_i.

Inspired from this method we propose a choice of vectors v_i which define the projection operator $\mathcal{V}$ and $\mathcal{W}$ to preserve the passivity and Hamiltonian structure through the *balanced reduction of* system (22):

Theorem 9: Define $\mathcal{V} : \mathbb{C}^n \mapsto \ell_2$ by

$$\mathcal{V}z = \sum_{i=1}^{n} v_i z_i \quad \forall z_i \in \mathbb{C}^n, \;\; i \in \mathbb{N} \tag{26}$$

with $v_i = (\delta_{i,1}, \delta_{i,2}, \cdots) \in \ell_2$ is the canonical unit vector. Consider the special choice $\mathcal{W} = \mathcal{V}^* Q_b$. Then a structure preserving approximation of the infinite-dimensional DPHS is a linear DPHS:

$$\left\{ \begin{array}{lcl} \dot{x}_n & = & M_n Q_n x_n + B_n u \\ y & = & B_n^* Q_n x_n \end{array} \right. \tag{27}$$

with

$$\begin{array}{c} M_n = \mathcal{V}^* Q_b M_b Q_b \mathcal{V} \quad Q_n = (\mathcal{V}^* Q_b \mathcal{V})^{-1} \\ B_n = \mathcal{V}^* Q_b B_b \end{array} \tag{28}$$

Proof: First we can show that

$$\mathcal{P} = \mathcal{V}(\mathcal{W}\mathcal{V})^{-1}\mathcal{W} = \mathcal{P}^2$$

is a projection. Next we choose $x_b \approx \mathcal{V}z_n$ and premultiplying (22) by the operator $\mathcal{W} = \mathcal{V}^* Q_b$. The finite-dimensional approximation becomes

$$\left\{ \begin{array}{lcl} \mathcal{V}^* Q_b \mathcal{V} \dot{z}_n & = & \mathcal{V}^* Q_b M_b Q_b \mathcal{V} z_n + \mathcal{V}^* Q_b B_b u(t) \\ y(t) & = & B_b^* Q_b \mathcal{V} z_n \end{array} \right. \tag{29}$$

We choose $x_n = \mathcal{V}^* Q_b \mathcal{V} z_n$. The matrix M_n can be separate in two parts, one part is skew symmetric and the other part is symmetric positive definite because

$$M_n + M_n^* \leq 0$$

and

$$J_n = \frac{1}{2}(M_n - M_n^*) \text{ and } R_n = -\frac{1}{2}(M_n + M_n^*) \tag{30}$$

■

Remark 10: In this projection method, the operator $\mathcal{V}$ is used to separate the state space X, and the special choice of $\mathcal{W} = \mathcal{V}^* Q_b$ ensure the finite dimensional approximation still has the port Hamiltonian structure and preserve the passivity. By using the finite dimensional PHS (27) and the LQG control Problem 4 associated with Theorem 5, one can then design a finite dimensional controller in order to stabilize the IDSHP (4).

V. APPLICATION TO THE CONTROL OF VIBRO-ACOUSTIC SYSTEM

In this section, we apply the proposed control design method to the 1-D vibro-acoustic system with active surface [21] depicted in Fig. 3. The simulation results are illustrated to show the effectiveness of the proposed method.

Fig. 3. Acoustic tube with active surface

The 1D port Hamiltonian representation of this model is given by:

$$\begin{array}{lcl} \dot{x}(t,\zeta) & = & \mathcal{M}\mathcal{Q}x(t,\zeta) + \mathcal{B}u_d(t) \\ y_d(t) & = & \mathcal{B}^* \mathcal{Q}x(t,\zeta) \end{array} \tag{31}$$

where $\zeta \in [0, 1]$ and:

$$\mathcal{M} = \begin{bmatrix} 0 & -\frac{\partial}{\partial \zeta} \\ -\frac{\partial}{\partial \zeta} & -f \end{bmatrix}, \mathcal{Q} = \begin{bmatrix} \frac{1}{\rho_0} & 0 \\ 0 & \frac{1}{\chi_s} \end{bmatrix} \tag{32}$$

The state variable is $x(t,\zeta) = [\theta(t,\zeta), \Gamma(t,\zeta)]^T$ with $\theta(t,\zeta)$ is the kinetic momentum, and $\Gamma(t,\zeta)$ is the volumetric expansion. The operator $\mathcal{B}$ is the distributed input operator and u_d and y_d are the distributed input and output which we discuss latter. The *total energy* is given by:

$$H = \frac{1}{2}\int_0^L \left(\frac{1}{\rho_0}\theta^2(t,\zeta) + \frac{1}{\chi_s}\Gamma^2(t,\zeta) \right) d\zeta = \frac{1}{2}\int_0^L \left(x^T \mathcal{Q} x \right) d\zeta \tag{33}$$

where ρ_0 is the air mass density, χ_s is the adiabatic compressibility coefficient, f is the air viscosity coefficient, $\mathcal{Q}$ is self-adjoint and coercive. The boundary port variables are defined as:

$$\begin{bmatrix} f_\partial \\ e_\partial \end{bmatrix} = \begin{bmatrix} \mathcal{Q}x(0,t) \\ \mathcal{Q}x(L,t) \end{bmatrix} = \begin{bmatrix} v(0,t) \\ v(L,t) \\ P(0,t) \\ P(L,t) \end{bmatrix} \tag{34}$$

where $v(t,\zeta) = \frac{1}{\rho_0}\theta(t,\zeta)$ is the velocity and $P(t,\zeta) = \frac{1}{\chi_s}\Gamma(t,\zeta)$ is the pressure. The boundary inputs are the pressures at $z = 0$ and at $z = L$. The outputs are the velocities at $z = 0$ and at $z = L$. These inputs and outputs can be derived from the boundary port variables (34) through [22]:

$$u_\partial = W_B \begin{bmatrix} f_\partial \\ e_\partial \end{bmatrix}, \qquad y_\partial = W_C \begin{bmatrix} f_\partial \\ e_\partial \end{bmatrix} \tag{35}$$

where the matrices W_B and W_C are defined by

$$W_B = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{36}$$

and

$$W_C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{37}$$

The state space is defined as $X = L_2((0,L);\mathbb{R}^2)$. The domain of the operator $\mathcal{A} = \mathcal{M}\mathcal{Q}$ is

$$D(\mathcal{A}) = \left\{ \mathcal{Q}x \in H^1((0,L);\mathbb{R}^2) \middle| \begin{bmatrix} \mathcal{Q}x(0) \\ \mathcal{Q}x(L) \end{bmatrix} \in \ker W_B \right\} \tag{38}$$

where $H^p((0,L);\mathbb{R}^n)$ defines the p order Sobolev space. This acoustic system is controlled through an active surface able to apply a distributed pressure over the interval $\zeta \in [0.7, 0.88]$ (see Fig. 4). This is equivalent to define a distributed input operator of the form:

$$\mathcal{B} = \begin{bmatrix} 0 \\ b(\zeta) \end{bmatrix} \text{ with } b(\zeta) = \begin{cases} 1 & \zeta \in [0.7, 0.88] \\ 0 & \text{else} \end{cases} \tag{39}$$

where $\mathcal{B} : \mathbb{C}^1 \mapsto X$. The power conjugated output y_d is the velocity on the same interval.

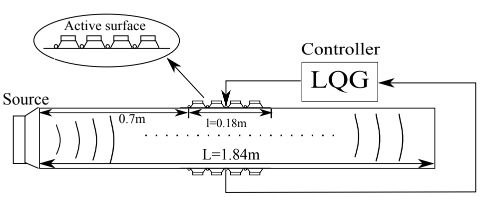

Fig. 4. Active surface and LQG control of acoustic tube

The physical parameters of the considered vibro-acoustic system are extracted from [21] and given in Table I.

In order to design the LQG controller of Theorem (5) and the final reduced order controller, we need to solve the operator Riccati equation. To this end, we use the structure preserving staggered grids finite difference method [21] to discretize the infinite dimensional system (31) in order to solve this equation. The spatial coordinate $[0, L]$ is divided into 50 points, hence the system has 100 state variables.

TABLE I

PHYSICAL PARAMETERS

Tube length L	$1.84\ m$
Air density ρ_0	$1.225\ kg/m^3$
Compressibility coefficient χ_s	$7.061e-6\ Pa^{-1}$
Active surface length l	$0.18\ m$
Air viscosity f	$1.8e-5\ Pa \cdot s$

First, we apply a sinusoidal pressure on the left side of the system by loudspeaker and measure the right side velocity as the output. In the Fig. 5 we can see the open loop response simulation result of the vibro-acoustic system.

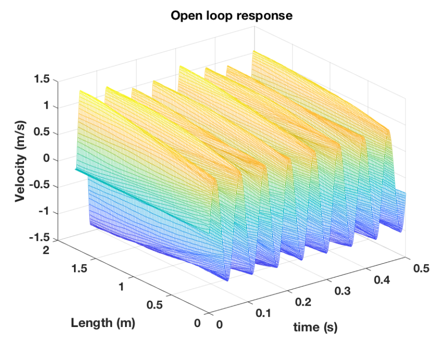

Fig. 5. Open loop response

We use Theorem (5) to design the controller and reduce the system choosing $\tilde{\mathbf{Q}} = \mathcal{Q}\mathcal{B}\mathcal{B}^*\mathcal{Q}$. We first simulate the system closed loop response with the full order controller (see Fig. 6) when the system is subject to the same input pressure. We can see in this figure that the wave that is propagating along the tube is attenuated when it reaches the active surface at $\zeta = 0.7$. We should keep in mind the full order controller has

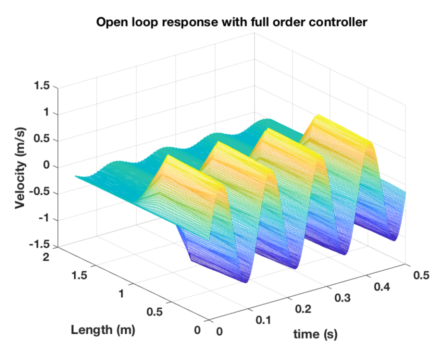

Fig. 6. Full order LQG controlled response

the same order as the system itself, *i.e.* $x_c \in \mathbb{R}^{100}$ in this case. In order to reduce the LQG controller, we have to check the different importance of the controller's state variable by the LQG singular values. The Fig. (7) shows the LQG singular values of the vibro-acoustic system. One can observe that the first two singular values are much larger than the other ones (almost three times). That means that the first two states of the balanced system play the most important role in the closed loop system. Hence we keep only two state variables in the reduced order LQG controller and reduce the original system with respect to these closed loop performances. The closed loop response using the reduced order LQG controller applied on the full order system is plotted in Fig. 8. It is worth noticing that the performances using the full order ($x_c \in \mathbb{R}^{100}$) and reduced order LQG controller ($x_{cr} \in \mathbb{R}^2$) are quite similar.

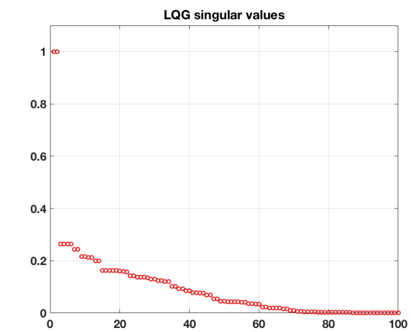

Fig. 7. LQG singular values

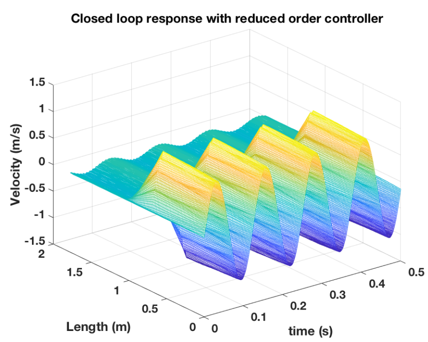

Fig. 8. Reduced order LQG controlled response

VI. CONCLUSION AND FUTURE WORKS

This paper proposes a reduced order LQG control design method for a class of infinite dimensional port Hamiltonian systems, that preserves the passivity properties and Hamiltonian structure of the system. It is based on an appropriate choice of the weighting functions during the LQG control design and closed loop model reduction. The effectiveness of this approach is illustrated on the control of a 1D vibro-acoustic system. A perspective is first to implement the designed controller on the experimental set up (Fig. 3) developed at FEMTO-ST institute. Second, the choice of the weighting operator will be investigated in order to make sure about the convergence of the LQG singular values.

VII. ACKNOWLEDGMENTS

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