Mixed Integer Linear Programming Approach to Optimize the Hybrid Renewable Energy System Management for supplying a Stand-Alone Data Center

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Abstract

The trend toward server-side computing and the exploding popularity of Internet services due to the increasing of demand for networking, storage and computation has created a world-wide energetic problem and a significant carbon footprint. These environmental concerns prompt to several green energy initiative aiming either to increase data center efficiency and/or to the use of green energy supply. In this regard, As part of the ANR DATAZERO project, many researchers are working to define main concepts of an autonomous green data center only powered by renewable energies. Thus, the present paper proposes a mixed integer linear program to optimize the commitment of a hybrid energy system composed of wind turbines, solar panels, batteries and hydrogen storage systems. The approach is used to supply a data center demand and takes the weather forecasts into account at the time of optimization. Different time window resolution are applied in order to verify the best time window for decision making.

Keywords— Renewable energy supply; Operational research; Integer linear programming; Hybrid energy system; Energy management; Optimization

1 Introduction

The computing capacity and the number of servers in data centers are more and more increasing to meet the soaring load demand of IT services, applications and storage. The high power consumption has led to many serious consequences such as increasing energy demand which has pushed the electrical power to enter a new evolution phase. It can be mainly identified by increasing interest about climate change, by a transition to a green renewable energy economy, as well as efficient use of energy (1).

In this context, this paper considers the use of renewable energy to displace fossil fuels. It consists in optimizing the management of a hybrid energy system composed of photovoltaic panels, wind turbines, battery storage system and a hydrogen storage system in order to supply the data center demand all over its life time. Moreover, latest research works on power management aimed at reducing data centers power consumption, are actually divided into two strategies. The first one consists in reducing the consumption of computing devices using different hardware and software technologies including workload scheduling (7), virtual machine (VM) management (17), cooling computing power balancing (3), etc. The second strategy, the one on which this paper takes part consists in optimizing the design of urban settlements by means of the exploitation of natural sources of energy and the development of building management systems (13) in order to make an efficient use of electrical energy.

Among many alternative energy sources used, solar and wind are considered as an attractive solution (12) to generate power on a large scale and are widely used for substituting fossil fuels in the electrical power process. However, due to their intermittent and variable nature, the integration of such sources is a challenge to reach a low-carbon society

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as it needs storage to enable satisfaction of the demand. Then, it is quite necessary to integrate multiple energy sources working together (batteries, biomass, wind, solar, hydrogen, geothermal) as the balance between generation and demand must be met in real-time (13). This is a common concern for both small/big power systems to operate islanded, as it needs an optimal management of the storage system (11). Nevertheless, nowadays, research tends to build bigger standalone buildings or datacenters 100% powered by green energy (9).

This paper presents, as part of the ANR DATAZEC research project, an effective management of a hybrid renewable energy system (HRES) in order to completely supply a standalone data center. We propose here a model based on Mixed Integer Linear Programming approach to optimize the commitment of energy sources. This is done in compliance with user demand with level maximization long-term energy stocks. This approach takes into account the season and forecasts weather when making decisions. For a better understanding of the models, a table of the notation used is presented in Table 1.

The remainder of this article is organized as follows: Section 2 describes the models of energy sources used in the optimization problem. Then, in section 3 the problem statement is explained and solved. Section 4 define the metric used to evaluate the obtained solutions and Section 5 illustrates different use case with the presentation of a solution. The paper ends with a conclusion and perspectives.

2 Model of a hybrid renewable energy system

The power system of the data center is composed of solar panels ($pv$), wind turbines ($wt$), fuel cells ($fc$), batteries, electrolyzers, hydrogen tanks and other appliances of energy conversion. In order to analyze and properly engage the hybrid power system, it is necessary to understand the pattern of each component. In the following, we consider an $H$ horizon discretized into $K$ periods of $\Delta t$ units of time such that $H = K \Delta t$. We assume that during a period $\Delta t$, for all $t$ with $k\Delta t \leq t < (k+1)\Delta t$ and $k \in [0, K-1]$, the parameters and variables of the model are constant.

The modeling of the components is described in the following according to the parameters of the system and the time step $k$ of the discrete work horizon $H$:

2.0.1 Wind Turbine Generator

Dong et al. (5) and many other researchers have used the same mathematical model for the output power $P_{wt,k}$ of the wind turbine. The same model will be used in our study.

2.0.2 Photovoltaic Panel

To modelize the output power of PV panels $P_{pv,k}$, a widely used model (16) assumes that the irradiance is proportional to the surface of the panels.

2.0.3 Batteries

The batteries are used for short term storage. The state of charge of the batteries is calculated for each $k$ with $k \in [1, K]$ with respect to the previous state of charge of the battery $SOC_{k-1}$, the self-discharge rate $\sigma$, the charging and discharging power $Pch_{k-1}$, $Pdch_{k-1}$, the charging and discharging efficiency $\eta_{ch}$, $\eta_{dch}$ as follows:

$$SOC_k = SOC_{k-1} \times (1 - \sigma) + Pch_{k-1} \times \eta_{ch} \times \Delta t - \frac{Pdch_{k-1}}{\eta_{dch}} \times \Delta t$$

As the battery storage system (BSS) cannot charge and discharge simultaneously, the value of $SOC_k$ is bounded by the minimum amount of $SOC_{min}$ and maximum energy $SOC_{max}$. So $\forall k \in [1, K]$ with $SOC_0 = SOC_{init}$:

$$SOC_k = \min\{SOC_{k-1} \times (1 - \sigma) + Pch_{k-1}\}$$

\[http://www.datazero.org\]
\[ \eta \Delta t \] if \( P_{ch_{k-1}} > 0 \)

\[ SOC_k = \max\{SOC_{k-1} \times (1 - \sigma) - \frac{P_{dch_{k-1}}}{\eta_{dch}}, \Delta t, SOC_{min}\} \text{ if } P_{dch_{k-1}} > 0 \]  

### 2.0.4 Electrolyzer

The operating power of the electrolyzer \( P_{ez_k} \) depends on the hydrogen mass flow \( Q_{ez_k} \) in (kg), and is bounded by \( P_{ez_{min}}, P_{ez_{max}} \) the operating range of the electrolyzer. This relation is defined for each time step \( k (k \in [0, K - 1]) \), with \( \eta_{ez} \) the electrolyzer efficiency and \( HHV h_2 \) the hydrogen higher heating value, as:

\[ P_{ez_k} \times \Delta t = \frac{HHV h_2 \times Q_{ez_k}}{\eta_{ez}} \]  

with \( P_{ez_{min}} \leq P_{ez_k} \leq P_{ez_{max}} \)

### 2.0.5 Fuel Cell

The hydrogen mass flow \( Q_{fc_k} \) depends on the output power of the fuel cell \( P_{fc_k} \) at any time during each period \( k (k \in [0, K - 1]) \) as in Equation (5) with \( \eta_{fc} \) the efficiency and \( LHV h_2 \) the low heating value of hydrogen:

\[ P_{fc_k} \times \Delta t = LHV h_2 \times Q_{fc_k} \times \eta_{fc} \]  

with \( P_{fc_k} \leq P_{fc_{max}} \)

### 2.0.6 Hydrogen tank

The level of hydrogen produced by the electrolyzer and consumed by the fuel cell in the tank is determined at any time during each period \( k \) with \( k \in [1, K] \) as follows:

\[ LOH_k = LOH_{k-1} + Q_{ez_k} - Q_{fc_k} \]  

with \( 0 \leq LOH_k \leq LOH_{max} \)

The previous models for each energy/power source are used to solve the tackle optimization problem.

### 3 Problem statement and resolution

The targeted data center is intended to be used all year long without resorting to the conventional power grid. In order to make it possible, the latter should maintain an energy level in its long-term storage in order to be able to operate even if days where the power from renewable sources is not enough to address the power demand. Indeed, the idea is that the most favorable days (let say in summer) has to compensate for the worst days (let say in winter). The challenge is therefore to smooth consumption over the year thanks to long-term storage (H\(_2\)) and over days thanks to the short term storage (batteries). For the tackled problem, we agree that a study of meteorological data over several years can predict what the level of hydrogen stocks should be at a macroscopic point of view. It is then a question of scheduling a commitment of the sources, and thus managing both the short term and long term storage, in order to stick to these forecasts.

The optimization problem is then to maximize the power production \( P_{prod_k} \) with maintaining the level of hydrogen \( LOH_k \) at the end of the horizon \( \mathcal{H} \) to make possible the seasonal compensation without wasting energy and to be able to handle the next season. The following sections describe constraints to respect the energy component usage and the data center power demand.
3.1 Flow conservation

In order to maintain a better use of the sources, the renewable energy produced \((P_{wt_k} + P_{pv_k})\) by wind turbines and photovoltaic panels during the whole time step \(k\) is used for three purposes:

- Satisfying of the data center load \((P_{load_k})\);
- Charging the hydrogen tanks thanks to electrolyzers \((P_{ez_k})\);
- Charging the batteries \((P_{ch_k})\).

In case of inability to produce enough renewable energy to meet the data center demand \((P_{load_k})\), additional electrical power is delivered by the batteries \((P_{dch_k})\) and fuel cells \((P_{fc_k})\). This use can be mathematically written by Equation (8) for each \(k \in [0, K - 1]\):

\[
P_{load_k} \leq P_{wt_k} + P_{pv_k} + (P_{fc_k} + P_{dch_k} - P_{ez_k} - P_{ch_k}) \times \eta_{inv}
\]

3.2 Mutual exclusion rules

Common sense rules have to allow the use of power components without any time restriction if the two following constraints are respected:

- When fuel cells are operating, it is only used to meet the demand and not to charge batteries.
- When batteries discharge, they are only used to meet the demand and not to produce hydrogen.

3.3 Mathematical Model

The resulting model can be viewed as a Constraint Satisfaction Problem (CSP) as a mathematical program that can be linearized as a Mixed Integer Linear Program (MILP) presented in the next section. The reader can find more explanations on the linearization in [10]. So we can optimally solve the addressed problem using an solver as Gurobi [8]. This allows to propose solutions by the Power module of the project: (i) identification of the profile that maximizes the power produced, (ii) computation of an optimal commitment around a target profile considering a given relax factor, and (iii) computation of an optimal commitment constrained by the load profile.

Therefore, the CSP is defined as follows (with \(k \in [0, K - 1]\) or \(k \in [0, K]\) for \(SOC_k\) and \(LOH_k\) variables):

\[
\begin{align*}
&P_{fc_k} \leq P_{fc\text{max}} \quad \forall k \in [0, K - 1] \\
&P_{ez} \leq P_{ez\text{max}} \quad \forall k \in [0, K - 1] \\
&SOC\text{min} \leq SOC_k \leq SOC\text{max} \quad \forall k \in [0, K] \\
&0 \leq LOH_k \leq LOH\text{max} \quad \forall k \in [0, K]
\end{align*}
\]

3.4 Obtained linear program

In order to be solved, the proposed CSP (9) was linearized following (2). The obtained linear program is detailed as in Equation (10).
The objective function consist in maximizing the power production $P_{prod_k}$ hour by hour to increase the chance to fulfill the data center power demand $P_{load_k}$.

\[
\begin{align*}
\text{maximize} & \quad \sum_{k=0}^{K-1} P_{prod_k} + \lambda LOH_k \\
\text{s.t. :} & \quad P_{prod_k} \leq Pre_k\eta_{invo} + (P_{fc_k} + P_{dch_k})\eta_{inv} \\
& \quad -(P_{ez_k} + P_{ch_k})\eta_{invo} \\
& \quad Pre_k = Pwt_k + Ppv_k \\
& \quad SOC_k = SOC_{k-1}(1 - \sigma) + \eta_{chk}P_{ch_{k-1}}\Delta t \\
& \quad -P_{dch_{k-1}}\Delta t/\eta_{dch} \\
& \quad P_{prod_k} \geq (1 - r_f) \times P_{load_k} \\
& \quad P_{ez_k} = HHV_{h2} \times Q_{ez_k}/\eta_{ez} / \Delta t \\
& \quad P_{fc_k} = LHV_{h2} \times Q_{fc_k}/\eta_{fc} / \Delta t \\
& \quad LOH_k = LOH_{k-1} + P_{ez_{k-1}} - P_{fc_{k-1}}/\eta_{tank} \\
& \quad P_{ch_k} \leq x_k \times P_{ch\text{max}} \\
& \quad P_{ch\text{max}} \geq 0 \\
& \quad P_{ch_k} \geq P_{ch_k}' \\
& \quad P_{ch_k} \geq (1 - x_k)P_{ch\text{max}} \\
& \quad P_{dch_k} \leq 0 \\
& \quad P_{dch_k} \leq (1 - x_k)P_{dch\text{max}} \\
& \quad P_{dch\text{max}} \leq P_{dch_k}' \\
& \quad P_{dch_k} \geq (1 - x_k)P_{dch\text{max}} \\
& \quad P_{ez_k} \leq y_k \times P_{ez\text{max}} \\
& \quad P_{ez\text{max}} \geq 0 \\
& \quad P_{ez_k} \geq (1 - y_k)P_{ez\text{max}} \\
& \quad 0 \leq P_{ez_k}' \leq P_{ez\text{max}} \\
& \quad P_{ez_k} \leq y_k \times P_{ez\text{min}} \\
& \quad Q_{ez_k} \leq Q_{ez_k}' \\
& \quad Q_{ez\text{max}} \geq 0 \\
& \quad Q_{ez_k} \leq Q_{ez_k}'(1 - z_k)Q_{ez\text{max}} \\
& \quad 0 \leq Q_{ez_k}' \leq Q_{ez\text{max}} \\
& \quad Q_{fc_k} \leq Q_{fc_k}' \\
& \quad Q_{fc\text{max}} \geq 0 \\
& \quad Q_{fc_k} \leq Q_{fc_k}'(1 - z_k)Q_{fc\text{max}} \\
& \quad 0 \leq Q_{fc_k}' \leq Q_{fc\text{max}} \\
& \quad u_k \leq x_k \\
& \quad u_k \leq y_k \\
& \quad 0 \leq 1 - x_k - y_k + u \\
& \quad u_k \geq 0 \\
& \quad v_k \leq x_k \\
& \quad v_k \leq z_k \\
& \quad 0 \leq 1 - x_k - z_k + v \\
& \quad v_k \geq 0 \\
\end{align*}
\]
Bounds:
\[ \forall k \in [0,K-1], P_{fc_k} \leq P_{fcmax} \]
\[ \forall k \in [0,K-1], Pez_k \geq Pez_{min} \]
\[ \forall k \in [0,K], SOC_{min} \leq SOC_k \leq SOC_{max} \]
\[ \forall k \in [0,K-1], 0 \leq LOH_k \leq LOH_{max} \]
\[ \forall k \in [0,K-1], Pch_k, Pdch_k, Pez_k, Pch'_k, Pdch'_k, Pez'_k \geq 0 \]
\[ \forall k \in [0,K-1], x_k, y_k, z_k \in \{0,1\} \]

Moreover, the long term storage $LOH_k$ should be strongly maintained rather than allowing waste if overproduction is too high. Thus, a high value for $\lambda$ is necessary to offset the fact that some of the energy produced is lost due to electrolyser and fuel cell efficiency.

Also, in order to match $P_{prod_k}$ with the data center load $P_{load_k}$ for each time step $k$, a relax factor $rf$ is used as in the constraint (11).

\[ P_{prod_k} \geq (1 - rf) \times P_{load_k} \quad (11) \]

In case of no existing solutions due to power production deficiency to supply the data center, the relax factor value between $0 \leq rf \leq 1$ is computed using a binary search approach in order to degrade the power response for the data center as less as possible. As the data center load is relaxed, another management is proposed. The solution is thus sub-optimal compared to the initial power demand, but an alternative feasible solution is proposed to overcome the intermittent nature of renewable sources. The evaluation of this degradation impact is evaluated in Section 5.

The obtained model used to provide a power profile that can match as close as possible with a power demand is the MILP (10) (We recall that additional explanations are available in (10)).

4 Metrics

In order to evaluate the solutions obtained by the MILP, some metrics are explained in this section:

4.1 Reliability

Due to the intermittent nature of the power produced by PV and WT, analyzing the reliability of the system is quite important. Actually, an electrical system is considered reliable when it is able to supply the required power. There are many methods for carrying out the reliability analysis: The Loss of Power Supply Probability (LPSP) defined in the following is one of the most used ones:

\[ LPSP = \frac{\sum_{k=0}^{K-1} \Delta t \left( s.t. \ P_{prod_k} < P_{load_k} \right)}{H} \quad (12) \]

where $H$ is the number of hours of the horizon. A LPSP equals to 0 means that the system is absolutely reliable and a LPSP equals to 1 means that the demand is not satisfied at all by the generated power.

4.2 Level of Autonomy (LA)

The Level of Autonomy (LA) is defined as the percentage of time where the demand is satisfied by the renewable energy solely. It is computed as defined in Equation (13).

\[ LA = \frac{\sum_{k=0}^{K-1} \Delta t \left( s.t. \ P_{fc_k} \geq P_{load_k} \right)}{H} \times 100 \quad (13) \]
4.3 Unused renewable energy (URE)

As the charging and discharging power of both system batteries and hydrogen are constants, the MILP enable, sometimes, to use all the energy produced by the renewable sources. Thus, this quantity of unused renewable energy is computed as:

\[
URE = \sum_{k=0}^{K-1} \left( \max(P_{prod_k} - P_{load_k}, 0) \times \Delta t \right)
\]  

(14)

4.4 Percentage of the Energy Produced to demand (PEP)

It consists in computation of amount power produced \(P_{prod_k}\) divided by the demand. This is mathematically proceeded by the following expression:

\[
PEP = \frac{\sum_{k=0}^{K-1} \min(P_{prod_k}, P_{load_k})}{\sum_{k=0}^{K-1} P_{load_k}}
\]  

(15)

5 Experiments and discussions

The Mixed Integer Linear Program is implemented using python and solved by GUROBI solver (8). The problem takes as inputs the weather conditions, power demand \(P_{load_k}\) and other information about the electrical power system sizing like number of wind turbines \(N_{wt}\), the solar panel surface \(N_{pv}\), battery storage capacity \(SOC_{max}\), the capacity of the hydrogen tank \(H_2\), the efficiency of each component. As an output, the solver gives a schedule of each time slot for supply-side source and energy storage usage, according to the availability of renewable power. The data representing the solar radiation and wind speed have been measured on an hourly scale from January 2004.
till December 2012 in a coastal area. To be more precise, the endogenous data of the solar radiation and wind speed
time series have been measured at Los Angeles (Latitude: 34.57, Longitude: -118.02, Time Zone: -8). These data has
been obtained respectively from the National Solar Radiation DataBase (NSRDB) (15) and from wind prospector of
the National Renewable Energy Laboratory (NREL) (6). From the IT part, the data center demand were obtained
from the traces of access logs from the World Cup web site of 1998 (4). Based on these data, a sizing study was
made and an average infrastructure as showed in the table is given. This sizing is an input the MILP needed to be
solved.

The same infrastructure sizing will be used in many management scenarios under different weather conditions
for a better illustration of the difference between the obtained results, but first, we proceed to the verification
of the proper functioning of the optimization strategy.

5.1 Management results

Figure 1 shows a solution obtained by solving the MILP on a given period \( H = 14 \) days = 336 h with a
time slot \( \Delta t = 1 \) h in the winter during the year 2010. This year was chosen for a better illustration of the
results. This solution consists of the evolution of each variable of the program, hour by hour, each visible
on a specific window of Figure 1. The first windows shows the data center demand \( P_{load} \) and the produced
power \( P_{prod} \), used to supply the data center, optimized by the MILP. The second window shows in details
the the solar panels \( P_{pv} \), the wind turbines \( P_{wt} \) and the total \( P_{re} \) produced. The third window shows both
charging \( P_{ch} \) and discharging power \( P_{dch} \) of the batteries followed by the variation of state of charge of
the batteries (SOC). The last two windows of Figure 1 show the actual level of hydrogen \( LOH \) computed
by the MILP. Finally, the electrolyzer power \( P_{ez} \) and fuel cell power \( P_{fc} \) obtained by the MILP is shown
to explain the evolution of \( LOH \).

This figure was chosen in winter in the worst conditions of the year 2010 to show the aim of the relax factor.
In fact, with an optimal relax factor \( rf = 0.02 \), the power supplied to the data center is relaxed (i.e., reduced
as less as possible) while a solution is possible because of the problem constraints. The first founded solution
is then displayed. It is used in order to propose an alternative power profile if no solution exists to answer to
the data center power demand. Thus, the latter could postpone or reschedule the jobs. Based on Figure 1
one can see that the battery state of charge comes back to its initial level each 24 hours because batteries are
only responsible of short term energy storage (daily compensation) to ensure the hourly lack of renewable
energy (e.g., solar night and day alternation) during the same day.

Also, as it is responsible of the seasonal offset, one can see that the level of hydrogen is decreasing as there is
not enough renewable energy to ensure the demand (no wind) and get stabilized at the end of window.

Thus, as the linear program is working properly, the following subsection shows different usage of this
program. Indeed, the management program should be able to find a solution under any weather conditions.
The metrics explained in Section 4 are used to verify the robustness of our approach.

5.2 Management scenarios results

In this part, the program is executed following different resolution time windows for the same horizon (1
year) under different weather conditions. In fact, the same sizing will be played from the year 2004 till 2012.
For each year, several resolution time windows are simulated with the weather condition of the year. One
simulation consists in iteratively solving the power management program for a given period length \( H \) as
many times as it is needed to cover the whole year. This resolution time window \( H \) is related to the weather
prediction time. Different cases of resolution time window are exploited to shows the impact of the decision
on the management proposed:

- 1st case: resolution window is equal to the horizon fixed (1 resolution for 1 year)
• 2nd case: resolution window is equal to 1 week (52 resolutions for 1 year)
• 3rd case: resolution window is equal to 3 days (121 resolutions for 1 year)
• 4th case: resolution window is equal to 1 day (365 resolutions for 1 year)

The aim of this comparison is to choose the best management time window using this approach (Should the decision be taken only 1 time per week, per three days or per days?). The chosen metrics are applied on each cases study. For all cases, an average value of the metrics is computed and displayed in the following results. Experimentation are performed on an Intel® Core™ i5-6200U CPU @ 2.30GHz × 4, 8GB RAM, 64-bit using Ubuntu 16.04 LTS as exploitation system. The solving time for each year with different resolution time windows will be also compared.

![Figure 2: Management of the renewable sources in the 3rd case for the year 2008](image)

In the following tables, the column LPSP is divided on two columns, the average and the standard deviation of the LPSP values obtained for each resolution in different simulations.

The table resume the metric results for a one year resolution with $H = 1$ year = 8760 h. The results of the relax factor shows quit high values as only one decision is taken for the whole year. This high relax factor has an impact on the LPSP which shows that, for example, during 2005, only 77% of the whole time was supplied by the system. Nevertheless, the infrastructure was able to give 94% of the energy produced to the demand during the year. It means that the difference between the load and the provided production is not big. Also, one can see that the level of autonomy $LA$ is around 60% for all the years and that the unused energy is quite different from a year to another.

After analyzing the results, taking only one decision for a whole year is actually not accurate due to the uncertainty of the weather conditions. Then, we decided to shorten the resolution window to 1 week (52 resolutions for 1 year), 3 days (121 resolutions for 1 year) and to 1 day (365 resolutions for 1 year). The resulting metric are summerized respectively in the tables 4, 5, 6.
In the three tables, one can see that the smaller is the resolution window, the smaller is the relax factor. In fact, one can see that for the years 2004, 2005, 2006, rf decreased from around 0.08 to 0 which improved the loss of power supply and made it capable to answer to the data center demand more often. Contrarily, one can see that the percentage of the energy produced to demand PEP degrades with the smaller time windows. In fact, this is due to the inability of the system to answer to the demand in some time steps (i.e., instead of treating the lack of overproduction during a year, it is treated in a day). Indeed, the LPSP values computed for the different horizon $H \in \{168, 72, 24\}$ are quite similar. Nevertheless, the standard deviation shows that the smaller is the resolution time window, the bigger is the periodic variation. To conclude, following to the metric results, the constraints of IT scheduling and the uncertainties of the weather prediction, the best resolution time windows is equal to 3 days as showed in Figure 2.

Based on Figure 2, one can see that around the hour 1000, the system is unable to meet the data center demand $P_{load}$: The fuel cell started working and the level of hydrogen decreases till reaching 0 kg. In this moment, the $P_{load}$ is relaxed by rf for those three days. This relaxation could also be translated by a purchase of hydrogen to fulfill the demand. Starting from the hour 4000 (beginning of June), one can see that the level of hydrogen increases till reaching it maximum level at around the hour 5500. $P_{prod}$ is then bigger than the demand. This unused renewable energy could be translated by selling hydrogen.

The resolution time of the MILP is in the order of seconds. For the one year time windows, the resolution time is equal to 32.48 s. The weekly resolutions takes around 40.5 s. The three days resolution time is equal to 44.8 s and the daily resolution time is equal to 55.7 s.

6 Conclusion and perspectives

In this article, we presented first an analysis of the main trade offs involved in powering data centers or stand alone infrastructure with hybrid renewable energy system. we also presented the ANR DATAZERO project, in which this work take part, that aims to provide a data center with only renewable energy. Second, from the models used for HRES, the problem statement and the constraint satisfaction problem due to it, was settled. Third, a mixed integer linear program is deduced from the CSP in order to to manage a hybrid renewable energy system presenting four optimization scenarios depending on the resolution time window. These problems have been illustrated by several simulations. The optimal model is convenient to address the power commitment of an 100% renewable energy data center, since the resolution is obtained within few seconds.

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### Table 1: Nomenclature

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{H}$</td>
<td>A given time window $[h]$</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>Interval of time between two time steps $[h]$</td>
</tr>
<tr>
<td>$k$</td>
<td>Index for one time step within $\mathcal{H}$</td>
</tr>
<tr>
<td>$K$</td>
<td>Number of time steps within $\mathcal{H}$</td>
</tr>
<tr>
<td>$P_{pwp_k}$</td>
<td>PV power $[kW]$</td>
</tr>
<tr>
<td>$P_{wt_k}$</td>
<td>WT power $[kW]$</td>
</tr>
<tr>
<td>$SOC_{max}$</td>
<td>Maximal state of Charge (SOC) $[kWh]$</td>
</tr>
<tr>
<td>$SOC_{min}$</td>
<td>Minimal SOC $[kWh]$</td>
</tr>
<tr>
<td>$SOC_{k}$</td>
<td>SOC at instant $k\Delta t$ $[kWh]$</td>
</tr>
<tr>
<td>$P_{ch_k}$</td>
<td>Power used to recharge battery $[kW]$</td>
</tr>
<tr>
<td>$P_{dch_k}$</td>
<td>Power discharged from battery $[kW]$</td>
</tr>
<tr>
<td>$LOH_{target_D}$</td>
<td>$H_2$ tank level targeted for a given day $D$</td>
</tr>
<tr>
<td>$LOH_{max}$</td>
<td>$H_2$ tank upper limit $[%]$</td>
</tr>
<tr>
<td>$LHV_{H_2}$</td>
<td>$H_2$ lower heating value $[kWh.kg^{-1}]$</td>
</tr>
<tr>
<td>$HHV_{H_2}$</td>
<td>$H_2$ higher heating value $[kWh.kg^{-1}]$</td>
</tr>
<tr>
<td>$LOH_{H_2}$</td>
<td>$H_2$ tank inventory level $[kg]$</td>
</tr>
<tr>
<td>$H_2_{max}$</td>
<td>Maximal tank capacity $[kg]$</td>
</tr>
<tr>
<td>$P_{ez_{max}}$</td>
<td>Electrolyzer power upper limit $[%]$</td>
</tr>
<tr>
<td>$P_{ez_{min}}$</td>
<td>Electrolyzer power lower limit $[%]$</td>
</tr>
<tr>
<td>$P_{ez_{k}}$</td>
<td>Power put into electrolyzer $[kW]$</td>
</tr>
<tr>
<td>$Q_{ez_{k}}$</td>
<td>Electrolyzer $H_2$ mass flow $[kg]$</td>
</tr>
<tr>
<td>$P_{fc_{min}}$</td>
<td>Minimum power to operate fuel cell $[kW]$</td>
</tr>
<tr>
<td>$P_{fc_{max}}$</td>
<td>Maximum power delivered by fuel cell $[kW]$</td>
</tr>
<tr>
<td>$P_{fc_{k}}$</td>
<td>Power delivered by fuel cell at period $k$ $[kW]$</td>
</tr>
<tr>
<td>$Q_{fc_{k}}$</td>
<td>Fuel cell $H_2$ mass flow $[kg]$</td>
</tr>
<tr>
<td>$\eta_{ch}$</td>
<td>Battery charge efficiency $[%]$</td>
</tr>
<tr>
<td>$\eta_{dch}$</td>
<td>Battery discharge efficiency $[%]$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Battery self-discharge rate $[%]$</td>
</tr>
<tr>
<td>$\eta_{tank}$</td>
<td>$H_2$ tank efficiency $[%]$</td>
</tr>
<tr>
<td>$\eta_{ez}$</td>
<td>Electrolyzer efficiency $[%]$</td>
</tr>
<tr>
<td>$\eta_{fc}$</td>
<td>Fuel Cell efficiency $[%]$</td>
</tr>
<tr>
<td>$\eta_{inv}$</td>
<td>Inverter efficiency $[%]$</td>
</tr>
<tr>
<td>$x_k$</td>
<td>Battery in use ($x_k = 1$) or not ($x_k = 0$)</td>
</tr>
<tr>
<td>$y_k$</td>
<td>Electrolyzer in use ($y_k = 1$) or not ($y_k = 0$)</td>
</tr>
</tbody>
</table>

### Table 2: Infrastructure sizing used in simulations

<table>
<thead>
<tr>
<th>Nwt</th>
<th>Npv</th>
<th>Pch</th>
<th>Pdch</th>
<th>SOCmax</th>
<th>Pez</th>
<th>Pfc</th>
<th>$H_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6650</td>
<td>980</td>
<td>555</td>
<td>5287</td>
<td>910</td>
<td>832</td>
<td>6382</td>
</tr>
</tbody>
</table>
Table 3: Metric for a one year simulation with $H = 1$ year = 8760 h

<table>
<thead>
<tr>
<th>year</th>
<th>relaxation</th>
<th>LA</th>
<th>LPSP Average</th>
<th>SD</th>
<th>URE</th>
<th>PEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>0.07</td>
<td>0.67</td>
<td>0.98</td>
<td>0</td>
<td>3.45</td>
<td>0.93</td>
</tr>
<tr>
<td>2005</td>
<td>0.08</td>
<td>0.69</td>
<td>0.77</td>
<td>0</td>
<td>84.94</td>
<td>0.94</td>
</tr>
<tr>
<td>2006</td>
<td>0.11</td>
<td>0.66</td>
<td>0.85</td>
<td>0</td>
<td>47.08</td>
<td>0.90</td>
</tr>
<tr>
<td>2007</td>
<td>0.31</td>
<td>0.64</td>
<td>0.75</td>
<td>0</td>
<td>82.54</td>
<td>0.77</td>
</tr>
<tr>
<td>2008</td>
<td>0.18</td>
<td>0.64</td>
<td>0.85</td>
<td>0</td>
<td>43.84</td>
<td>0.78</td>
</tr>
<tr>
<td>2009</td>
<td>0.27</td>
<td>0.63</td>
<td>0.83</td>
<td>0</td>
<td>62.16</td>
<td>0.78</td>
</tr>
<tr>
<td>2010</td>
<td>0.48</td>
<td>0.60</td>
<td>0.72</td>
<td>0</td>
<td>121.51</td>
<td>0.67</td>
</tr>
<tr>
<td>2011</td>
<td>0.43</td>
<td>0.64</td>
<td>0.66</td>
<td>0</td>
<td>155.09</td>
<td>0.73</td>
</tr>
<tr>
<td>2012</td>
<td>0.3</td>
<td>0.61</td>
<td>0.79</td>
<td>0</td>
<td>60.94</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Table 4: Metric for a one year simulation with $H = 1$ week = 168 h

<table>
<thead>
<tr>
<th>year</th>
<th>relaxation</th>
<th>LA</th>
<th>LPSP Average</th>
<th>SD</th>
<th>URE</th>
<th>PEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>0</td>
<td>1</td>
<td>0.06</td>
<td>0.04</td>
<td>84.36</td>
<td>0.67</td>
</tr>
<tr>
<td>2005</td>
<td>0</td>
<td>1</td>
<td>0.05</td>
<td>0.04</td>
<td>158.81</td>
<td>0.69</td>
</tr>
<tr>
<td>2006</td>
<td>0</td>
<td>1</td>
<td>0.05</td>
<td>0.03</td>
<td>129.46</td>
<td>0.67</td>
</tr>
<tr>
<td>2007</td>
<td>0.06</td>
<td>0.93</td>
<td>0.20</td>
<td>0.31</td>
<td>56.70</td>
<td>0.64</td>
</tr>
<tr>
<td>2008</td>
<td>0.02</td>
<td>0.97</td>
<td>0.18</td>
<td>0.29</td>
<td>56.89</td>
<td>0.64</td>
</tr>
<tr>
<td>2009</td>
<td>0.04</td>
<td>0.95</td>
<td>0.26</td>
<td>0.35</td>
<td>43.21</td>
<td>0.63</td>
</tr>
<tr>
<td>2010</td>
<td>0.10</td>
<td>0.89</td>
<td>0.31</td>
<td>0.39</td>
<td>45.65</td>
<td>0.60</td>
</tr>
<tr>
<td>2011</td>
<td>0.08</td>
<td>0.91</td>
<td>0.26</td>
<td>0.37</td>
<td>85.30</td>
<td>0.64</td>
</tr>
<tr>
<td>2012</td>
<td>0.12</td>
<td>0.93</td>
<td>0.30</td>
<td>0.37</td>
<td>1.55</td>
<td>0.61</td>
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</tbody>
</table>

Table 5: Metric for a one year simulation with $H = 3$ days = 72 h

<table>
<thead>
<tr>
<th>year</th>
<th>relaxation</th>
<th>LA</th>
<th>LPSP Average</th>
<th>SD</th>
<th>URE</th>
<th>PEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>0</td>
<td>1</td>
<td>0.06</td>
<td>0.05</td>
<td>84.59</td>
<td>0.67</td>
</tr>
<tr>
<td>2005</td>
<td>0</td>
<td>1</td>
<td>0.05</td>
<td>0.05</td>
<td>159.24</td>
<td>0.69</td>
</tr>
<tr>
<td>2006</td>
<td>0</td>
<td>1</td>
<td>0.05</td>
<td>0.05</td>
<td>129.81</td>
<td>0.67</td>
</tr>
<tr>
<td>2007</td>
<td>0.04</td>
<td>0.95</td>
<td>0.17</td>
<td>0.27</td>
<td>54.05</td>
<td>0.64</td>
</tr>
<tr>
<td>2008</td>
<td>0.02</td>
<td>0.98</td>
<td>0.15</td>
<td>0.24</td>
<td>57.52</td>
<td>0.64</td>
</tr>
<tr>
<td>2009</td>
<td>0.04</td>
<td>0.95</td>
<td>0.20</td>
<td>0.30</td>
<td>42.42</td>
<td>0.63</td>
</tr>
<tr>
<td>2010</td>
<td>0.08</td>
<td>0.91</td>
<td>0.28</td>
<td>0.37</td>
<td>44.27</td>
<td>0.59</td>
</tr>
<tr>
<td>2011</td>
<td>0.06</td>
<td>0.93</td>
<td>0.22</td>
<td>0.33</td>
<td>85.46</td>
<td>0.64</td>
</tr>
<tr>
<td>2012</td>
<td>0.05</td>
<td>0.94</td>
<td>0.23</td>
<td>0.32</td>
<td>1.55</td>
<td>0.61</td>
</tr>
</tbody>
</table>
Table 6: Metric for a one year simulation with $\mathcal{H} = 1 \text{ day} = 24 \text{ h}$

<table>
<thead>
<tr>
<th>Year</th>
<th>Relaxation</th>
<th>LA</th>
<th>LPSP Average</th>
<th>LPSP SD</th>
<th>URE</th>
<th>PEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>0</td>
<td>1</td>
<td>0.07</td>
<td>0.09</td>
<td>84.13</td>
<td>0.67</td>
</tr>
<tr>
<td>2005</td>
<td>0</td>
<td>1</td>
<td>0.05</td>
<td>0.06</td>
<td>158.37</td>
<td>0.69</td>
</tr>
<tr>
<td>2006</td>
<td>0</td>
<td>1</td>
<td>0.06</td>
<td>0.07</td>
<td>129.09</td>
<td>0.66</td>
</tr>
<tr>
<td>2007</td>
<td>0.03</td>
<td>0.96</td>
<td>0.09</td>
<td>0.26</td>
<td>53.61</td>
<td>0.64</td>
</tr>
<tr>
<td>2008</td>
<td>0.01</td>
<td>0.98</td>
<td>0.08</td>
<td>0.20</td>
<td>56.71</td>
<td>0.64</td>
</tr>
<tr>
<td>2009</td>
<td>0.02</td>
<td>0.97</td>
<td>0.09</td>
<td>0.25</td>
<td>40.72</td>
<td>0.63</td>
</tr>
<tr>
<td>2010</td>
<td>0.06</td>
<td>0.93</td>
<td>0.08</td>
<td>0.34</td>
<td>43.98</td>
<td>0.60</td>
</tr>
<tr>
<td>2011</td>
<td>0.05</td>
<td>0.94</td>
<td>0.08</td>
<td>0.29</td>
<td>84.65</td>
<td>0.64</td>
</tr>
<tr>
<td>2012</td>
<td>0.03</td>
<td>0.96</td>
<td>0.1</td>
<td>0.28</td>
<td>1.55</td>
<td>0.61</td>
</tr>
</tbody>
</table>